Name: Kashif Ahmed

Id: 14225

Subject: Basic Maths

Submitted To: Sir Raza Ahmed

Major Assignment

Q1 Transpose to make x the subject.

a. $y = 3x$

$x = y/3$

b. $y = 1/x$

$x = 1/y$

c. $y = 7x - 5$

$7x = y + 5$

$x = \frac{y+5}{7}$

d. $y = \frac{1}{2}x - 7$

$2y + 7 = x$

$x = 2y + 7.$

Q2 Solve the following

a. $\frac{1}{2}x + \frac{3}{2}(x-4) = 6$

$-21x + 12 = -6 - 3x$

Sol:-

$1/2\,x + \frac{3}{2}(x-4) = 6 \rightarrow eq\,i$

$1/2\,x + \frac{3}{2}x - \frac{12}{2} = 6$

$1/2\,x + \frac{3}{2}x - 6 = 6$

$\frac{1x + 3x}{2} = 12$

$\frac{4x}{2} = 12$

$2x = 12$

$x = 12/2$

$x = 6$

$-21x + 12 = -6 - 3x \rightarrow eq\,ii$

$-21x + 3x = -12 - 6$

$-18x = -18$

$x = 1$

So $x = 6, 1$

b. $4x + \frac{1}{2}(2x-4) = 18$

$-1 - 7m = -8m + 7$

Sol:-

$4x + 1/2\,(2x-4) = 18 \rightarrow eq\,i$

$4x + \frac{2x}{2} - 4/2 = 18$

$4x + 1x - 2 = 18$

$5x = 18 + 2$

$5x = 20$

$x = 4$

$-1 - 7m = -8m + 7 \rightarrow eq\,2$

$-1 - 7 = -8m + 7m$

$-8 = -1m$

$m = 8$

So $x = 4$, $m = 8$.

Q3

$2x + y = 2$

$3x + 7y = 14$

Sol:-

$2x + y = 2 \rightarrow i$

$3x + 7y = 14 \rightarrow ii$

Multiply 7 to eq i

$7(2x + y) = 7(2)$

$14x + 7y = 14 \rightarrow eq\ iii$

Subtract eq iii from ii

$$\begin{array}{r} 3x + 7y = 14 \\ \underset{-}{+} 14x \underset{-}{+} 7y = \underset{-}{+} 14 \\ \hline -11x = 0 \end{array}$$

$x = 0$

Put $x = 0$ in eq i

$2(0) + y = 2$

$y = 2$

$x + 5y = 15$
$-3x + 2y = 6.$

Sol:-

$x+5y = 15 \rightarrow$ eq i
$-3x+2y = 6 \rightarrow$ eq ii

Multiply 3 to eq i

$3x + 15y = 45 \rightarrow$ eq iii

Add eq ii & iii

$$\begin{array}{r} 3x + 15y = 45 \\ -3x + 2y = 6 \\ \hline 17y = 51 \end{array}$$

$y = 51/17$

$y = 3$

Put $y = 3$ in eq 1

$x + 5(3) = 15$

$x = 0.$

So $y = 3, x = 0.$

$-2x+4y=-16$

$y=-2$

Sol:-

$-2x+4y=-16 \rightarrow i$

$y=-2 \rightarrow ii$

Put $y=-2$ eq i

$-2x+4(-2)=-16$

$-2x-8=-16$

$-2x=-16+8$

$-2x=-8$

$x=4$

So $y=-2$, $x=4$.

Q4 If $A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$ Find $|A|$

Finding $|A|$

$A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix}$

$= (3 \times 3) - (-2 \times 1)$

$= 9 + 2$

$= 11.$

Q 3

Write each product as a single matrix.

i. $\begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}$

Sol

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ +1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} (3\times1)+(1\times0)+(-1\times1) & (3\times-1)+(1\times2)+(-1\times0) \\ (0\times1)+(-1\times0)+(2\times1) & (0\times-1)+(-1\times2)+(2\times0) \end{bmatrix}$$

$$\begin{bmatrix} 3+0-1 & -3+2+0 \\ 0+0+2 & 0-2+0 \end{bmatrix}$$

$$\begin{bmatrix} 3-1 & -3+2 \\ 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & -2 \end{bmatrix}$$

