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Section : B

Q 1 Part A

Forces on immersed bodies:

A body Which is widly immersed in a homogeneous fluid may be subjected to two kind of forces using relative motion between body and fluid.

These forces are termed as drag and lift depending on weather force is parallel or at right angles to motion.

Drag forces on submerged body can have two components.

① **Pressure Drag F_P:** ⇒ It is equal to the integration of components in the direction of motion of all pressure forces ~~exil~~ exerted on surface of the body.

$$F_P = C_P \rho \frac{V^2}{2} A \quad C_P \text{ - depends on shape}$$

② **Friction Drag:-** It is equal to integration of components of shears stress along the surface of the body in direction of motion.

$$F_f = C_f \rho \frac{V^2}{2} BC \quad C_f \rightarrow \text{depends on viscosity}$$

Friction Drag of Boundary layer.

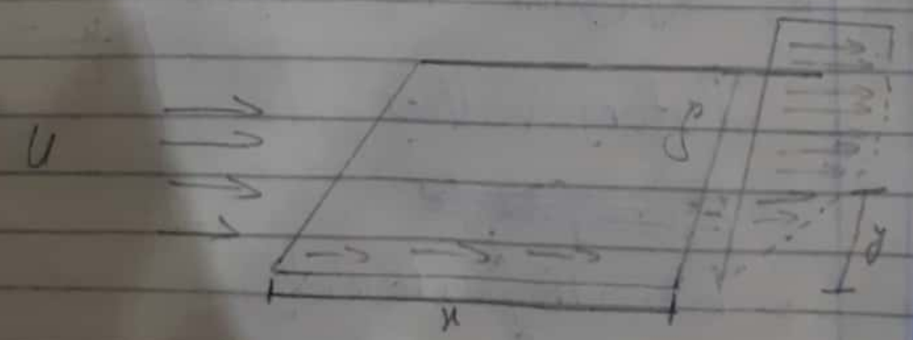
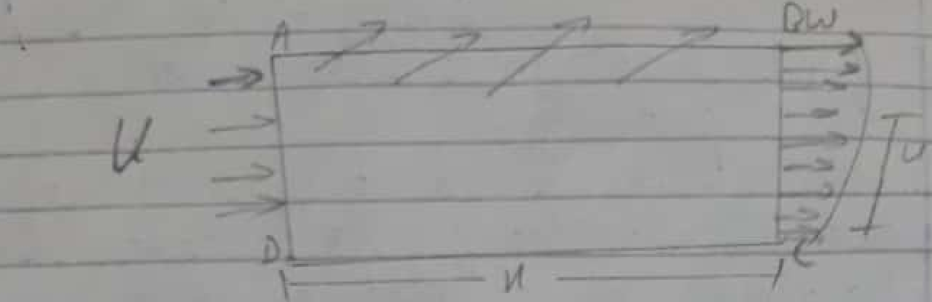


Fig shows growth of boundary layer along one side of smooth plate in steady flow of ~~inc~~ incompressible fluid consider a control volume



Where δ is distance from boundary layer to plate - U is undisturbed velocity

Now - F_x = - drag = rate of momentum in x-direction leaving through BC + rate of momentum in x direction leaving through AB - rate of momentum in x direction entering through DA.

According to impulse momentum principle.

$$\Sigma F = \frac{d(mv)}{dt} = \frac{(\rho \times vol) \times v}{dt} = \rho Q V$$

$$\Sigma F_x = \rho Q_2 V_2 - \rho Q_1 V_1$$

(A) $\rightarrow \rho U (U B \delta)$

(C) $\rightarrow \rho B \int_0^{\delta} u^2 dy$

(B) $\rightarrow \rho (U B \delta - B \int_0^{\delta} u\, dy) U$

$\frac{u}{U} = f\left(\frac{y}{\delta}\right)$

$\therefore \eta = y/\delta$

$\frac{u}{U} = f(\eta)$

Putting value

~~$F_x = \rho B U^2 \delta$~~

$$F_x = \rho B \int_0^{\delta} u(U-u)\, dy$$

Solving this,

$F_x = \int B u^2 \delta \alpha$ Where α is a function of boundary layer velocity distribution only.

