ID : 7966

SECTION : B

SEMESTER : 4^{th}

DEPT : Civil

Q1 Solve the following objective type questions

i) The order of matrix A is $m \times p$ and the order of B is $p \times n$. Then the order AB is?

The order of matrix is equal to the no of its rows multiply by no of columns $AB = m \times n$

ii) The number of non-zero rows in an Echelon form

1 non zero row in echelon form

iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then a=?

Determinant of B $= (1 \times a) - (4 \times 2) = 0$

$|B| \Rightarrow a - 8 = 0$

$\boxed{a = 8}$

iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix} \Rightarrow 2i(-i) - (i)(i)$ $\qquad \because i^2 = -1$

$= -2i^2 - i^2 \Rightarrow -2(-1) - (-1) = 3$

$\boxed{|A| = 3}$

v) The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is?

When the diagnols are same and other are non-zero, so know as scalar matrix.

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is (scalar matrix)

vi) Solution of $dy/dx + 2xy = y$?

$y^2 = 2x - 4x^2 + 2c$

$\because dy/dx = y - 2xy$

$dy/dx = y(1-2x)$

$ydy = dx(1-2x)$

$\int ydy = \int (1-2x)\,dx$

$y^2 = 2x - 4x^2 + 2c$

vii) The order and degree of differential equation $\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is ?

Order = 1

Degree = 3

viii) The order and degree of D.E

$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is ?

Order and degree is not defined as its not a polynomial.

ix) The D.E $2\frac{dy}{dx} + x^2y = 2x+3$, $y(0)=5$ is ?

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5$$

x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Expand by R_1

$$|A| = +1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c-b)$$

$$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b.$$

Q2) PART A:-

Express the determinant.

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

As the product ~~rule~~ of factors which are linear in a, b, c:

Solution:-

$$= \begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}$$

expand by R_1

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

∴ abc common

$$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$\Rightarrow abc[bc(c-b) - ac(c+a) + ab(b-a)]$$

Ans.

Q2) PART B:

$$\begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$$

Solution:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

As we know that

$$(A - \lambda I) = 0 \quad -(A)$$

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Now Determinant

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow (B)$$

Again

$$\begin{vmatrix} 3-\lambda & 1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ expand by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda) - (-1)(-1)\big) + 1\big(((-1)(2-\lambda)) - (-1)(-1)\big) - 1\big((-1) - (-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\lambda+\lambda^2-1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - ~~4(-\lambda~~(4-\lambda).$$

$$= \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \quad \text{— (i)}$$

$\Rightarrow$ Now

$$+1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

expand by C_1

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6u-8} \quad \text{—— (ii)}$$

Now

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

expand by C_1

$$= -\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -\left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right)$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \quad \text{— (iii)}$$

put eq (i), (ii) (iii) in (B)

$$= (2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right] - \lambda^2+6\lambda-8-\lambda^2+6\lambda-8$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8.$$

$$= \lambda^4-10\lambda^3+32\lambda^2-32\lambda = 0.$$

By synthetic division

we get.

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16) = 0.$$

$(\lambda = 0)$

$\lambda - 2 = 0$

$\lambda = 2$

$\lambda^2 - 8\lambda + 16 = 0$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$(\lambda-4)(\lambda-4) = 0.$

$\lambda = 4, \lambda = 4.$

$\lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4$ Ans

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Q3 $(x^2+3y^2)dx - 2xy\,dy = 0$ $x=2,\ y=6$

Solution:-

$(x^2+3y^2)dx - 2xy\,dy = 0$

$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$

Divide b/s by $2xy\,dy$

we get,

$\frac{(x^2+3y^2)dx}{2xy\,dy} = \frac{2xy\,dy}{2xy\,dy}$ ∵ By cross

$\Rightarrow \frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$

$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$

$dy/dx = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right]$ — (i)

Let $y = vx$

Diff $dy = v\,dx + x\,dv$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$ — (ii)

put eq (ii) in (i)

$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{x}{xv} + 3\frac{vx}{x}\right)$

$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$

Multiplying both side by 2

$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$

$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$

$2x\frac{dv}{dx} = \frac{1}{v} + v$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

multiplying both sides by $\frac{dx}{dv}$

$$2x\,dv = \frac{1+v^2}{v}\,dx$$

multiply $\frac{v}{x(1+v^2)}$

$$\frac{v}{1+v^2}\,dv = \frac{1}{x}\,dx$$

Integrating:-

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx + c$$

$$\ln(1+v^2) = \ln x + \ln c$$

Take e on b/s

$$e^{\ln(1+v^2)} = e^{\ln(xc)}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

put $v = y/x$

$$1+(y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \quad \text{— (iii)}$$

put $x = 2,\ y = 6$ in eq(iii)

$$4+36 = 8c$$

$$c = \frac{40}{8} = 5 \quad \text{put in (iii)}$$

so

$x^2+y^2=5x^3$

$y^2=5x^3-x^2$

$y^2=x^2(5x-1)$

Taking under root ($\sqrt{\ }$)

$\sqrt{y^2}=\sqrt{x^2(5x-1)}$

$y=+x\sqrt{5x-1}$, $y=-x\sqrt{5x-1}$ or

$\boxed{y=\pm x\sqrt{5x-1}}$ Ans.