Q#1) Find the root of the equation:-

$x^3 + 3.993 \times 10^{-4} = 0.165x^2$, use Newton Raphson Method with $x_0 = 0.02$

Solution:-

Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad \because f(x_0) = 0.00033, \quad \because f'(x_0) = -0.0054$$

$$= 0.02 - \frac{-0.00033}{-0.0054}$$

$$\boxed{x_1 = 0.081}$$

$$\rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad \because f(x_1) = -0.00016, \quad f'(x_1) = -0.00070$$

$$= 0.081 - \frac{-0.00016}{-0.00070}$$

$$\boxed{x_2 = 0.058}$$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \qquad \because f(x_2) = 0.000030$$

$$f'(x_2) = 0.0090$$

$$= 0.058 - \frac{0.000030}{-0.0090}$$

$$\boxed{x_3 = 0.061}$$ Ans...

Q#2 use the number $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the lagrange interpolation Polynomial for $f(x) = 1/x$ at $x = 3$.

Solution: putting value in the function

$$y_0 = 0.5, \quad y_1 = 0.364, \quad y_2 = 0.25$$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

Putting values.

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)}(0.364) +$$

$$\frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\boxed{y = 0.33}$$

Q#3 Complete the divided difference table for the Given and construct the interpolating polynomial that uses all this data.

$x = 1.0 \quad 1.3 \quad 1.6 \quad 1.9 \quad 2.2$

$y = 0.7651977 \quad 0.6200860 \quad 0.4554022 \quad 0.2818186 \quad 0.1103623$

	x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}\cdots, x_i)$	$f(x_{i-4}\cdots, x_i)$
x_0	1	0.7651977				
			-0.4837056			
x_1	1.3	0.6200860		-0.108734		
			-0.548946		0.0658785	
x_2	1.6	0.4554022		-0.04944433		-0.0028049
			-0.578612		0.06261255	
x_3	1.9	0.2818186		0.006818		
			-0.571521			
x_4	2.2	0.1103623				

i) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = -0.4837056}$.

ii) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = -0.548946}$.

iii) $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$.

iv) $f(x_3, x_4) = \dfrac{f(x_4) - f(x_3)}{x_4 - x_3} = \dfrac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$\boxed{f(x_3, x_4) = -0.571521}$

2nd Divided Difference:

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$\boxed{f(x_0, x_1, x_2) = 0.108734}$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$\boxed{F(x_1, x_2, x_3) = -0.0494433}$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{0.571501 - (-0.578612)}{2.2 - 1.6}$$

$$\boxed{f(x_2, x_3, x_4) = 0.006818}$$

(Pto)

Third divided difference:

$$f(x_0,x_1,x_2,x_3) = \frac{f(x_1,x_2,x_3) - f(x_0,x_1,x_2)}{x_3 - x_0}$$

$$= \frac{0.0494433 - (-0.108734)}{1.9 - 1}$$

$$\boxed{F(x_0,x_1,x_2,x_3) = 0.0658785}$$

$$F(x_0,x_1,x_2,x_3) = \frac{f(x_2,x_3x_4) - f(x_1,x_2,x_3)}{x_4 - x_1}$$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$\boxed{f(x_0,x_1,x_2,x_3) = 0.0625255}$$

Fourth Divided Difference:

$$f(x_0,x_1,x_2,x_3,x_4) = \frac{0.0625255 - 0.0658785}{2.2 - 1}$$

$$\boxed{= -0.0028049}$$

(Pto)

$$f(u) = f(u_0) + (u-u_0)f(u_0,u_1) + (u-u_0)(u-u_1)f(u_0,u_1,u_2) + (u-u_0)(u-u_1)(u-u_2)f(u_0,u_1,u_2,u_3) + (u-u_0)(u-u_1)(u-u_2)(u-x_3)f(u_0,u_1,u_2,u_3,x_4)$$

$$= 0.7651977 + (u-1)(-0.4837056) + (u-1)(u-1.3)(-0.1087734) + (u-1)(u-1.3)(u-1.6)(0.0658785) + (u-1)(u-1.3)(u-1.6)(u-1.9)(-0.0028049)$$