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Section	B
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Q1 ID - 7752. Pg - 1.

$$kx + cx + m\ddot{x} = P(t)$$

In our case system is Undamped ($c=0$)
undergoing free vibration $P(t) = 0$.

Hence general equation EOM become

$$kx + m\ddot{x} = 0$$

$$k = 3EI/l^3$$

$$= \frac{3 \times 29000 \times (150)^3}{(10 \times 12)^3} = 7.55 \text{ k/in}$$

$$k = 90600 \text{ lb/ft.}$$

in order to determine the chances of mistake
during calculation it is more appropriate to
use fundamental units like lb, ft, sec, leg.m.

$$k = 7.55 \text{ k/in} = 90600 \text{ lb/ft.}$$

$$m = \frac{7752 \text{ lb} \cdot \text{sec}^2}{32.2 \text{ ft}} = 240.74 \text{ slug.}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{90600}{240.74}} = 19.39 \frac{\text{rad}}{\text{sec}}$$

Q1 7732.

$$T_n = 2\pi / \omega_n = 2(3.14) / 19.39 = \frac{0.324}{\cancel{0.328}}$$

Substituting the corresponding value
in eq-1 $90600u + 240.74 = 0$

$$u(t) = u(0) \cos(\omega_n t) + \frac{u'(0)}{\omega_n} \sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24} \text{ Ft and } u'(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) * \cos(19.39t) + 0 =$$

Equivalent static force at any time 't'
is

$$F(t) = k \cdot u(t) = \frac{90600 + \cos(19.39t)}{24}$$

$$F_{st(t)} = 3775 \cancel{04} * \cos(19.38t) = 3562.08$$

Amplitude of dynamic displacement

$$u = \sqrt{\left(u(0)\right)^2 + \left(\frac{u'(0)}{\omega_n}\right)^2} = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$k u_0 = 90600 * \frac{1}{24} = 3775 \text{ lb.}$$

(b) T_n ? ^D
(3)

As seven required are completed in "3.57" sec Thus time required to complete one cycle:

$$= 7/3.57 = 1.96 \text{ sec}$$

$$T_n = 1.96 \text{ sec.}$$

$$\text{Now } \omega_n = \omega_n \sqrt{1 - \zeta^2}$$

$$= 2\pi / \omega_n = \frac{2\pi}{\omega_n} (\sqrt{1 - \zeta^2})$$

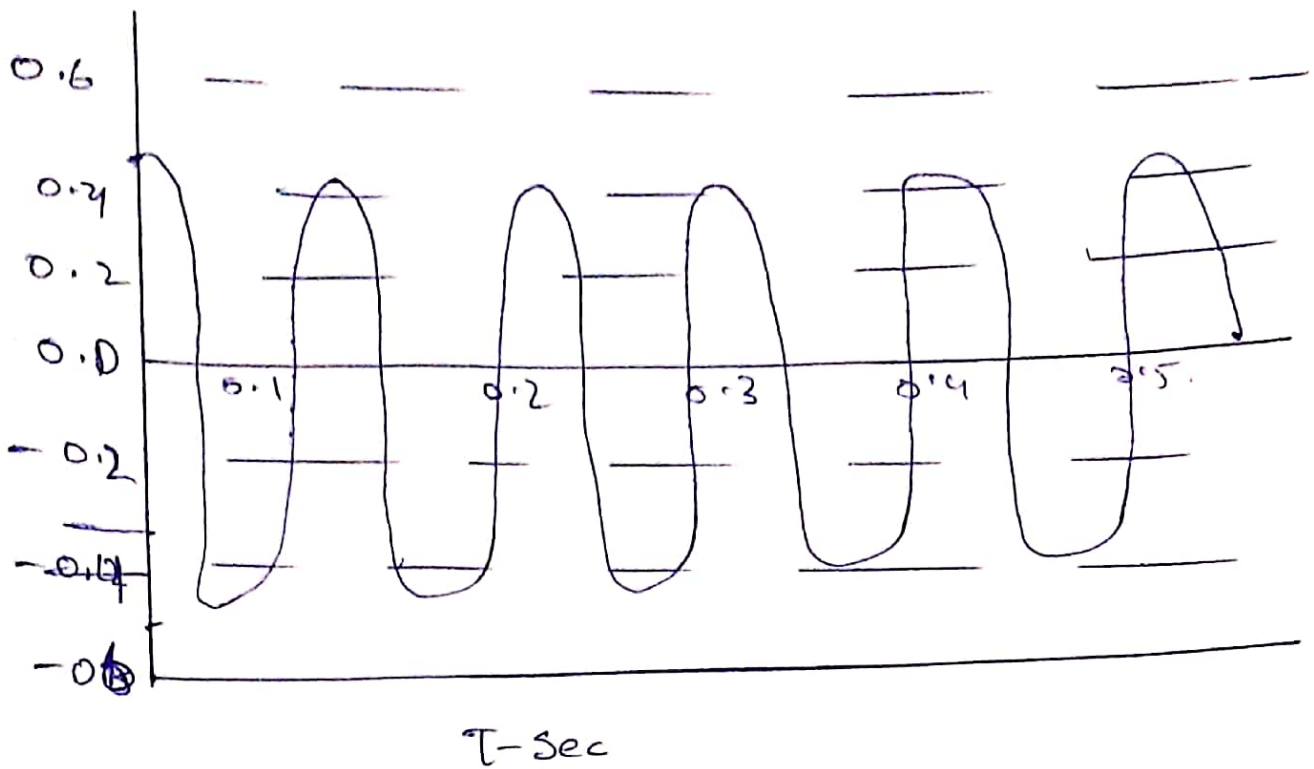
$$\text{As } T_n = T_n / \sqrt{1 - \zeta^2}$$

