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Sec: "C"

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Dept: Civil Engineering (B.S)

Subject: Intro to Structural Dynamic & Earth quake Engg

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Question no 1

Solution:-

$K = \frac{3EI}{L^3}$

$K = \frac{3 \times 2900 \frac{K}{in^2} \times 150 in^4}{(10 \times 12 in)^2}$

$K = 7.55 K/in = 90625 lb/ft$

$m = \frac{7772 lb\, sec^2}{32.2 ft}$

$= 241.36 \frac{lb\, sec}{ft}$

$\omega_n = \sqrt{K/m} = \sqrt{96025/241.36} = 19.95$

$T_n = 2\pi/\omega_n = 2\pi/19.95 = 0.314\ sec$

Substituting the Corresponding value in eq-1

$90625\ddot{u} + 241.36\ddot{u} = 0$

Where "k" is in lb/ft and "m" is in lb sec/ft^2

General Solution to the EOM for undamped free vibration is

$U(t) = U(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$

$u(0) = 1/2 = 1/24 ft$ and $\dot{u}(0) = 0$

$u(t) = (1/48)^{*} \cos(19.95t) + 0$

$= (1/24)^{*} \cos(19.95t)$

Equivalent static for at any time "t" is

$f_s(t) = k \cdot u(t) = \frac{90625 * \cos(19.95t)}{24}$

$f_s(t) = 3776 \cos(19.95)$

Amplitude of dynamic displacement u_o for Undamped free vibration

$u_o \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/w_n\right)^2\right]} = \sqrt{((1/24)^2 + 0)} = 1/24 ft$

Amplitude of equivalent static force

$ku = 90625 * 1/24 =$ **3776 lb**

X ——— X

Question #2

Solution

E.O.M for damped free vibration is

$ku + c\dot{u} + m\ddot{u} = 0$

It is known from problem no 1

that $k = 90625$ l/ft and $m = 241.36$ lbsec²/ft

Now

$c = \zeta \times 2m\omega_n = 2 \times 241.36 \times 19.95 \times 0.013$

| c = ~~125.0794 lbsec/ft~~ 125.1934 |

By substituting value of k c and m in eq ①

$90625u + 125.1934\dot{u} + 241.36\ddot{u} = 0$

Solution to the E.O.M for damped free vibration

is $U(t) = e^{-\zeta\omega_n t}\left[U(0)\cos(\omega_D t) + \frac{1}{\omega_D}\{\dot{u}(0) + u(0)\}\sin(\omega_D t)\right]$

| $\omega_D = 19.95$ rad/sec |

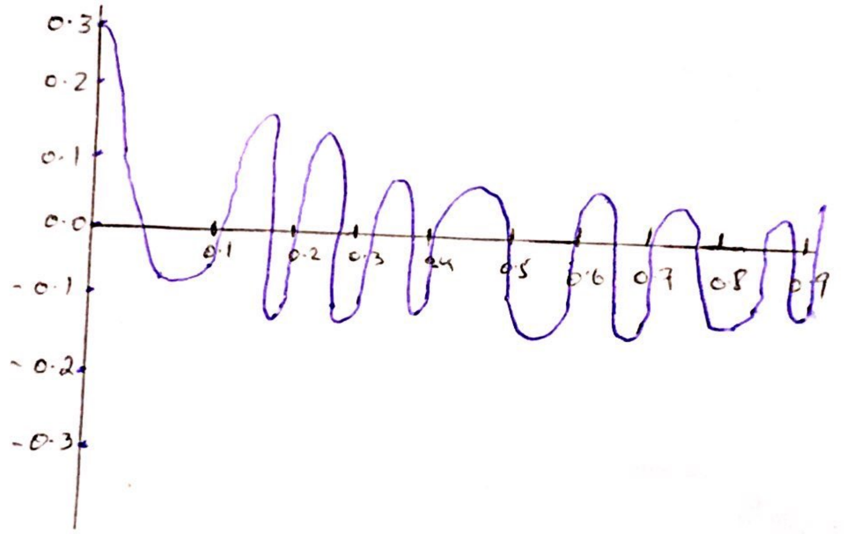
$e^{-0.013 \times 19.95t}\left(1/24 \times \cos(19.95t) + \frac{1}{19.95} \times [0 + 1/24 \times 0.013 \times 19.95 \times \sin 19.95t)\right]$

