Department of Electrical Engineering

Assignment

Date: 13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th

Instructor: **Total Marks:** 30

Student Details

Name: JUNAID UR REHMAN **Student ID: 11484**

<table>
<tr><td rowspan="3">Q1.</td><td>(a)</td><td>Consider the following analog signal

$$x_a(t) = 3cos100\pi t + 4sin200\pi t$$
i. Determine the minimum sampling rate required to avoid aliasing.
ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.
iii. What is the analog signal $y(t)$ we can reconstruct from the samples if we use ideal interpolation?</td><td>Marks 5
CLO 1</td></tr>
<tr><td>(b)</td><td>Consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$
This is signal is sampled at the rate $F_s = 2Hz$.

i. Draw the sampled signal.
ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i .
iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.</td><td>Marks 5
CLO 1</td></tr>
<tr><td>(a)</td><td></td><td>Marks 5</td></tr>
</table>

Q2.		Determine the response of the system to the following input signal with given impulse response $x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$	CLO 2
	(b)	Compute the convolution y(n) of the following signal $x(n) = \begin{cases} \alpha^{n+1}, & -3 \leq n \leq 5 \\ 0, & elsewhere \end{cases}$ $h(n) = \begin{cases} 2^{n}, & 0 \leq n \leq 4 \\ 0, & elsewhere \end{cases}$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{3})^{n}, & n \geq 0 \\ (\frac{1}{4})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \geq 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

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Q1 (a) 4

Consider the following analog signal.
$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

(1) Determine the minimum sampling rate required to alloid aliasing?

Ans #

minimum Sampling rate

$f_s \geq 2 f_{max}$

$\boxed{f = \frac{w}{2\pi}}$

$F_1 = \frac{100\cancel{\pi}}{2\cancel{\pi}}$

$\boxed{F_1 = 50 Hz}$

$F_2 = \frac{200\cancel{\pi}}{2\cancel{\pi}}$

$\boxed{F_2 = 100 Hz}$

So

F_2 is max (greater than f_1)

$\boxed{f_s \geq 2 \times 100 Hz}$

Sampling frequency to aviod alaising

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(2)

(2) Suppose that the Signal is sampled at the rate $F_s = 100Hz$ What is the discrete time signal obtained after Sampling? Also explain the effect of this Sampling rate on the newly generated discrete time Signal?

Sol #
$F_s = 100Hz$
So F_1 becomes

$$F' = \frac{F1}{F_s} = \frac{180}{100} = 0.5\ Hz$$

F_2 become

$$F2' = \frac{F_2}{100} = \frac{100}{100} = 1\ Hz$$

So
$$\omega_1' = 2\pi f_1' \qquad W_2' = 2\pi f_2'$$
$$W_1' = 2\pi \times 0.5 \qquad W_2' = 2\pi \times 1$$
$$\boxed{W_1' = \pi} \qquad \boxed{W_2 = 2\pi}$$

$$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$$

The Signal becomes

$$x[n] = 3\cos \pi n + 4\sin 2\pi n$$

The effect of this Sampling rate on the newly generated discrete times Signal is that there will be no Aliasing phenomena means that there will not present unwaid component in the re construction of the Signal. we construct the origional signal

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$\omega_1 = 100\pi$

$F_2 = 200\pi$

$F_1 = \frac{100\pi}{2\pi}$

$f_2 = 100\pi$

$\boxed{F_1 = 50}$

(3) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Ans# folding frequency of the sampled signal is

$$\text{folding frequency} = F_s/2 \Rightarrow \frac{100}{2}$$

$$= 50 \text{ Hz}$$

We have frequency of the original signal
$f_1 = 50 \text{ Hz}$, $2f_2 = 100 \text{ Hz}$

Both the frequency are either equal or greater the folding frequency

Hence for ideal interpolation we can construct the original signal

$$x_2(t) = 3\cos 100\pi + 4\sin 200\pi t.$$

The original signal is constructed because we are sampling frequency at nyquiest rate

We can also re construct the signal for sampling frequency above the nyquist rate.

