

Name: Sajid Ahmad
ID: 12671

Program: BE (E)
Subject: Calculus
Module: 10th semester

①

Q# 1
①

Estimate $\int x^4 \sqrt{1-x^2} dx$

Sol:-

let:

$$1 - x^2 = u$$

$$\frac{d}{dx} (1 - x^2) = \frac{d}{dx} u$$

$$-2x = \frac{du}{dx}$$

$$x dx = -\frac{1}{2} du$$

Now

$$= \int (u)^{\frac{1}{2}} \cdot \left(-\frac{1}{2}\right) du$$

$$= -\frac{1}{2} \int 4^{\frac{1}{4}} du$$

$$= -\frac{1}{2} \cdot \frac{4}{5} \cdot 4^{\frac{5}{4}} + C$$

$$\left. \begin{aligned} \therefore \frac{1}{4} + 1 \\ = \frac{5}{4} \end{aligned} \right\}$$

P.T.O

Name: Sajid Ahmad

ID: 12671

②

Program: BE (E)

Subject: Calculus

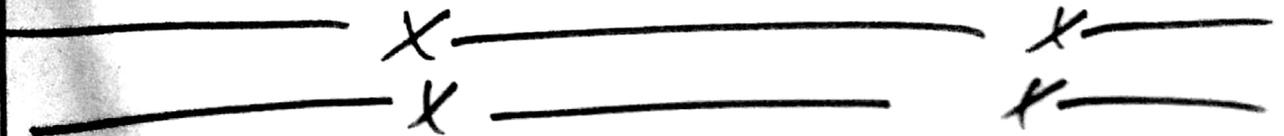
Module: 10th Semester

$$= \frac{-2}{5} 4^{5/4} + C$$

By back Substitution

$$= \frac{-2}{5} (1 - 0^2) 5^{5/4} + C$$

Ans.



Name: Sajid Ahmad

Program: BE (E)

ID: 12671

Subject: Calculus

Module: 10th Semester

Q# 1
B

Estimate $\int_0^1 x^3 (1+x^4)^3 dx$

using substitution method.

Sol: $\int_0^1 x^3 (1+x^4)^3 dx$ — (1)

let

$$t = 1 + x^4$$

$$\frac{dt}{dx} = 4x^3 dx$$

$$\frac{dt}{4} = x^3 dx$$
 — (2)

so put in above @ Eq (1)

$$\frac{1}{4} \int_0^1 t^3 dt$$

$$\frac{1}{4} \left(\frac{t^4}{4} \right) \Big|_0^1$$
 Now solve the limit

$$\frac{1}{16} (1^4 - 0^4)$$

$$\frac{1}{16} (1) =$$

$$\boxed{\frac{1}{16}} \text{ Ans.}$$

Name: Sajid Ahmad

Program: BECE

ID: 12671

Subject: Calculus

Module: 10th Semester

(4)

Q # 2

Part (A)

Sol:-

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$

$$(x^2 + 3x) + y^2 + z^2 - 4z + 1 = 0$$

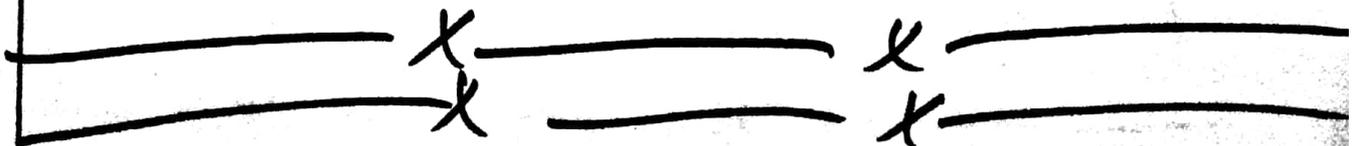
$$\left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) + (y - 0) + \left(z^2 - 4z + \left(\frac{-4}{2}\right)^2\right) \\ = -1 + \left(\frac{3}{2}\right)^2 + \left(\frac{-4}{2}\right)^2$$

$$\left(x + \frac{3}{2}\right)^2 + (y - 0)^2 + (z - 2)^2 = \frac{21}{4}$$

So $(x_0, y_0, z_0) = \text{Centre}$

$$= \left(\frac{3}{2}, 0, 2\right)$$

Radius $\Rightarrow a = \sqrt{\frac{21}{4}}$ Ans.



