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Section:- B

Subject:- Differential
Equations

Semester:- 9Th

Q 1 (a)

$w = \sin(x+ct)+\cos(2x+2ct)$

$$\frac{\partial w}{\partial t} = \cos(x+ct)\cdot c - \sin(2x+2ct)\cdot 2c$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) - 2c\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 2c\cos(2x+2ct)\cdot 2c$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)} \quad -(1)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct)\cdot 2$$

$$\frac{\partial^2 w}{\partial x^2} - \sin(x+ct) - 4\cos(2x+2ct)$$

$$c^2\frac{\partial^2 w}{\partial x^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct).$$

(1) and (2) $\quad \frac{\partial^2 w}{\partial t^2} = c^2\frac{\partial^2 w}{\partial x^2}$

①

Q=1 ⓑ

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

i) $w = \sin(x+ct) + \cos(2x+2ct)$

ii) $w = \tan(2x+ct)$

$$\frac{\partial w}{\partial t} = \sec^2(2x+ct)\frac{\partial}{\partial t}(2x+ct)$$

$$= c\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c\,2\sec(2x+ct)\frac{\partial}{\partial t}\sec(2x+ct)$$

$$= 2c^2\sec(2x+ct)\sec(2x+ct)\tan(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2\sec^2(2x+ct)\tan(2x+ct)$$

(2)

$$\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 2 \cdot 2\sec(2x+ct) \cdot \sec(2x+ct) \tan(2x+ct) \cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)$$

$$2c^3\sec^2(2x+ct)\tan(2x+ct) \neq c^2 8\sec^2(2x+ct)\tan(2x+ct)$$

So not Stisfy

Q No → 2

Given function is

$$f(x) = \begin{cases} x ; & -\pi < x \le 0 \\ 2x ; & 0 \le x \le \pi \end{cases}$$

we have to find the fourier co-efficients, a_0, a_n & b_n

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{0} x dx + \frac{1}{\pi}\int_{0}^{\pi} 2x dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$\frac{1}{\cancel{\pi}}\left[0 - \frac{\pi^{\cancel{2}}}{2}\right] + \frac{\cancel{2}}{\cancel{\pi}}\left[\frac{\pi^{\cancel{2}}}{\cancel{2}} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2}} \text{ — } (1)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

②

$$= \frac{1}{\pi}\int_{-\pi}^{0} (x \cos nx \, dx) + \frac{1}{\pi}\int_{0}^{\pi} (2x \cos nx)\, dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right.\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left[\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

$$\text{So } a_n = \begin{cases} \dfrac{-2}{\pi n^2} ; & \text{if } n \text{ is odd} \\ 0 ; & \text{if } n \text{ is even} \end{cases} \quad \text{— ⑤}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin nx\, dx = \frac{1}{\pi}\int_{-\pi}^{0} x \sin nx\, dx + \frac{2}{\pi}\int_{0}^{\pi} x \sin nx$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

3 ←

$$\boxed{b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] = \frac{-3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}}$$

So The required fourier is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2x-1)}{(2n-1)^2} + 3\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}$$

Q.No → 3

Given $y'' - 4y' + 13y = 8\sin 3x$

we have to find

$$y = y_c + y_p$$

for y_c the charaterestic (axilary eqn) is

$$m^2 - 4m + 13 = 0$$

$$\Rightarrow m^2 - 4m + 13 = 0$$

$$\Rightarrow m = \frac{4 \pm \sqrt{16 - 52}}{2} \Rightarrow m = \frac{4 + 6i}{2}$$

$$\Rightarrow 2 \pm 3i \qquad \alpha = 2 \; \& \; B = 3$$

So $y_c = e^{2x}\{c_1 \cos 3x + c_2 \sin 3x\}$

for y_p let

$$y_p = \text{Imag}\left(\frac{1}{m^2 - 4m + 13} 8e^{3ix}\right)$$

$$= 8\,\text{Imag} \frac{e^{3ix}}{(x^2) - 4(3i) + 13}$$

$$= 8\,\text{Imag} \frac{e^{3ix}}{-9 - 12i + 13}$$

②

$$8\,\text{Imag}\ \frac{e^{3ix}}{4-12i}$$

$$y_p = 2\,\text{Imag}\ \frac{e^{3ix}}{(1-3i)} \times \frac{1+3i}{1+3i}$$

$$= \frac{2\,\text{Imp}(1+3i)\,e^{3ix}}{(1)^2-(3i)^2}$$

$$= \frac{2}{10}\,\text{Imp}\,(1+3i)\,(\cos 3ix + i\sin 3x)$$

$$= \frac{1}{5}\,(i\sin 3x + 3\cos 3x)$$

The genral solution is

$$y = y_c + y_p$$

$$= c_1 e^{2x}\cos 3x + c_2 e^{2x}\sin 3x + \frac{1}{5}(\sin 3x + 3\cos 3x)$$

Now the intial condition $y(0)=1$

$$y(0) = c_1 + \frac{1}{5}(3)$$

$$1 = c_1 + 3/5 \qquad c_1 = 1 - 3/5 = \frac{2}{5}$$

$$\boxed{c_1 = 2/5}$$

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Again use the anothr intial Condition $y'(0)=2$

$$y' = c_1 2e^{2x}\cos 3x + c_1 e^{2x}(-3\sin 3x) + c_2 2 e^{2x}\sin 3x$$
$$+ c_2 e^{2x}(3\cos 3x) + \frac{2}{10}(\cos 3x - \sin 3x)$$

$$y'(0) = 2c_1 + 0 + 0 + c_2 3(1) + \frac{2}{10}(1 - 3(0))$$

$$2 = 2c_1 + 3c_2 + 2/10$$

$$2 = 2(2/5) + 3c_2 + 1/5$$

$$2 = 4/5 + 3c_2 + \frac{1}{5}$$

$$2 = \frac{4}{5} + \frac{1}{5} + 3c_2$$

$$2 = 1 + 3c_2$$

$$2 - 1 = 3c_2 \Rightarrow \boxed{c_2 = 1/3}$$

so General solution is

$$y = 2/5\, e^{2x}\cos 3x + \frac{1}{3} e^{2x}\sin 3x + \frac{1}{5}(\sin 3x + 3\cos 3x)$$

Ans

Q:- No (4)

$$(D^2 - DD')z = \cos x \cos 2y$$

The given PDE can be rewrite as:

$$D(D-D')z = \cos x \cos 2y$$

In CF is given by:

$$CF = \phi_1(y) + \phi_2(y+x)$$

While its PI is given by:

$$PI = \frac{1}{(D^2 - DD')} \cdot \frac{1}{2}\{\cos(x+2y) + \cos(x-2y)\}$$

$$= \frac{1}{2}\left[\frac{1}{(-1+2)}\cos(x+2y) + \frac{1}{(-1-2)}\cos(x-2y)\right]$$

$$= \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

Hence the Complete Solution OF The given PDE is given by

$$z = \phi_1(y) + \phi_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

ANS