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Q1b# Consider a discrete time Signal which is given by

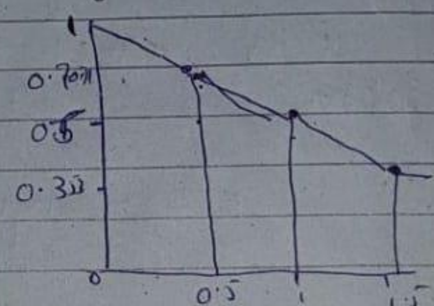
$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled at the rate $F_s = 2$ Hz

(i) Draw the sampled Signal?

Sol #

x_n	$= 0.5^n$
0	1
0.5	0.707.
1	0.5.
1.5	0.353



$T = 0.5$ sec

(ii) The samples of the signals are intended to carry 3 bit per sample. Determine the quantization level and quantization resolution to quantized the sampled Signal achieved in part i.?

Sol#

$L = 2^n$

$n = \text{bits} = 3$

$L = 2^3 = 8$ levels

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1 - 0}{8} = 0.125$$

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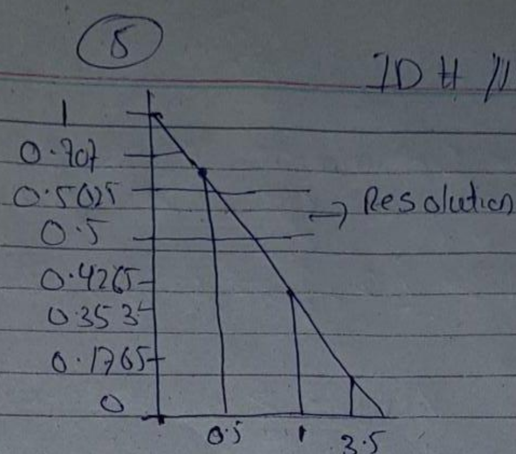

(iii) Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form?

Sol#

	Discret time Signal	truncation	Rounding	error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

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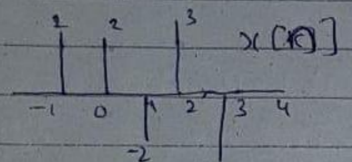
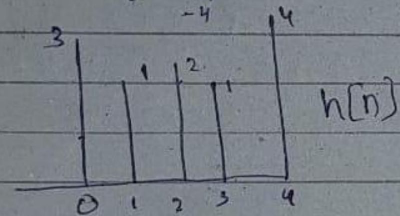
Q2(a)# Determine the response of the system to the following input signal with given impulse response

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}, \quad h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

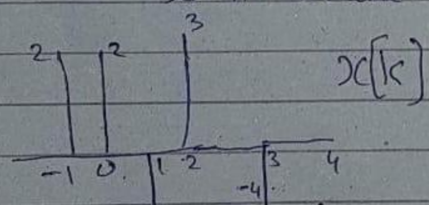
SOL#

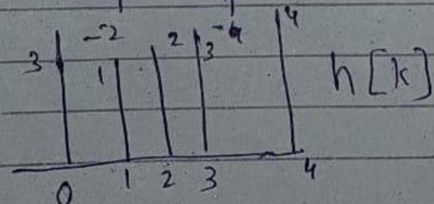
$$X[n] = \sum_{k=\infty}^{-\infty} x[k]\,h[n-k]$$





Step(1)# Convert x(n) & h[n] into x[k] & h[k]
So it become





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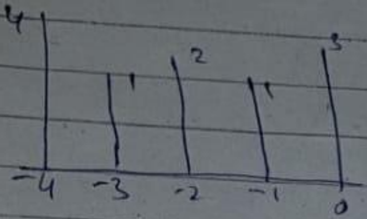
Step No 2 folded Signal $h(-k)$.



$h[-k]$.

