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Programe: BE (E)
subject: Calculus
Module: 10 th semester

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Q# 1 (A) Estimate $\int \theta^4 \sqrt{1-\theta^2}\, d\theta$

Sol:-

let:

$$1-\theta^2 = u$$

$$\frac{d}{d\theta}(1-\theta^2) = \frac{d}{d\theta} u$$

$$-2\theta = \frac{du}{d\theta}$$

$$\theta\, d\theta = \frac{-1}{2} du$$

Now

$$= \int (u)^{\frac{1}{4}} \cdot \left(-\frac{1}{2}\right) du$$

$$= \frac{-1}{2} \int u^{\frac{1}{4}} du$$

$$\because \frac{1}{4} + 1 = \frac{5}{4}$$

$$= \frac{-1}{2} \cdot \frac{4}{5} u^{\frac{5}{4}} + C$$

P.T.O

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$$= \frac{-2}{5} u^{5/4} + C$$

By back Substitution

$$\boxed{= \frac{-2}{5}(1-\theta^2)^{5/4} + C}$$

Ans.

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Q# 1
B

Estimate $\int_0^1 x^3(1+x^4)^3\,dx$

using substitution method.

Sol:-

$$\int_0^1 x^3(1+x^4)^3\,dx \quad \text{—— (1)}$$

let

$$t = 1 + x^4$$

$$\frac{dt}{dx} = 4x^3\,dx$$

$$\frac{dt}{4} = x^3\,dx \quad \text{—— (2)}$$

so put in above @ Eq (1)

$$\frac{1}{4}\int_0^1 t^3\,dt$$

$$\frac{1}{4}\left(\frac{t^4}{4}\right)\Big|_0^1$$ Now solve the limit

$$\frac{1}{16}(1^4 - 0^4)$$

$$\frac{1}{16}(1) = \boxed{\frac{1}{16}}$$ Ans.

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Q # 2
Part A

Sol:-

$$x^2+y^2+z^2+3x-4z+1=0$$

$$(x^2+3x)+y^2+z-4z+1=0$$

$$\left(x^2+3x+\left(\frac{3}{2}\right)\right)+(y-0)+\left(z^2-4z+\left(\frac{-4}{2}\right)^2\right)$$

$$=-1+\left(\frac{3}{2}\right)^2+\left(\frac{-4}{2}\right)^2$$

$$\left(x+\frac{3}{2}\right)^2+(y_0)^2+(z-2)^2=\frac{21}{4}$$

So (x_0, y_0, z_0) = centre

$$=\left(\frac{3}{2}, 0, 2\right)$$

$\boxed{\text{Radius } a = \sqrt{\frac{21}{4}}}$ Ans.

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Q#12 (B)

$y = \sqrt{x}\ ,\ 0 \le x \le 4.$

Soln:-

Given that:

$y = \sqrt{x}$

$0 \le x \le 4 \Rightarrow a \le x < b.$

As $V = \int_a^b \pi y^2 \, dx$

$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$

$V = \pi \int_0^4 x \, dx = \pi \left. \frac{x^2}{2} \right|_0^4$

$V = \frac{\pi}{2} \left[(4)^2 - 0 \right] = 8\pi$

$\boxed{= 8\pi \text{ Ans}}$

Q#3 (A) If $A = 2i - 4j + \sqrt{5}k$ and $B = -2i + 4j - \sqrt{5}k$ then illustrate the vector proje A.B

Sol:

$A = 2i - 4j + \sqrt{5}k$

$B = -2i + 4j - \sqrt{5}k$

Projection A B = ?

By Dot Product.