Now to find local shear stress

$$\tau = \frac{F_x}{Area} \Rightarrow \tau_0 = \frac{dF_x}{B \cdot dx}$$

$$F_x = \int B u^2 \delta \alpha$$

$$\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$$

Laminar Boundary layer:. In Case of laminar flow.

$$\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0} = \frac{\mu}{\delta}\left(\frac{du}{dy}\right) = \frac{\mu u}{\delta}\left(\frac{d(u/U)}{d\eta}\right)_{\eta=0}$$

By solving

$$\tau_0 = \frac{\mu u \beta}{\delta} \quad \text{(i)}$$

Equating $\Rightarrow \tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$

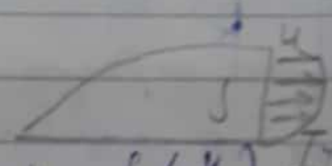
$$\delta d\delta = \frac{\mu \beta}{\rho u \alpha} dx$$

Solving it

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho u \alpha} x + C$$

At $x = 0$, $\delta = 0$, $C = 0$

$$\delta = \sqrt{\frac{2 \mu \beta x}{\rho u \alpha}} = \sqrt{\frac{2\beta}{\alpha}} \cdot \frac{x}{\sqrt{R_x}}$$



$$\frac{u}{U} = f\left(\frac{y}{\delta}\right)$$

$$\eta = \frac{y}{\delta}$$

$$\therefore \frac{u}{U} = f(\eta)$$

$$u = U f(\eta)$$

$$R_x = \frac{x U \rho}{\mu}$$

Experimently $\beta = 1.63$, $\alpha = 0.133$

Putting Value in eqn ① $\rightarrow \frac{\delta}{x} = \sqrt{\frac{2 \times 1.63}{0.133}} \times \frac{1}{\sqrt{R_x}}$

$$= \frac{4.91}{\sqrt{R_x}}$$

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

Where R_x may be called the local Reynold's number. It should be noted that R_x increases linearly in down stream direction.

Now,

$$F_f = B \int_0^L \tau_0 \, dx \Rightarrow$$

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

$$R_x = \frac{x U \rho}{\mu}$$

Thus, $F_f = 0.664 B \sqrt{\rho \mu L U^3}$

Where $F_f = C_f \, \rho \frac{V^2}{2} BL$

Equating both

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

Where R is based on characteristic length of whole plate. The laminar boundary layer will remain laminar if R_x is of about 50900.

P#05

Turbulent Boundary layer:.

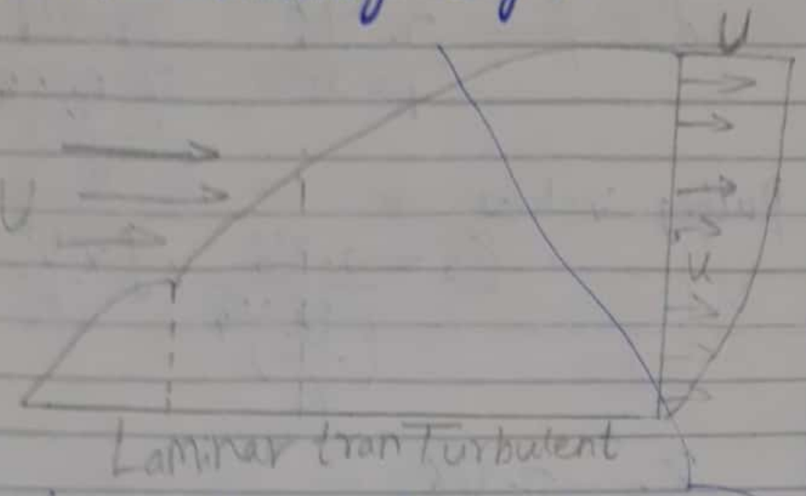


Fig shows the velocity distribution of boundary layer which is steeper near wall and flatty through out remainder of layer.

- The shear stress is greater in turbulent layer than laminar.

- Thus, $\tau_o = \frac{f \rho V^2}{8}$ where "V" is always velocity

To obtain relation b/w average and max velocity we have $\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{f}}$

Where $f = 0.028$ from Moody's chart

Thus,

$$U = 1.235V$$

Now, $\tau_o = \frac{f \rho V^2}{8}$

Where, $V = \frac{U}{1.235}$

and $f = \frac{0.316}{R^{0.25}} = \frac{0.316}{\left(\frac{DV}{\nu}\right)^{1/4}}$

Putting values.

$$\tau_o = \frac{0.316}{\left(\frac{DV}{\nu}\right)^{1/4}} \times \rho \times \frac{V^2}{8}$$