Step No 3 Find $y[0] = ?$

$$y[0] = \sum_{k=-1}^{0} x[-1]h[-1] + x(0)$$

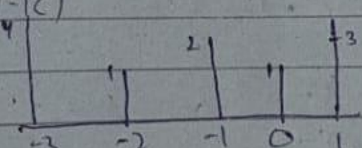
$$= 2 \times 1 + (1)(3)$$

$$= 2 + 3$$

$$\boxed{= 5}$$

For $n=1$

$h(1-k)$



Step No 4 $y[1] = ?$

$$y[1] = \sum_{k=-1}^{1} x[k]\, h[1-k]$$

$$= x[-1)\, k(-1) + x(0)\, h(0) + x(1) \cdot h(1)$$

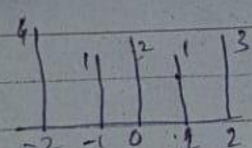
$$= (2)(2) + (1)(1) + (3)(-2)$$

$$= 4 + 1 - 6$$

$$= -1$$

$n = 2$

$h(2-k)$



Step No 5 $y[2] = ?$

$$y[2] = \sum_{k=-1}^{2} x(k]\, h(2-k]$$

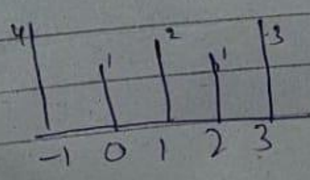
$$= x[-1]\, h(-1) + x(0) + h(0) + x(1)\, h(1) + x(2)\, h(2)$$

$$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$$

$$= 2 + 2 - 2 + 9$$

$$= 11$$

$n = 3$



$h(3-k)$

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Step No 6 $y[3] = ?$

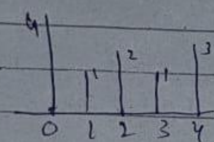
$$y[3] = \sum_{k=1} x[k] \times h(3-k]$$

$$= 2\times4 + 1\times1 + -2\times2 + 3\times1 + 4$$

$$= 4 + 1 - 4 + 3 - 12$$

$$= -8$$

$n = 4$



$h(4-k)$

Step No 7 $y[4] = ?$

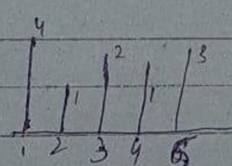
$$y[4] = \sum_{k=0} x(k) \times h[4-k]$$

$$= (1\times4) + (-2)\cdot(1) + (3)(2) + (-4)\cdot(1)$$

$$= 4 - 2 + 6 - 4$$

$$= 4$$

$n = 5$



$h(5-k)$

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Step # 8

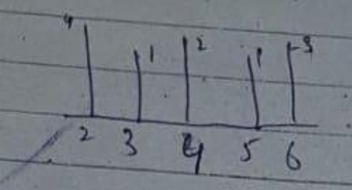
$$y[5] = \sum_{k=1} x[k] \times h[5-k]$$

$$= (-2)(4) + (3)(1) + (4)(2)$$

$$= -8 + 3 - 8$$

$$= -13$$

$n = -6$



Step #9

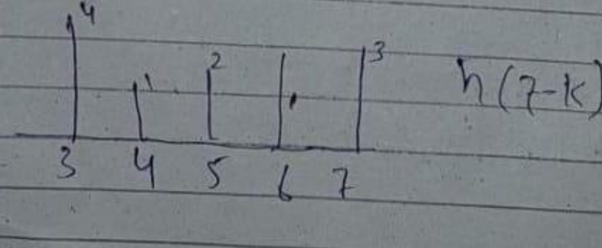
$$y[6] = \sum_{k=2}^{k=} x(2)\, h(2) + x(3)\, h(3)$$

$$y[6] = (3)(4) + (1)(-4)$$

$$= 12 - 4$$

$$= 8$$

$n = 7$



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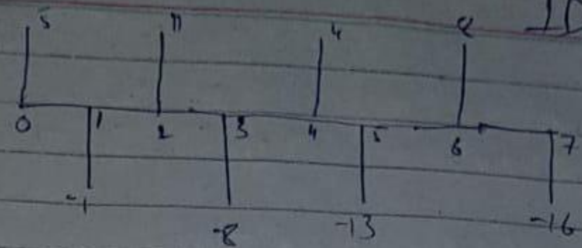
$y[n] =$



Answer

Q2(b) # Compute the convolution $y(n)$ of the following signal.

$$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0 & \text{else wher} \end{cases}$$

$$h[n] = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & \text{elsewhere.} \end{cases}$$