$$T_n = T_n (\sqrt{1 - \zeta^2})$$

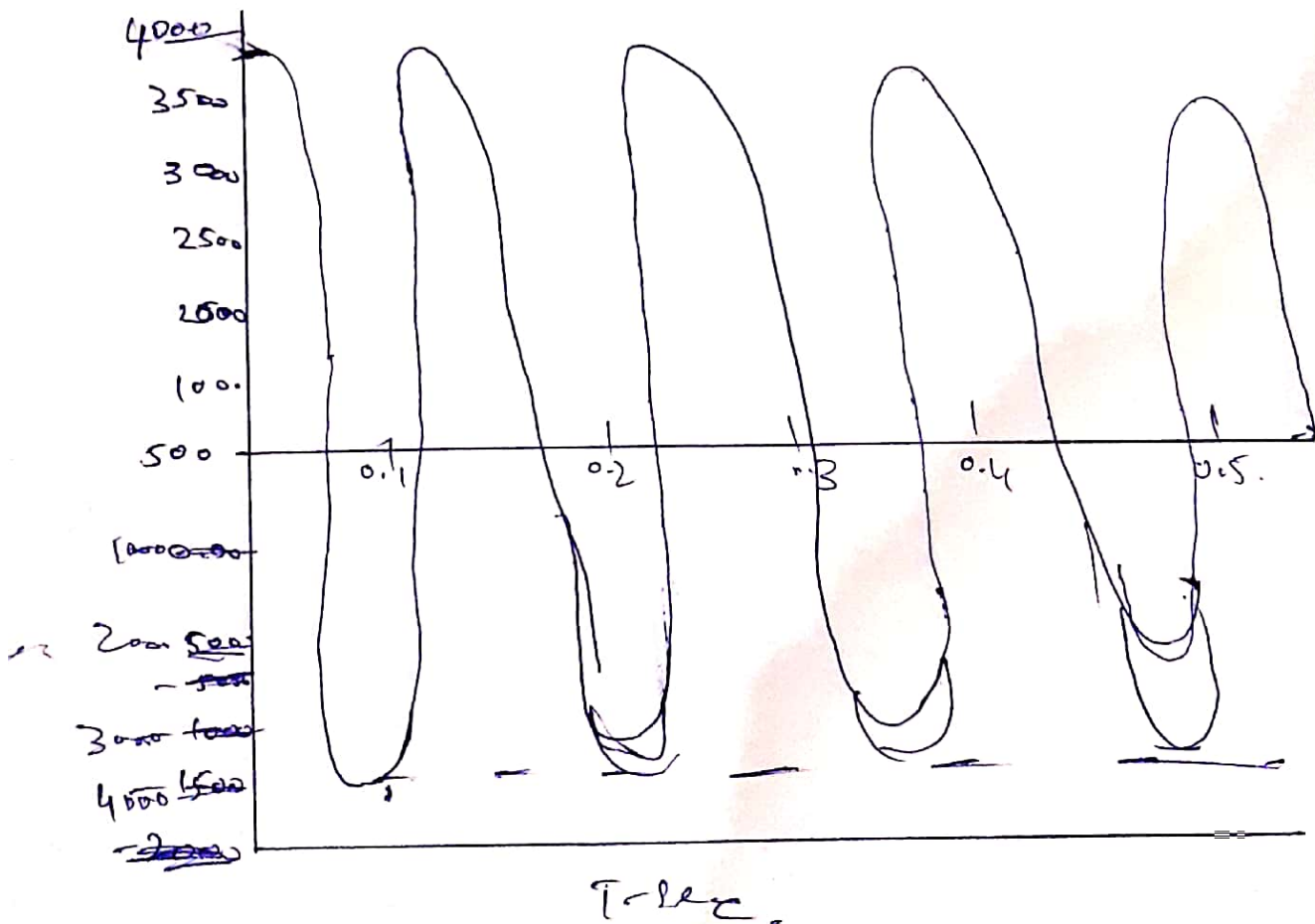
$$= 1.96 (\sqrt{1 - (0.0462)^2})$$

$T_n = 1.957$ " Natural Period of Undamped vibration

Undamped Free vibration



Undamped Free Vibration



~~Q2~~

Sol.

$$E = 29000 \text{ ksi}$$

$$I = 150 \text{ in}^4$$

$$F_{st} = 7752 \text{ lb}$$

$$\text{Take } c_0 = 5\%$$

EOM for damped free Vibration

$$Ku + Cu + mu = 0 \rightarrow (1)$$

l_b is known from equation 2.

$$k = 90600 \text{ lb/ft}$$

$$m = 240.74 \text{ slug}$$

$$m = 240.74 \text{ slug}$$

$$C = C \times 2m\omega_n$$

$$C = 0.05 \times 2(240.74)(19.39)$$

$$C = 466.794$$

