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ID # 7692

Section: A

BE(c)

Subject: Intro to structural dynamics & Earthquake Engineering.

Instructor: Engr- Yaseen.

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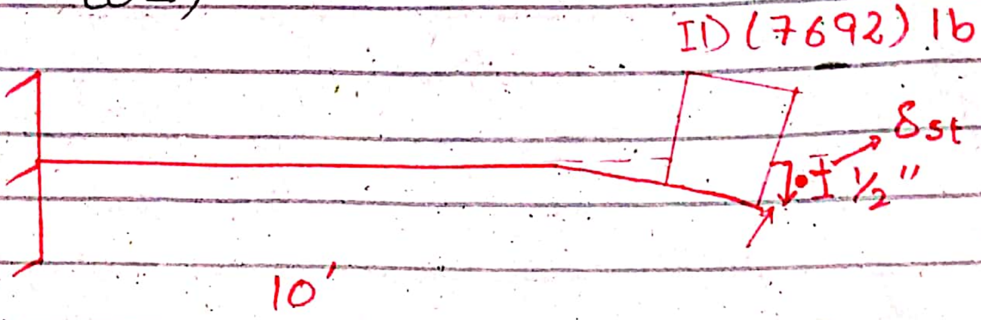
Q. No. (01)



Solution:-

The general E.O.M for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t).$$

In our case system is undamped (c=0) undergoing free vibration (P(t)=0)

Hence general EOM becomes $ku + m\ddot{u} = 0$ ...(i)

$$K = 3EI/L^3$$

$$K = \frac{3 * 29000\ \text{K/in}^2 * 150\ \text{in}^4}{(10 * 12\ \text{in})^3}$$

$$K = 7.55\ \text{K/in}.$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or Kg, m, sec.

$$K = 7.55\ \text{K/in} = 90625\ \text{lb/ft}.$$

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$$m = \frac{7692 \text{ lb}}{32.2 \text{ ft/sec}^2}$$

$$m = 238.88 \text{ slug}$$

$$W_n = \sqrt{K/m}$$

$$W_n = \cancel{\sqrt{90625/}} \sqrt{\frac{90625}{238.88}}$$

$$W_n = 19.48 \text{ rad/sec}$$

$$T_n = 2\pi / w_n = 2\pi / 19.48$$

$$T_n = 0.323 \text{ sec}$$

Substituting the corresponding values in eq - I.

$$90625\, u + 238.88\, \ddot{u} = 0$$

Where "K" is in lb/ft and 'm' is in lb sec/ft^2

General solution to the EOM for undamped free vibration is,

$$u(t) = u(0) \cos(w_n t) + \dot{u}(0)/w_n \sin(w_n t)$$

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$u(0) = \frac{1}{2}'' = \frac{1}{24}$ ft and $\dot{u}(0) = 0$

$u(t) = ((\frac{1}{24}) * \cos(19.48t) + 0)$

$u(t) = (\frac{1}{24}) * \cos(19.48t)$.

Equivalent static force at any time "t" is

$f_s(t) = K.u(t)$.

$$f_s(t) = \frac{90625 * \cos(19.48t)}{24}$$

$f_s(t) = 3776.04 \cos(19.48t)$.

Amplitude of dynamic displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{\left((u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right)}$$

$$u_0 = \sqrt{((\frac{1}{24})^2 + 0}$$

$u_0 = \frac{1}{24}$ ft.