Where, $V = \frac{U}{1.235}$

& $D = 2\delta$

$$\tau_o = 0.316 \left[\frac{2\delta}{\nu} \times \left(\frac{u}{1.235}\right)\right]^{1/4} \times \frac{\rho}{8} \times \left(\frac{U}{1.235}\right)^2$$

$$\tau_o = \frac{0.023 \rho u^2}{\left(\frac{\delta u}{\nu}\right)^{1/4}} \quad \text{—} \ (i)$$

As we have general equation

$$\tau_o = \rho u^2 \alpha \frac{d\delta}{du} \quad \text{—} \ (2)$$

Equating (i) and (2) and integrating for boundary layer condition.

if $u = 0$, $\delta = 0$

$$\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Uu}\right)^{1/5} \times u$$

For, $\alpha = 0.0972$

$$\delta = \frac{0.377}{(R_u)^{1/5}} \times u \quad \text{--- (3)}$$

Putting (3) in equ (i)

$$\tau_o = 0.0587 \rho \frac{U^2}{2} \left(\frac{\nu}{U u}\right)^{1/5}$$

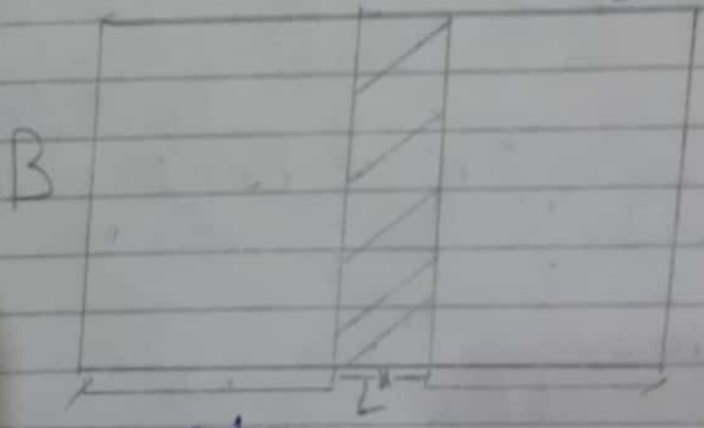
Now,

$$F_f = B \int_0^L \tau_o \, du$$

$$F_f = 0.0735 \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right)^{1/5} \times BL$$

As we have

$$F_f = C_f \rho \frac{U^2}{2} \times BL$$



Equating B/s

$$C_f = \frac{0.0735}{R^{1/5}} \quad \because (500{,}000 < R < 10^7)$$

For $R > 10^7$

$$C_f = \frac{0.455}{\log R^{2.58}}$$

Q 1
Part B Derivation:.

$$E = y + \frac{V^2}{2g}$$

Assuming "q" is discharge per unit width "b".
Thus, $q = \frac{Q}{b}$

Average velocity will be

$$V = Q/A = \frac{qb}{b \times y} = \frac{q}{y} \text{ or } q = V \times y$$

Now, $E = y + \frac{V^2}{2g} = y + \frac{q^2}{y^2 \times 2g}$

⇒ Lets consider how E will vary with y if 'q' remains constant.

Thus, $(E - y) = \frac{q^2}{2g \times y^2}$

$$\Rightarrow (E-y)y^2 = \frac{q^2}{2g} \text{ (constant)}$$

For particular "q" there will be 3 kind of values of "y" with two roots positive and one negative thus two values represent two y different situation as "slow and deep" and a fast and shallow. The point dividing the flow is critical point where energy is minimum and depth is critical depth.

→ The relation of critical depth is

$$E = y + \frac{1}{2g} \times \frac{q^2}{y^2}$$

For minimum specific energy, $\frac{dE}{dy} = 0$

Thus, $\frac{dE}{dy} = 1 - \frac{2q^2}{2g(y)^3} = 0$

Thus, $\frac{q^2}{g(y^3)} = 1 \Rightarrow q^2 = gy^3$ or $\left(\frac{q^2}{g}\right)^{1/3} = y_{cr}$

As,

$$q = V_{cr} y \quad \text{Thus}, \; q^2 = gy^3$$

$$\rightarrow (V_{cr} y)^2 = gy^3 \Rightarrow V_{cr}^2 = g\, y_{cr}$$

$$V_{cr} = \sqrt{g y_{cr}}$$

or. $y_{cr} = \frac{V_{cr}^2}{g}$

→ As E is minimum at critical point

Thus, $\frac{y_{cr}}{2} = \frac{V_{cr}^2}{2g}$

$$E_{min} = y_{cr} + \frac{V_{cr}^2}{2g} \rightarrow \frac{y_{cr}}{2} = \frac{3}{2} y_{cr}$$