$$U(t) = e^{-0.2593}\left[0.0418\cos 19.95t) + 0.0005\,\sin(19.95t)\right]$$

$$f_s(t) = K \cdot U(t) = 90625 \times Ut$$

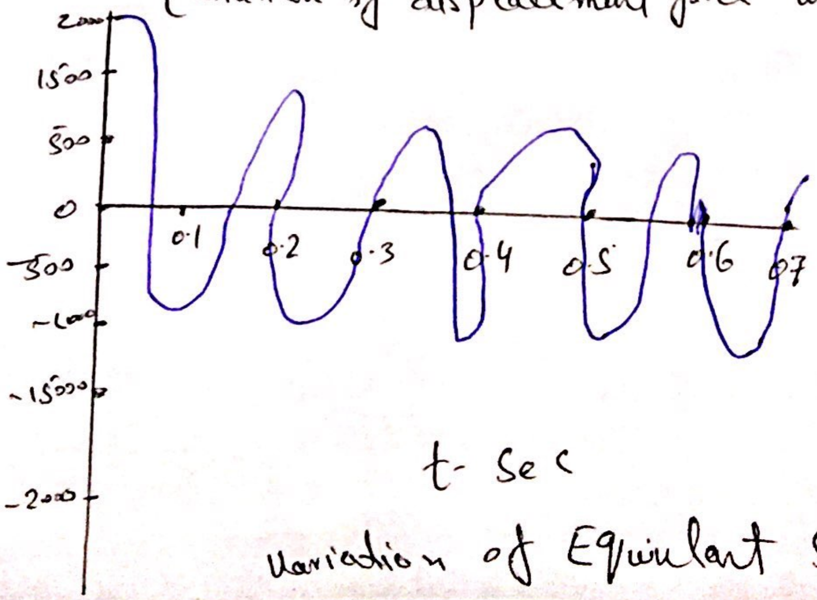
$$f_s(t) = e^{-0.2593}\left[3715.625 \times \cos(19.95) + 45.312 \times \sin(19.95t)\right]$$

Damped free Vibration



t-sec

(Variation of displacement force with time



t- sec

Variation of Equivalent Static force with time

Given Data

Force = 60 kips

Displacement of tank = $\left(\frac{ID}{1000}\right)$

$= \frac{7772}{1000} = 7.772$

time taken to Complete 7 cycles = 3.57 sec

Amplitude of displacement

= 2.286 cm

= 0.9"

Req:

1) Damping ratio ?

$\zeta = \frac{1}{2\pi j} \ln\left(\frac{U_i}{U_{i+1}}\right)$

$\zeta = \frac{1}{2\pi j} \ln\left(\frac{7.772}{0.9}\right)$

$= \zeta = \frac{2.15}{2\pi(7)}$

$\zeta = 0.0488 = 4.88\%$

2) Natural Period of Undamped Vibration, $T_n = ?$

$$T_D = \frac{3.57}{7}$$

$$T_D = 0.51 sec$$

Now

$$W_D = W_n\sqrt{(1-\xi^2)}$$

$$\frac{2\pi}{W_D} = \frac{2\pi}{(W_n\sqrt{1-\xi^2})}$$

$$T_D = \frac{T_n}{(1-\xi)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0489)^2}$$

$$\Rightarrow T_n = 0.5094 = 0.51 sec$$

$$\Rightarrow \boxed{T_n = 0.51 sec}$$

(3) Stiffness of Structure, $K = ?$

$$K = 60 \times \frac{\cos 60°}{2} = \boxed{18000 lb/ft}$$

(4) Weight of the tank, $W = ?$

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\left(\frac{w}{g}\right)}} = \sqrt{\frac{kg}{w}}$$

$$\Rightarrow \quad w_n^2 = \frac{K \times g}{W}$$

$$W = \frac{K \times g}{w_n^2}$$

Also, $W_n = \frac{2\pi}{T_n}$

$$W = \frac{K \times g}{\frac{4\pi^2}{T_n^2}}$$

$$W = K \times g \times \frac{T_n^2}{4\pi^2}$$

$$W = 18000 \times 32.2 \times \frac{(0.51)^2}{4\pi^2}$$

$$\boxed{W = 3818.64 \text{ lb} = 3.81 K}$$

(5) $c = ?$

It is known that $\xi = \frac{c}{2m\omega_n}$

$$c = \xi \times 2m\omega_n$$

$$c = \xi \times 2m(2\pi/T_n)$$

$$c = 0.0488 \times 2 \times \left(\frac{3818.64}{32.2}\right)\left(\frac{2\pi}{0.51}\right)$$

$$c = 142.59 \text{ lb}\cdot\text{sec/ft}$$

(6) No of Cycle to reduce displacement amplitude from 7.772" to 0.5", $J = ?$

$$J = \frac{1}{2\pi\xi} \ln\left(\frac{U_1}{U_{j+1}}\right)$$

$$J = \frac{1}{2\pi 0.0488} \ln\left(\frac{7.772''}{0.5''}\right)$$

$$J = 8.94 \text{ or } 9 \text{ cycles}$$

———— X ———— X ————