Substituting Values of k, c, M
in eq ① we get.

$$90600 + 466.794 + 240.74 = 0$$

Sol The s.o.m for damped

$$u(t) = e^{-\zeta \omega_n t}$$

$$\left[u(0) \cos(\omega_n t) + \frac{1}{\omega_n} [u'(0) + \zeta \omega_n u(0)] \sin \omega_n t \right]$$

$$\omega_n = 19.39 \text{ rad/sec.}$$

$$u(t) = e^{-0.05 \times 19.39 t}$$

$$\left[\frac{1}{24} \times \cos(19.392t) \times \frac{1}{19.392} \right]$$

$$\left[0 + \frac{1}{24} \times 0.05 \times 19.39 \times \sin(19.39t) \right]$$

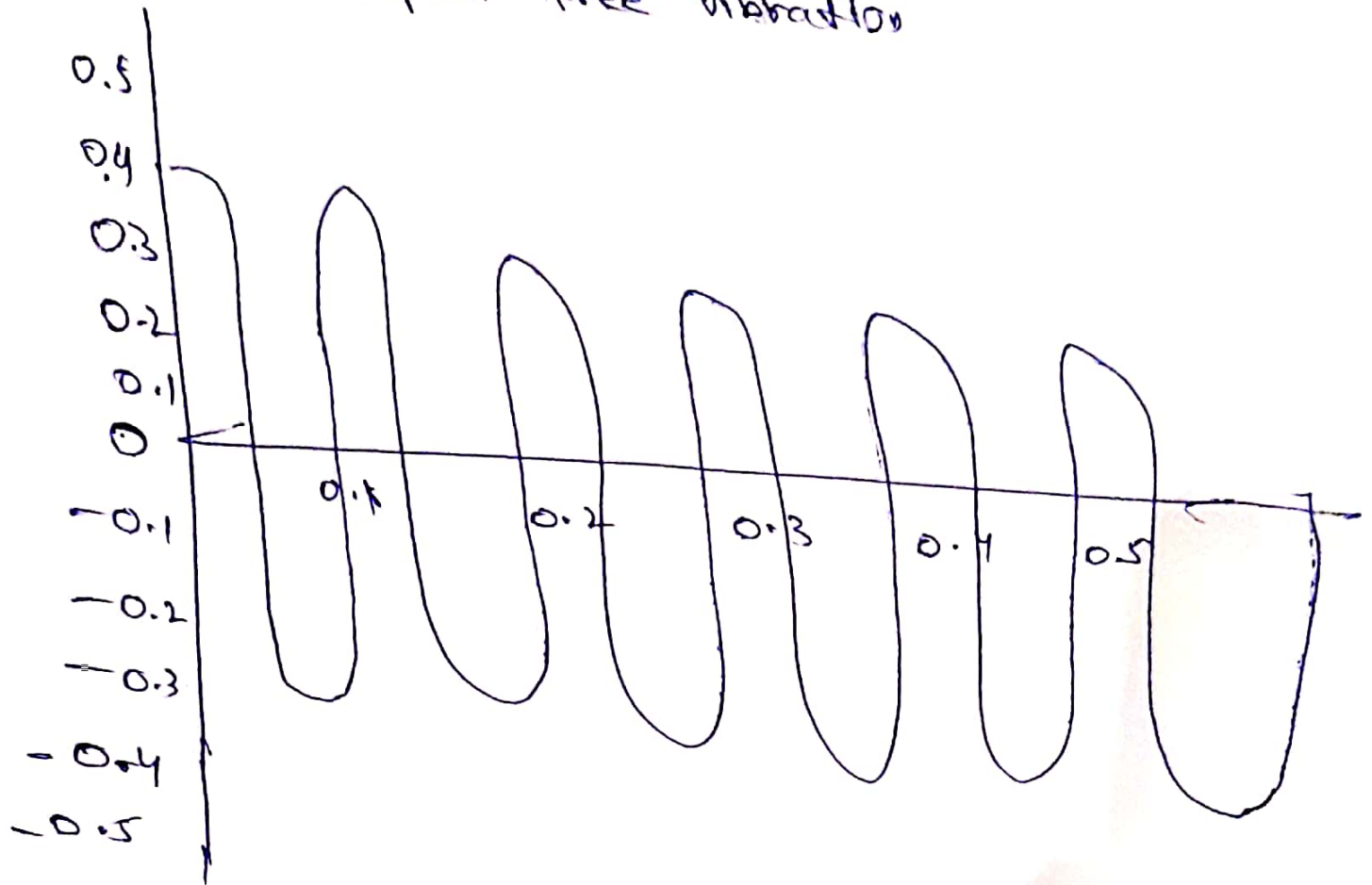
$$u(t) = e^{-0.972t} \left[0.0414 \times \cos 19.392 + 0.051 + 0.041 \times 0.05 \times \sin 19.49 \right]$$