Amplitude of equivalent static force,

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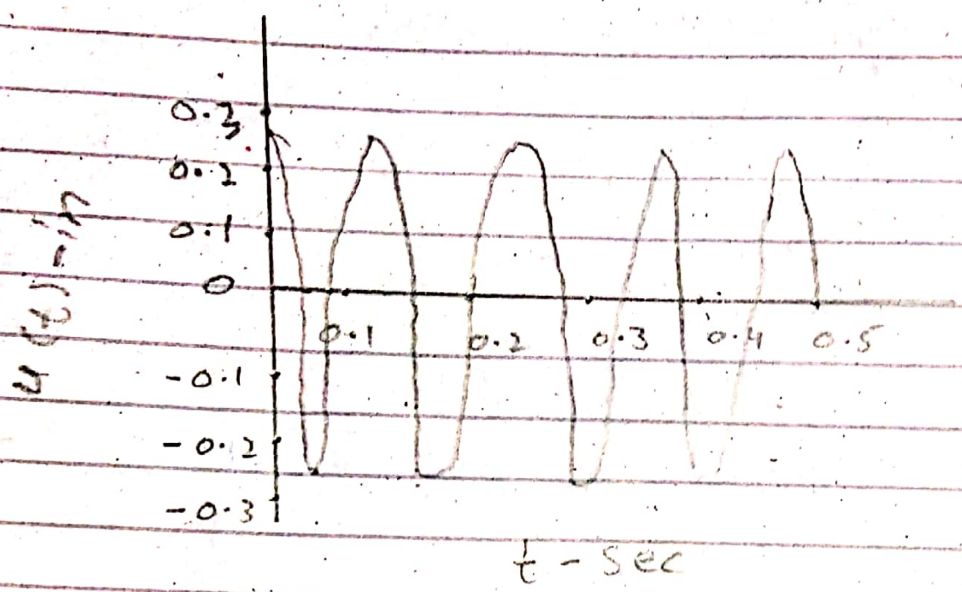
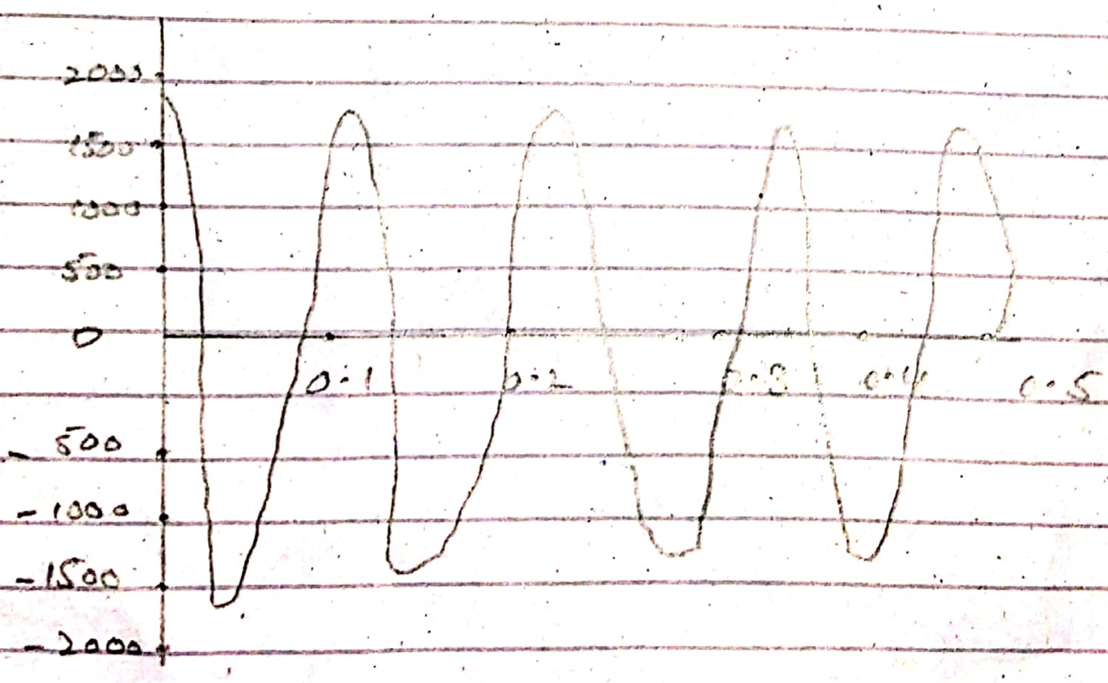
$$K_{40} = 90625 * \frac{1}{24}$$

$$K_{40} = 3776.04 \text{ lb.}$$



undamped force vibration



Undamped free vibration

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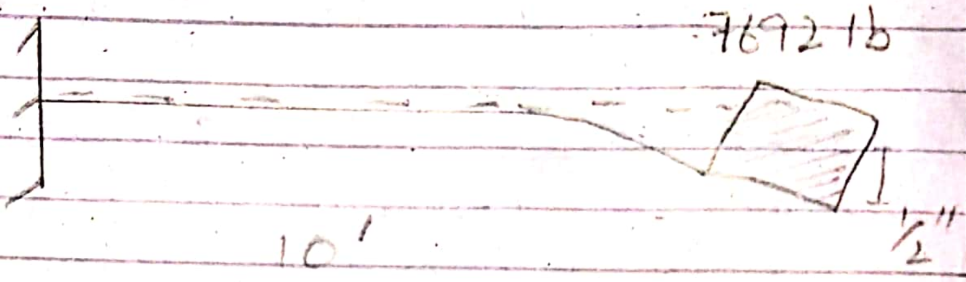
Solution:-



* E.O.M for damped free vibrations is;

$$Ku + c\dot{u} + m\ddot{u} = 0 \quad --- \quad (i).$$

* It is known from (ques-1).