$E_{min} = \frac{3}{2} y_{cr}$ or $y_{cr} = \frac{2}{3} E_{min}$

Subcritical	Critical	Supercritical
$y > y_{cr}$	$y = y_{cr}$	$y < y_{cr}$
$V < V_{cr}$	$V = V_{cr}$	$V > V_{cr}$

A#11

Q 2
Sol:- **Given data:-**

$$Q = 3.5\ m^3/s$$
$$S_o = 0.0008$$
$$n = 0.0219$$
$$b = 7886\ mm = 7.886\ m$$

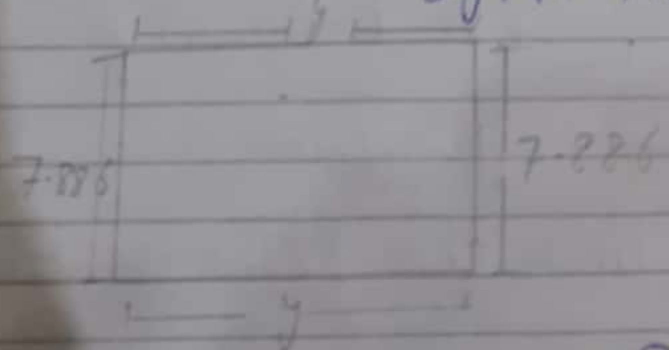
Required data:-

$$y = ?$$
$$y_{cr} = ?$$
$$V_{cr} = ?$$

$$Q = \frac{1}{n} \cdot A \cdot R_h^{2/3} \cdot S_o^{1/2} \quad —①$$

$$A = y \times b$$
$$= y \times 7.886$$

$$R_h = \frac{A}{P} = \frac{7.886y}{2y + 15.772}$$



Putting values in equ ①

$$3.5 = \frac{1}{0.0219} \times 7.886y \times \left(\frac{7.886y}{2y+15.772}\right)^{2/3} (0.0008)^{1/2}$$

P#12

$y = 0.7198$

$y = 719.8\,mm$

→ Now $y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$

$q = Q/b = \frac{3.5}{7.886} = 0.444\ m^2/s$

$y_{cr} = \left(\frac{0.444^2}{9.81}\right)^{1/3}$

$y_{cr} = 0.272\ m$

$y_{cr} = 272\,mm$

→ $V_{cr} = ?$

$V_{cr} = \sqrt{g y_{cr}}$

$V_{cr} = \sqrt{9.81 \times 0.272}$

$\boxed{V_{cr} = 1.63\,m/s}$

Since, $y > y_{cr}$

Thus, the flow is subcritial.

Q 3
Sol Given data:-

Wide, $B = 200\,mm$

Length, $L = 800\,mm$

Specific gravity $= 0.89$

Undistributed Velocity, $U = 5$

Kinetic Viscosity, $\nu = 0.93 \times 10^{-4}\,m^2/s$

Solution:-

As we know that

$$R = \frac{LU}{\nu}$$

$\nu = 0.93 \times 10^{-4}\,m^2/s$
$L = 0.80$
$U = 5$

putting these Value

$$R = \frac{0.80 \times 5}{0.93 \times 10^{-4}} = 43010 < 500{,}000$$

Thus, $$C_f = \frac{1.328}{\sqrt{R}}$$

$$= \frac{1.328}{\sqrt{43010}} = 0.0064$$

Now, $F_f = C_f \rho \frac{V^2}{2} \times BL$

P#14

$$F_f = 0.006 \times 6.89 \times 1000 \times \frac{(5)^2}{2} \times 0.20 \times 0.80$$

$$F_f = 53.4$$

$$\frac{\delta}{x} = \frac{491}{\sqrt{R_x}} \quad \text{at} \quad x = 1$$

$$\delta = \frac{491}{\sqrt{43010}} \times 80cm$$

$$\delta = 189\ cm$$

$$F_f = 0.664 \times B\sqrt{SU \cdot Lv^3}$$

$$= 0.664 \times 0.20 \sqrt{\frac{0.89 \times 1000 \times 1000 \times 0.89 \times 0.93 \times}{0.80 \times (5)^3}}$$

$$\boxed{F_f = 11.39N}$$

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