Name: Sajid Ahmad

Program: BECE

ID: 12671

Subject: Calculus

Module: 10th Semester

5

Q#2

$$y = \sqrt{x}, \quad 0 \leq x \leq 4.$$

(B)

Solo:

Given that:

$$y = \sqrt{x}$$

$$0 \leq x \leq 4 \Rightarrow a \leq x < b.$$

$$\text{As } V = \int_a^b \pi y^2 dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$V = \pi \int_0^4 x dx = \pi \left. \frac{x^2}{2} \right|_0^4$$

$$V = \frac{\pi}{2} [(4)^2 - 0] = 8\pi$$

$= 8\pi$ Ans

(6)

Q#3

(A)

If $A = 2i - 4j + \sqrt{5}k$ and $B = -2i + 4j - \sqrt{5}k$
then illustrate the vector proje A, B

Sol:

$$A = 2i - 4j + \sqrt{5}k$$

$$B = -2i + 4j - \sqrt{5}k$$

Projection $AB = ?$

By Dot Product.

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$B \cdot A = (-2i \cdot 2i) + (4j \cdot -4j) + (-\sqrt{5}k \cdot \sqrt{5}k)$$

$$B \cdot A = -4 - 16 - 5$$

$$B \cdot A = -25$$

Now

$$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$A \cdot A = \cancel{(2i - 4j + \sqrt{5}k)} \cdot \cancel{(2i - 4j + \sqrt{5}k)}$$

$$A \cdot A = (2i \cdot 2i) + (-4j \cdot -4j) + (\sqrt{5}k \cdot \sqrt{5}k)$$

$$A \cdot A = 4 + 16 + 5$$

(7)

$$A \cdot A = 25$$

$$\text{So } \text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A} \right) A$$

~~Proj A B~~ = By putting the values.

$$= \left(\frac{-25}{25} \right) (2i - 4j + \sqrt{5}k)$$

$$= (-1)(2i - 4j + \sqrt{5}k)$$

$$= -2i + 4j - \sqrt{5}k$$

$$\text{Proj}_A B = -2i + 4j - \sqrt{5}k \quad \text{Ans.}$$



8

Q#4

$$y = -x^2 + 5x - 4, [0, 2]$$

Sol^o-

Given that

$$y = f(x) = -x^2 + 5x - 4$$

and

$$[a, b] = [0, 2]$$

As $a = 0$

$$b = 2$$

So Area under graph will be

$$A = \int_a^b f(x) dx$$

putting the values.

$$A = \int_0^2 (-x^2 + 5x - 4) dx$$

Now Solve the Integration

9

we get

$$A = \left(-\frac{x^3}{3} + \frac{5x^2}{2} - 4x \right) \Big|_0^2$$

$$A = \left(-\frac{1}{3} (2)^3 + \frac{5}{2} (2)^2 - 4(2) - (0) \right)$$

$$A = \left(-\frac{1}{3} (8) + \frac{5}{2} (4) - 8 \right)$$

$$A = -\frac{8}{3} + \frac{20}{2} - 8$$

$$A = \frac{-2}{3}$$

$$A = -0.6$$

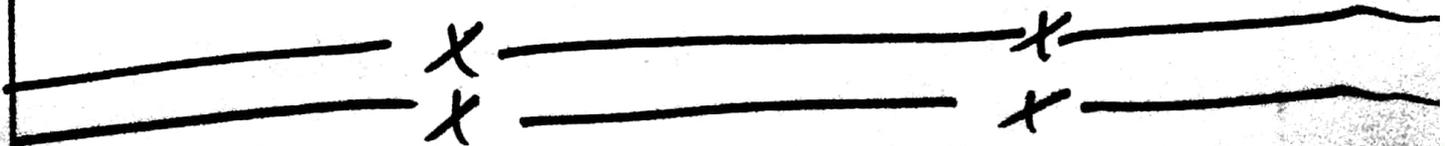
$$\begin{aligned} & \boxed{\text{R.W}} \\ & = \frac{-(2 \times 8) + (3 \times 20) - (8 \times 6)}{6} \\ & = \frac{-16 + 60 - 48}{6} \\ & = \frac{-4}{3} \\ & = -\frac{2}{3} \end{aligned}$$

As Area is Never in Negative

So we take the value positive

$$\boxed{A = 0.6}$$

Ans.