Sol # $x(n) = \{\alpha^{-2}, \alpha^{-1}, \alpha; \alpha^{1}; \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$

and

$h[n] = \{1, 2, 4, 6, 16\}$

$$y[n] = \sum_{k=0}^{4} h(k)\, x(n-k)$$

Therefore

$y(-2) = \alpha^{-2}$

$y(-1) = x(-\alpha) + x(1) = \alpha^{-2} + \alpha^{-1}$

$y(0) = h(1)\,x(-2) + h(x)\,h(-9) + h(x)\,h(\alpha)$

$= 9 - \alpha^{-2} + 9 \cdot \alpha^{-2} + 9 \cdot 1 + \alpha^{-3} + 9$

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$y(1) = \alpha^{-2} + \alpha^{-3} + 1 + h(9)\, x(4-3$

$= \alpha^{-2} + \alpha^{-1}\, 9 + h(3) + x(1) = x^{-2} + x^{-1} + 1 + 2\alpha^{1}$

$y(2) = \alpha^{-2} + \alpha^{-1} + 1 + 2\alpha^{1} + h(2)\, x(2)$

$= \alpha^{2} + \alpha^{-3} + 1 + 2\alpha^{1} + 4\alpha^{2}$

$y(3) = \alpha^{-2} + \alpha^{-1} + 9 + 2\alpha^{1} + 4\alpha^{2} + 6\alpha^{3}$

$y(4) = \alpha^{-2} + \alpha^{-1} + 9 + 2\alpha^{1} + 4\alpha^{2} + 6\alpha^{3} - h(4) \cdot x(4)$

$= \alpha^{-2} + \alpha^{-3} + 9 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4}$

$y(5) = 1 + 2\alpha^{1} + 4\alpha^{2} + 8\alpha^{3} + 16^{4} + 5$

$y(6) = 4\alpha^{2} + 8\alpha^{3} + 16\alpha^{4} + \alpha^{5} + \alpha^{6}$

$y(7) = 8\alpha^{3} + \alpha^{4} + \alpha^{5} + \alpha^{6}$

$y(8) = 16\alpha^{4} + \alpha^{5} + \alpha^{6}$

$y(9) = \alpha^{5} + \alpha^{6}$

$y(10) = \alpha^{6}$

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Q3 Determine the z-transform of the following Signal and also sketch its Region of Convergence (ROC)?

1) $x(n) = \begin{cases} (\frac{1}{4})^n , & n \geq 0 \\ (\frac{1}{3})^{-n} & n < 0 \end{cases}$

(2) $x(n) = \begin{cases} (1/2)^n - 3^n , & n \geq 0 \\ 0 & \text{else where} \end{cases}$

Sol# (1)

Writing in the form of z transform.

$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=\infty}^{0} \left[\frac{1}{3}\right]^n z^{-n} - 1$

Using geometric Series

$= \frac{1}{1 - 1/4 \, z^{-1}} + \sum_{n=0}^{\infty} \left[\frac{1}{3}\right]^n z^n - 1$

$= \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - 2/3 \, z} - 1$

$= \frac{1 - 1/3 z + 1 - 1/4 z^{-1}}{(1 - 1/4 z^{-1})(1 \cdot 1/4 z^{-1})} - 1$

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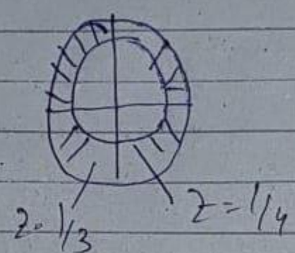
$$= \frac{1-\frac{1}{3}z+1\cdot\frac{1}{4}z^{-1}-(1\cdot\frac{1}{4}z^{-1})(1\cdot\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{\cancel{1}-\frac{1}{3}z+1-\frac{1}{4}z^{-1}\cancel{1} : 1+\frac{1}{3}z+\frac{1}{4}z^{-1}+\frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{13/12}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

Hence the ROC is $\frac{1}{4} < |z| < 3$



(2) $x(n) = \sum_{n=0}^{\infty} (1/2)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$

using geometric series to simplify it

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$= \frac{\cancel{1}-3z^{-1}-\cancel{1}+\frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

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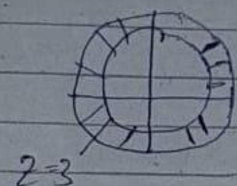
$$= \frac{-\frac{5}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - 3z^{-1}\right)}$$

The ROC is $|z| > 3$



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