NAME :- ASFANDYAR KHAN

ROLL # 7399.

SUBJ :- DIFFRENTIAL EQUATION

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QNo: 1

$f(t) = 1+t \quad -\pi \leq t \leq \pi$

Here we use formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \rightarrow eq\ 1$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left(t + \frac{t^2}{2} \right)_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi^2}{2} \right) \right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2} \right)$$

$$\boxed{a_0 = \frac{1}{2\pi} (2\pi + \pi^2)}$$

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$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)(\cos nt)\, dt$$

$$a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n} \Big|_{-\pi}^{\pi} - \int \left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left(\cancel{(1+t)\frac{\sin nt}{n}}^{\,0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\, dt$$

$$b_n = \frac{1}{\pi}\left(1+t\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt \cdot \frac{d}{dt}(1+t)\, dt\right)\right)$$

$$bn = \frac{1}{\pi}\left((1+t)(-\cos nt)\Big|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt\ (1)}{n}\right)\right)$$

$$bn = \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \left(\cancel{\frac{\sin n\pi}{n^2}}^{0}\Big|_{-\pi}^{\pi}\right)\right)$$

$$bn = \frac{-1}{n\pi}\left(((1+\pi)(\cos n\pi)) - ((1+(\pi)))\right)(\cos n\pi)$$

$$bn = \frac{-1}{n\pi}\left(\cos n\pi + \pi\cos n\pi - \cos n\pi + \pi\cos n\pi\right)$$

$$bn = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi}\cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$.

$$bn = \frac{2}{n}(-1)^{n+1}$$

So equation become

$$f(x) = \frac{1}{2\pi}(2\pi+\pi^2) + 0 + \sum_{n=1}^{\infty}\frac{2}{n}(-1)^{n+1}\sin t$$

QNo : 2

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen Values = ?

Step = 1

We have

$$(A - \lambda I)x = 0$$

A = given matrik

Step = 2

We have ; a characteristic equation is :

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 1-\lambda \end{vmatrix}$$

Step - 3

$$\lambda^3 - \left(\begin{array}{c}\text{Sum of} \\ \text{diagnol} \\ \text{element}\end{array}\right)\lambda^2 + \left(\begin{array}{c}\text{sum of} \\ \text{diagnol} \\ \text{minor}\end{array}\right)\lambda - |A| = 0 \rightarrow B$$

Sum of diagnol element $= 1+1+2 = 4$

Sum of diagnol minor $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B).

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \longrightarrow (C)$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

By putting values in eq (c)

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

Using quardratic formula.

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{-(4)^2 - 4(1)(3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2}$$

$$= \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$$

Solution

QNo : 3

Sol.
$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad R_4 \; R_2.$$

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 0 & 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1+4/5 \\ 0 & -1 & -6/5 & +4/5 \\ 0 & 2 & -1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 3/5 \\ -1 \end{bmatrix} \quad -\frac{1}{5} \times R_3$$

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 0 & -1 & 6/5 & 4/5 \\ 0 & 0 & 7/5 & 8/5 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 3/5 \\ 1/5 \end{bmatrix} \quad 5 \times R_3 \text{ and } 5 \times R_4$$

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 0 & -5 & 6 & 4 \\ 0 & 0 & 7 & 8 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 3 \\ 1 \end{bmatrix} \quad 5R_3 \text{ and } 5R_4$$

$$\left[\begin{array}{cccc} 1 & 0 & 4/5 & 2/5 \\ 1 & -1 & 2 & 1 \\ 0 & -5 & 6 & 4 \\ 0 & 0 & 7 & 8 \end{array}\right] \left[\begin{array}{c} 3/5 \\ 1 \\ 3 \\ 1 \end{array}\right] \quad \frac{1}{5} \times R_1$$

$$\left[\begin{array}{cccc} 1 & 0 & 4/5 & 2/5 \\ 0 & -1 & 6/5 & 1/5 \\ 0 & -5 & 6 & 4 \\ 0 & 0 & 7 & 8 \end{array}\right] \left[\begin{array}{c} 3/5 \\ 2/5 \\ 3 \\ 1 \end{array}\right] \quad R_2 \times 5$$

$$\left[\begin{array}{cccc} 1 & 0 & 4/5 & 2/5 \\ 0 & -5 & 6 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 7 & 8 \end{array}\right] \left[\begin{array}{c} 3/5 \\ 2 \\ 1 \\ 1 \end{array}\right] \quad R_3 - R_2$$

$$\left[\begin{array}{cccc} 1 & 0 & 4/5 & 2/5 \\ 0 & -5 & 6 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 7 & 8 \end{array}\right] \left[\begin{array}{c} 3/5 \\ 2 \\ 1 \\ 1 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ \frac{1}{7} \times R_3 \\ \frac{1}{3} \times R_4. \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 4/5 & 2/5 \\ 0 & -5 & 6 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} 3/5 \\ 2 \\ -4/7 \\ 1/3 \end{array}\right] \quad C_2 \times -5$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 \\ 0 & 1 & 6 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/5 \\ 26/21 \\ -11/21 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4/5 & 2/5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/5 \\ 31/21 \\ -11/21 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 1/2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/4 \\ 31/21 \\ -11/21 \\ 1/3 \end{bmatrix} \quad 5/4 \times R_1.$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 126/84 \\ 31/21 \\ -11/21 \\ 1/3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 \\ 31/21 \\ -11/21 \\ 1/3 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = \left(3/4,\ 31/21,\ -11/21,\ \frac{1}{3}\right)$$

$$x = \frac{3}{4}$$

$$y = \frac{31}{21}$$

$$z = -11/21$$

$$m = \frac{1}{3}$$

Q No : 4

$$\frac{d^2u}{dt^2} = c^2 \frac{d^2u}{dx^2}$$

$u(x,t) = \sin(x+2t)$ is solution of

$$\frac{d^2u}{dt^2} = c^2 \frac{d^2u}{dx^2}$$

it will satisfy the above equation

$$\frac{du}{dt} = \cos(x+2t) \cdot \frac{d}{dt}(x+2t)$$

$$\frac{du}{dt} = 2\cos(x+2t)$$

again

$$\frac{d^2u}{dt^2} = -2\sin(x+2t) \cdot \frac{d}{dt}(x+2t)$$

$$\Rightarrow \frac{d^2u}{dt^2} = -4\sin(x+2t) \rightarrow A$$

Now;

$$\frac{du}{dx} = \cos(x+2t)$$

$$\frac{d^2u}{dx^2} = -\sin(x+2t)$$

$$\Rightarrow \frac{d^2u}{dx^2} = -\sin(x+2t) \longrightarrow (B)$$

comparing (A)' & (B)

$$c = 2.$$

$$\Rightarrow -4\sin(x+2t) = -c^2 \sin(x+2t)$$

$$\Rightarrow -4\sin(x+2t) + c^2 \sin(x+2t) = 0$$

This is possible if $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$\Rightarrow 0 = 0.$$

Thus $u(x,t) = \sin(x+2t)$
is the solution of I-D wave equation

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