$$f_s(t) = k \cdot u(t) = 90600 \times u(t).$$

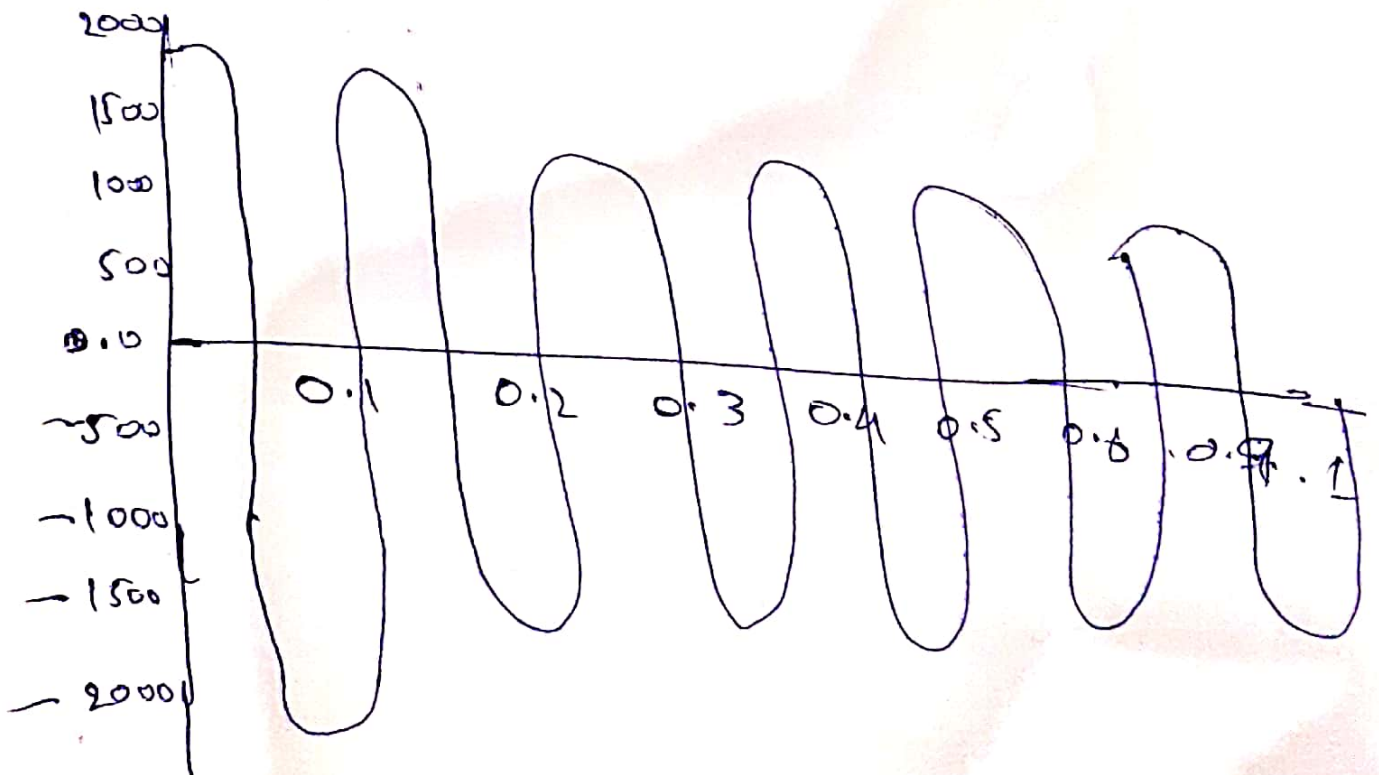
$$f_s(t) = 1c u(t) = 90600 \times u(t).$$