Name: Sa'jid Ahmad

Paragvone: BECE

ID: 12671

(10)

Subject: Calculus

Module: 16th Semester

Q#5

(A)

$$A = i - 2j - 2k$$

$$B = 6i + 3j + 2k$$

Sol:

$$A = i - 2j - 2k$$

$$|A| = \sqrt{1 + 4 + 4} = \text{~~3~~}$$

$$= \text{~~3~~} \sqrt{9} = 3$$

$$|A| = 3$$

$$B = 6i + 3j + 2k$$

$$|B| = \sqrt{36 + 9 + 4}$$

$$|B| = \sqrt{49}$$

$$|B| = 7$$

$$\theta = \cos^{-1} \left(\frac{A \cdot B}{|A| \cdot |B|} \right)$$

$$\theta = \text{~~30}~~$$

Name: Sajid Ahmad

Programme: BE (E)

ID: 12671

(11)

Subject: Calculus

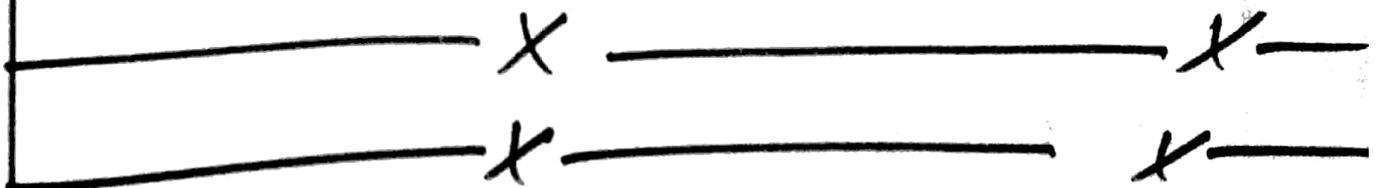
Module: 16th Semester

$$\theta = \cos^{-1} \left(\frac{(i-2j-2k) \cdot (6i+3j+2k)}{3 \times 7} \right)$$

$$\theta = \cos^{-1} \left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{21} \right)$$

$$\theta = \cos^{-1} \left(\frac{-4}{21} \right)$$

$$\theta = 100.9805^\circ \quad \text{Ans}$$



Name: Sajid Ahmad

Program: BECE

Subject: Calculus

Module: 10th Semester

ID: 12671

12

Q# 5

B

Change into a spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$

Sol:

$$x^2 + y^2 + (z-1)^2 = 1$$

$$(\rho \sin \phi \cos \theta)^2 + (\rho \sin \phi \sin \theta)^2$$

$$+ (\rho \cos \phi - 1)^2 = 1$$

$$\rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta$$

$$+ \rho^2 \cos^2 \phi + 1 - 2\rho \cos \phi = 1$$

$$\rho^2 \sin^2 \phi (\cos^2 \theta + \sin^2 \theta) + \rho^2 \cos^2 \phi + 1 - 2\rho \cos \phi = 1$$

$$\rho^2 \cos \phi = 1$$

$$\rho^2 (\sin^2 \phi) + \rho^2 \cos^2 \phi - 2\rho \cos \phi = 1 - 1$$

$$\rho^2 (\sin^2 \phi + \cos^2 \phi) - 2\rho \cos \phi = 0$$

$$\rho^2 = 2\rho \cos \phi \quad \boxed{\rho = 2 \cos \phi} \text{ Ans.}$$

Name: Sajid Ahmad

Program: BE(E)

Subject: Calculus

Module 10 the Spheroid

RD: 12671

13

$$r = \rho \sin \phi$$

$$x = r \cos \theta = \rho \sin \phi \cdot \cos \theta$$

$$y = r \sin \theta = \rho \sin \phi \sin \theta$$

$$z = \rho \cos \phi$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

Equation Relation spherical
Co-ordinate Equation.