$K = 90625$ lb/ft and $m = 238.88$ slug

$W_n = 19.48$, $m = 238.88$ lb.sec²/ft.

$\Rightarrow C = \zeta * 2m W_n = 2 * 238.88 * 19.48 * \zeta$

($\zeta = 0.03 - 0.05$ with considerable cracking the damping ratio)

$\Rightarrow C = 2 * 238.88 * 19.48 * 0.05$

$C = 465.34$ lb.sec/ft.

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* By substituting values of k, c and m in eq (i) we get.

$$90625\,u + 465.34\,\dot{u} + 238.88\,\ddot{u} = 0$$

* Solution to the E.O.M for damped free vibration is;

$$u(t) = e^{-\xi \omega_n t}\left(u(0)\cos(\omega_D t) + \frac{1}{\omega_D}(\dot{u}(0) + u(0)\xi\omega_n).\sin(\omega_D t)\right).$$

$$\omega_D = 19.48 \text{ rad/sec}.$$

$$u(t) = e^{-0.05*19.48t}\left(\frac{1}{24} * \cos(19.48t) + \frac{1}{19.48} * \left(0 + \frac{1}{24} * 0.05 * 19.48 * \sin(19.48t)\right)\right).$$

$$u(t) = e^{-0.974t}\left(0.0417 * \cos(19.48t) + 0.513 * (0.041 * \sin(19.48t))\right)$$

$$u(t) = e^{-0.974t}\left(0.0417 * \cos(19.48t) + 0.021 * \sin(19.48t)\right)$$

$$f_s(t) = k.u(t) = 90625 * u(t).$$

$$f_s(t) = e^{-1.373t}\left(3779.1\cos(19.48t) + 1903.12 * \sin(19.48t)\right).$$

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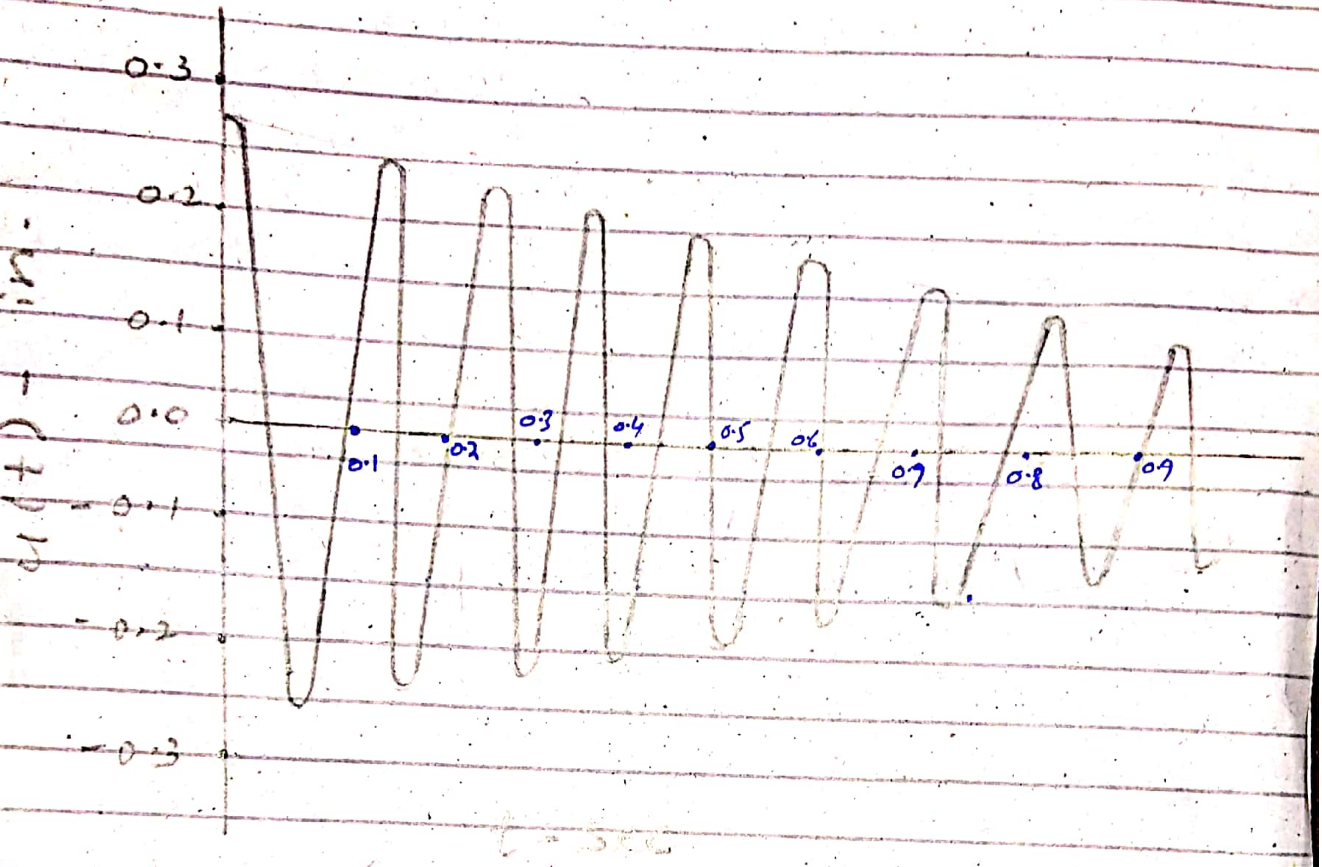
Damped free vibration:



variation of displacement with time.

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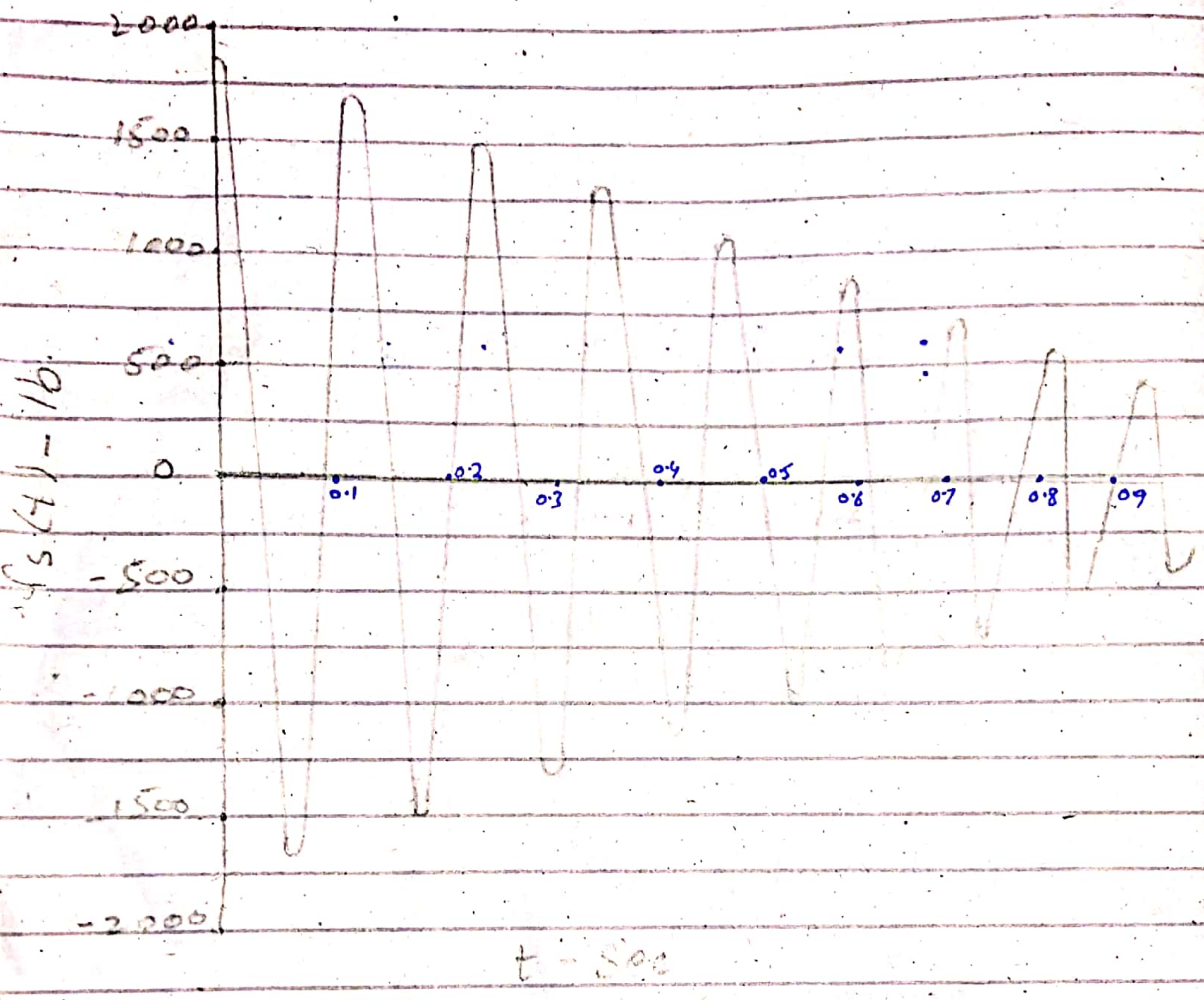
Damped Free vibration:



Variation of equivalent static forces with time.

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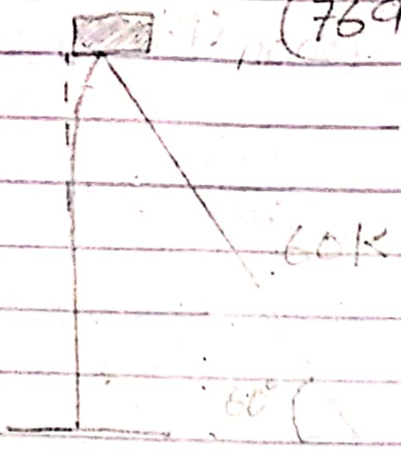
Q. No. (03).

$(7692/1000)$ inches



Solution:→

$(7.692'' \text{ or } 7.7'')$

$u_1 = 7692/1000 = 7.7''$

After $j = 7$, $u_{j+1} = u_6 = 2.286\,cm$
$= 0.9''$

a). ζ = Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$7 = \frac{1}{2\pi\zeta} \ln(7.7/0.9)$$

$$\zeta = 0.049 = 4.9\,\%$$

b). $T_n = ?$

7 cycles of vibrations are completed in 3.57 sec.

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Time required to complete one cycle = $3.57 / 7 = T_D$

$T_D = 0.51$ sec.

Now

$\omega_D = \omega_n \sqrt{(1 - \xi^2)}$

$2\pi / \omega_D = 2\pi / (\omega_n \sqrt{1 - \xi^2})$

$\Rightarrow T_D = \dfrac{T_n}{\sqrt{1 - \xi^2}}$

$\Rightarrow T_n = T_D * \sqrt{1 - \xi^2}$

$\Rightarrow T_n = 0.51 * \sqrt{1 - (0.049)^2}$

$\Rightarrow T_n = 0.5094 = 0.51$ sec.

C). $K = ?$

$K = \dfrac{60 * \cos 60°}{7.7} = 3.9$ k/in.

$K = 46800$ lb/ft.

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d). weight of the tank, W = ?

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{(W/g)}} = \sqrt{\frac{K*g}{W}}$$

$$\Rightarrow \omega_n^2 = K*g / W$$

$$\Rightarrow W = K*g / \omega_n^2 .$$

Also

$$\omega_n = 2\pi / T_n$$

$$W = Kg / \left(\frac{4\pi^2}{T_n^2}\right) = Kg * \frac{T_n^2}{4\pi^2}$$

$$W = \frac{46800 * 32.2 * (0.51)^2}{4\pi^2}$$

$$W = 9928.5 \text{ lb}$$

$$W = 9.93 \text{ K}$$

e). $c = ?$

It is known that $\zeta = \frac{c}{2m\omega_n}$

$$\Rightarrow c = \zeta * 2m\omega_n = \zeta * 2m * (2\pi / T_n).$$

$$c = 0.049 * 2 * 2 * (\pi / 0.51)\left(\frac{9928.5}{32.2}\right)$$

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$\Rightarrow C = 372.27$ lb.sec/ft.

f). No. of cycles to reduce displacement amplitude from 7.7 in to 0.5" ., $j = ?$

$$j = \frac{1}{2\pi \zeta} \ln \left(\frac{u_1}{u_{j+1}} \right)$$

$$\Rightarrow j = \frac{1}{2 * \pi * 0.049} \ln \left(\frac{7.7}{0.5} \right)$$

$\Rightarrow j = 8.69$ or 9 cycles.