$B \cdot A = (-2j + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$B \cdot A = (-2j \cdot 2j)(4j \cdot -4j)(-\sqrt{5}k \cdot \sqrt{5}k)$

$B \cdot A = -4 - 16 - 5$

$\boxed{B \cdot A = -25}$

Now

$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$\cancel{A \cdot A = (2i - 4j +}$

$A \cdot A = (2i \cdot 2i)(-4j \cdot -4j)(\sqrt{5}k \cdot \sqrt{5}k)$

$A \cdot A = 4 + 16 + 5$

$$\boxed{A \cdot A = 25}$$

So $\text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A}\right) A$

~~$\text{Proj}_A B$~~ = By putting the values

$$= \left(\frac{-\cancel{25}}{\cancel{25}}\right)(2i - 4j + \sqrt{5}K)$$

$$= (-1)(2i - 4j + \sqrt{5}K)$$

$$= -2i + 4j - \sqrt{5}K$$

$$\boxed{\text{Proj}_A B = -2i + 4j - \sqrt{5}K}$$ Ans.

Q#4 $y = -x^2 + 5x - 4$, $[0, 2]$

Sol:-

Given that

$$y = fx = -x^2 + 5x - 4$$

and

$$[a, b] = [0, 2]$$

As $a = 0$

$b = 2$

So Area under graph will be

$$A = \int_a^b f(x)\,dx$$

putting the values.

$$A = \int_0^2 (-x^2 + 5x - 4)\,dx$$

Now Solve the Integration

we get

$$A = \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x\right)\Big|_0^2$$

$$A = \left(-\frac{1}{3}(2)^3 + \frac{5}{2}(2)^2 - 4(2) - (0)\right)$$

$$A = \left(-\frac{1}{3}(8) + \frac{5}{2}(4) - 8\right)$$

$$A = \frac{-8}{3} + \frac{20}{2} - 8$$

$$A = \frac{-2}{3}$$

$$A = -0.6$$

R.W

$$= \frac{-(2\times 8)+(3\times 20)-(8\times 6)}{6}$$

$$= \frac{-16+60-48}{6}$$

$$= \frac{-\cancel{4}\,2}{\cancel{6}\,3}$$

$$= -\frac{2}{3}$$

As Area is Never in Negative so we take the value positive

$$\boxed{A = 0.6}$$ Ans.

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Paragram: BE(E)

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Q#5

A

$A = i - 2j - 2k$

$B = 6i + 3j + 2k$

Sol:-

$A = i - 2j - 2k$

$|A| = \sqrt{1+4+4}$ =

$= \sqrt{9} = 3$

$\boxed{|A| = 3}$

$B = 6i + 3j + 2k$

$|B| = \sqrt{36+9+4}$

$|B| = \sqrt{49}$

$\boxed{|B| = 7}$

$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A| \cdot |B|}\right)$

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$$Q = \cos^{-1}\left(\frac{(i-2j-2k)\cdot(6i+3j+2k)}{3\times 7}\right)$$

$$Q = \cos^{-1}\left(\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right)$$

$$Q = \cos^{-1}\left(\frac{-4}{21}\right)$$

$\boxed{Q = 100.9805^\circ}$ Ans

—— X —————— X ——

—— X —————— X ——

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Q# 5 (B) Change into a spherical coordinate equation for the sphere $x^2+y^2+(z-1)^2=1$

Sol:->

$$x^2+y^2+(z-1)^2=1$$

$$(\rho\sin\phi\cos\theta)^2+(\rho\sin\phi\sin\theta)^2$$
$$+(\rho\cos\phi-1)^2=1$$

$$\rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta$$
$$+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\phi^2+1-2\rho\cos\phi=1$$

$$\rho^2(\sin^2\phi)+\rho^2\cos^2\phi-2\rho\cos\phi=1-1$$

$$\rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$

$$\rho^2=2\rho\cos\phi \qquad \boxed{\rho=2\cos\phi} \text{ Ans.}$$

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$r = \rho \sin \phi$

$x = r \cos\theta = \rho \sin\theta \cdot \cos\theta$

$y = r \sin\theta = \rho \sin \phi \sin\theta$

$z = \rho \cos \phi$

$\rho = \sqrt{x^2 + y^2 + z^2}$

Equation Relation sphereal Co-ordinate Equation.

—— X —— X ——

—— X —— X ——