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QNO1 ⇒ find PQ where P is the point in three - dimensional space with Coordinates (4, 1, 3) & the point Q with coordinates (1, 2, 4). find the distance b/w P & Q, find the position vector of the point dividing PQ in the ratio 1:3.

Sol

Coordinate of $P = (4, 1, 3)$

$OP = 4i + 1j + 3k$

or $OQ = \vec{OQ} - \vec{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= 3i + 1j + 1k \longrightarrow ①$

Now distance between P & Q $= |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9+1+1}$

$= \sqrt{11} \longrightarrow ②$

Let M be the point which divided PQ in ratio 1:3, then by ratio theorem Position vector of $M = \vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= 13i + 5j + 13k \longrightarrow ③$$

Hence eq ①, ② & 3 are the required soln:

QNo2 ⇒ Evaluate

$$\int \frac{4x^3+10x+4}{2x^2+x}$$

Sol ⇒

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

$$\begin{array}{r|l} 2x^2+x & \begin{array}{l} \quad\ \ 2x-1 \\ \hline \cancel{4x^3}+10x+4 \\ \cancel{\pm 4x^3} \quad \pm 2x^2 \\ \hline -\cancel{2x^2}+10x+4 \\ \cancel{\mp 2x^2} \mp x \\ \hline 11x+4 \end{array} \end{array}$$

So

$$2x-1+\frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1 \int \frac{11x+4}{2x^2+x} \longrightarrow (1)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \longrightarrow ②$$

Now find

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow Ⓐ$$

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \longrightarrow Ⓐ$$

$$\frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$11x+4 = A(2x+1) + Bx \longrightarrow ③$$

Put $x = 0$ in ③

$$\boxed{4 = A}$$

Now put $x = -\frac{1}{2}$ in (3)

$$11\left(-\tfrac{1}{2}\right)+4 = B\left(-\tfrac{1}{2}\right)$$

$$\frac{-11}{2}+4 = \frac{-B}{2}$$

$$\frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$$

$$-3 = -B \Rightarrow \boxed{B = 3}$$

Putting the value of A & B in (A)

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking Integral on both sides.

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \frac{3}{2}\ln|2x+1|$$

Putting these value in (2)

$$= x^2 - x + 4\ln|x| + \frac{3}{2}\ln(2x+1)$$

Now put these value in (1)

$$\boxed{\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C}$$

QNo3 => Evaluate

$$a)\ \int_0^2 x^2 e^x\,dx$$

$$b)\ \int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$$A)\ \int_0^2 x^2 e^x\,dx$$

Now First Find Integration

$$= \int x^2 e^x\,dx$$

$$= x^2 \int e^x dx - \int\left(\int e^x dx \frac{d}{dx} x^2\right) dx$$

$$= \cancel{x^2 \int e^x dx - \int}$$

$$= x^2 e^x - \int e^x (2x)\,dx$$

$$= x^2 e^x - 2\int x\, e^x\,dx$$

$$= x^2 e^x - 2\left[x\int e^x dx - \int\left(\int e^x dx \frac{d}{dx} x\right) dx\right]$$

$= x^2e^x - 2\left[xe^x - \int e^x dx\right]$

$= x^2e^x - 2xe^x + 2e^x$

Now put limits.

$= \left| x^2e^x - 2xe^x + 2e^x \right|_0^2$

$= \left(2^2e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)\right)$

$= \left(4e^2 - 4e^2 + 2e^2 - 2\right)$

$= \boxed{2e^2 - 2 \quad Ans}$

QNO3 B) a)

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

First find Integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \longrightarrow (1)$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\boxed{2dy = \frac{1}{\sqrt{x}}\,dx}$$ put in (1)

$$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

put limits

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{2} - \cos 1)$$

$$\boxed{= -2\cos\sqrt{2} + 2\cos(1)}$$ Ans

QNo4 $\Rightarrow$ Verify that

$$U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Satisfies the three dimensional laplace's equation.

Sol $\Rightarrow$ The laplace eq in 3d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \longrightarrow (a)$$

So, $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = \frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial^2 u}{\partial x^2} = -x(x^2+y^2+z^2)^{-3/2}\Big]$$

$$\cancel{\frac{\partial^2 u}{\partial u^2} = 3x(x^2+y^2+z^2)^{-5/2}} \; (x^2+y^2$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(\frac{-3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 U}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \longrightarrow (1)$$

Now

$$\frac{\partial U}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$\frac{\partial U}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 U}{\partial y^2} = \left[-y\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 U}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2} \longrightarrow (2)$$

$$\frac{\partial U}{\partial z} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial U}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 U}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} \quad (x^2+y^2+z^2)^{-3/2} \longrightarrow (3)$$

Putting eq 1, 2, and 3 in eq (A)

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$+ 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2)+3z^2-(x^2+y^2+z^2)\right]$$

$$=(x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2}-\cancel{x^2}-\cancel{y^2}-z^2+3y^2-\cancel{x^2}-\cancel{y^2}-z^2+3z^2-\cancel{x^2}-\cancel{y^2}-z^2\right]$$

$$=(x^2+y^2+z)^{-5/2}\,(0)=0$$

So the given $U(x,y,z)$ is solution of laplace equation