$$f_s(t) = e^{-0.972t} [3775 \cos(19.39) + 94.34 \sin(19.39)]$$

Damped free vibration



Damped free vibrator.



Q3
Solve

Given data

$$\text{Force} = 60 \text{ Kips}$$

$$u_1 = \frac{7752}{1000} = 7.752 \text{ in.}$$

After, $J = 7$ (cycle)

$$\text{Completed} = 3.57 \text{ sec}$$

$$u_{J+1} = 2.286 \text{ cm} = 0.9 \text{ in}$$

Ignore The Vertical Vibration

Required

- (a) Damping Ratio
- (b) Natural Period of Undamped Vibration
- (c) Stiffness of structure
- (d) Weight of tank
- (e) Damping Coefficient
- (f) Number of cycle to reduce the displacement Amplitude to 0.5".

(a)

$$\zeta = \text{Damping ratio} = 0.9 \quad (2)$$

$$\text{As } \zeta = \frac{1}{2\pi n} \ln \left[\frac{u_x}{u_{j+1}} \right]$$

By Putting values

$$\zeta = \frac{1}{2(3.14)n} \ln \left[\frac{7.752}{0.9} \right]$$

$$(b) \quad \zeta (7 \times 2 \times 3.14) = 2.148$$

$$\zeta (43.96) = 2.148$$

$$\zeta = \frac{2.148}{43.96}$$

$$= 0.0488$$

$$\zeta = 4.88\%$$

(c) Stiffness of structure $k = ?$

$$A = k = \frac{F \cos \theta}{\Delta}$$

$$k = \frac{60 \cdot \cos(60)}{\Delta} \quad F = 60 \text{ kips}$$

$$\theta = 60^\circ$$

$$k = 15 \text{ k/in}$$

$$k = 18000 \text{ k/ft}$$

(d) weight of tank $w = ?$

$$\text{As } \omega_n = \sqrt{k/m} = \sqrt{k/w/g}$$

$$= \sqrt{k \cdot g/w}$$

$$\Rightarrow \omega_n^2 = k \cdot g/w \Rightarrow (w = k \cdot g/\omega_n^2)$$

By putting values of $\omega_n = 2\pi/T_n$

$$w = k \cdot g / (4\pi^2/T_n^2) = k \cdot g \left(\frac{T_n^2}{4\pi^2} \right)$$

$$w = \frac{18000 \text{ lb}}{\text{ft}} \cdot \frac{32.2 \text{ ft}}{\text{sec}^2} \left(\frac{(1.957)^2}{4(3.14)^2} \right)$$

$$W = 56284.75 \text{ lb} = 56.2841 \text{ k lb}$$

(E) Damping Co-efficient $c = ?$

It is known that $g = \frac{c}{2m\omega_n}$

$$\Rightarrow c = g (2m\omega_n) = g (2m (2\pi / T_n))$$

By Putting Value.

$$c = 0.0462 \left(2 \left(\frac{56284}{32.2} \right) \right) (2(3.14))$$

$$c = 518.286 \text{ (lb/see) / } \#F.$$

(F) No of cycle to reduce displacement at titude from " 6.872 into 0.5 in.

$$j = ? \quad j = \frac{1}{2\pi g} \ln \left(\frac{41}{4j+1} \right)$$

$$= \frac{1}{2(3.14)(0.0462)} \ln \left(\frac{7.752}{0.9} \right)$$

$$j = 3.446 \text{ in.}$$

$$j = 7.23 \text{ cycles.}$$