Q5

(ii) $\begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$

Sol:-

$$\begin{bmatrix} 3 & -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} (3\times1)+(-2\times2)+(2\times-2) \end{bmatrix}$$

$$\begin{bmatrix} 3-4-4 \end{bmatrix}$$

$$\begin{bmatrix} 3-8 \end{bmatrix}$$

$$\begin{bmatrix} -5 \end{bmatrix}$$

Q5 (iii)

$$\begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} (2\times-1)+(-2\times-1)+(-1\times-1) & (2\times-2)+(-2\times-1)+(-1\times-2) & (2\times5)+(-2\times3)+(-1\times4) \\ (1\times-1)+(1\times-1)+(-2\times-1) & (1\times-2)+(1\times-1)+(-2\times-2) & (1\times5)+(1\times3)+(-2\times4) \\ (1\times-1)+(0\times-1)+(-1\times-1) & (1\times-2)+(0\times-1)+(-1\times-2) & (1\times5)+(0\times3)+(-1\times4) \end{bmatrix}$$

$$\begin{bmatrix} -2+2+1 & -4+2+2 & 10-6-4 \\ -1-1+2 & -2-1+4 & 5+3-8 \\ -1+0+1 & -2+0+2 & 5+0-4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q5 (iv)

$$\begin{bmatrix} -1 & -2 & 5 \\ -1 & -1 & 3 \\ -1 & -2 & 4 \end{bmatrix}\begin{bmatrix} 2 & -2 & -1 \\ 1 & 1 & -2 \\ 1 & 0 & -1 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} (-1\times 2)+(-2\times 1)+(5\times 1) & (-1\times -2)+(-2\times 1)+(5\times 0) & (-1\times -1)+(-2\times -2)+(5\times -1) \\ (-1\times 2)+(-1\times 1)+(3\times 1) & (-1\times -2)+(-1\times 1)+(3\times 0) & (-1\times -1)+(-1\times -2)+(3\times -1) \\ (-1\times 2)+(-2\times 1)+(4\times 1) & (-1\times -2)+(-2\times 1)+(4\times 0) & (-1\times -1)+(-2\times -2)+(4\times -1) \end{bmatrix}$$

$$\begin{bmatrix} -2-2+5 & +2-2+0 & 1+4-5 \\ -2-1+3 & +2-1+0 & 1+2-3 \\ -2-2+4 & +2-2+0 & 1+4-4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Q6 If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

Find $A^2 + BC$

Sol:-

First A^2

$$A \times A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times 1) + (4\times 2) & (1\times 4) + (4\times 1) \\ (2\times 1) + (1\times 2) & (2\times 4) + (1\times 1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix}$$

Now BC

$$BC = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (-3\times 1) + (2\times 0) & (-3\times 0) + (2\times 2) \\ (4\times 1) + (0\times 0) & (4\times 0) + (0\times 2) \end{bmatrix}$$

$$BC = \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0+0 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$A^2 + BC$

$$= \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 9-3 & 8+4 \\ 4+4 & 9+0 \end{bmatrix}$$

$$BC = \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$$

Q7

Show if $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

a) $(A+B)(A+B) \neq A^2 + 2AB + B^2$

Sol:-

First $(A+B)(A+B)$

$$A+B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 2+0 \\ 0-1 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$(A+B)(A+B) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (0 \times 0) + (2 \times -1) & (0 \times 2) + (2 \times 3) \\ (-1 \times 0) + (3 \times -1) & (-1 \times 2) + (3 \times 3) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \ -2 & 0 \ +6 \\ 0 \ -3 & -2 \ +9 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix} \longrightarrow \text{eq } 1$$

Now $A^2 + 2AB + B^2$

$$A \times A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1 \times -1) + (2 \times 0) & (-1 \times 2) + (2 \times 1) \\ (0 \times -1) + (1 \times 0) & (0 \times 2) + (1 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \ +2 \\ 0 & 0 \ +1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(B) $(A+B)(A-B) \neq A^2 - B^2$

First $(A+B)(A-B)$

$$A+B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 2+0 \\ 0-1 & 1+2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

So $A-B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -1-1 & 2-0 \\ 0+1 & 1-2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix} \rightarrow eq\ 1$$

Now $A^2 - B^2$

$$A \times A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (-1\times -1)+(2\times 0) & (-1\times 2)+(2\times 1) \\ (0\times -1)+(1\times 0) & (0\times 2)+(1\times 1) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & -2+2 \\ +0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B \times B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times 1)+(0\times -1) & (1\times 0)+(0\times 2) \\ (-1\times 1)+(2\times -1) & (-1\times 0)+(2\times 2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ -1-2 & 0+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 0-0 \\ 0+3 & 1-4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix} \rightarrow e$$

From eq 1 & 2 it is clear

$(A+B)(A-B) \neq A^2 - B^2$

Q8 show that

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} (-1\times a)+(2\times b)+(3\times c) \\ (2\times a)+(1\times b)+(0\times c) \\ (3\times a)+(5\times b)+(-1\times c) \end{bmatrix}$$

$$= \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix} \longrightarrow \text{Hence proved.}$$