~~Alugging~~ Plugging in the constant gives

$$\Psi(s)\ \frac{50}{81} + \frac{5/9}{s^2} + \frac{31/81}{s-9} = \frac{2}{s-1}$$

By tasting the inverse transform the

$$y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{2t} - 2e^{t}$$

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Paper Differential equation

Q1 Part 1) The second defint order differentiale equation is written as when d(x) = 0 the equation is called homogeneous The other non other called non ~~homo~~ homogeneous To a nonhogenous equation we associde the so called associated homogeneous equation

Q1 b slove the following 2nd order linear homogenious non homogenous differention equation

1) $4y'' - 6y' + 7y = 0$

sol

First finding

$4\lambda^2 - 6\lambda + 7 = 0$

$\lambda = \frac{6 \pm \sqrt{(-6)^2 - 4(4)(7)}}{2(4)}$

$\lambda = \frac{6 \pm \sqrt{36-112}}{8}$ = $\lambda = \frac{6 \pm \sqrt{-76}}{8}$

Q4 Solve The following I IVP using laPlace Tranform

i) $y'' - 10y' + 9y = 5t$ $\qquad y(0) = -1 \quad y'(0) = 2$

First tastin from of every term

$\mathcal{L}\{y''\} \; 10\mathcal{L}\{y'\} + 9\mathcal{L}(y) - \mathcal{L}\{5t\}$

By formula

$s^2 Y_s - sy(0) - y'(0) - 10[sY_s - y(0)] + 9Y_s$

Now put initial auluation

$(s^2 - 10s + 9)(s) + s + 2 = 5/s^2$

solve for $y(s)$

$$y(s) = \frac{5}{s^2(s+4)(1+)} + \frac{19 - s}{(s-9)(s-1)}$$

$$y(s) = \frac{5 + 12s^2 - s^2}{s^2(s-9)(s-1)}$$

The Partial fraction of tranform will

$$(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{3-9} + \frac{D}{s-1}$$

$5 + 12s^2$ = for contants

$s = 0 \qquad 5 = 9B \qquad = B = 5/9$

$s = 1 \qquad 16 = -8D \qquad = D = -2$

$s = 4 \qquad 248 = 648C \qquad C = 34$

$s = 2 \qquad 45 = -14A + \frac{934}{8} \qquad A = 5$

Difine laPlace transformation along wi example find the laPlace if trantion of the given function

$f(t) = 6e^{-5t} + e^{3t} + 5t^{3} - 9$

sol $F(s) = 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$

$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s} - \frac{9}{s}$

Answer

$g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

sol

$g(s) = \frac{4s}{s^2(4)^2} - \frac{9 \cdot 4}{s^2+(4)^2} + \frac{2s}{s^2+(10)^2}$

$\frac{4s}{s^2+16} - \frac{36}{s^2+11} + \frac{2s}{s^2+100}$

Ans

iii $h(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

$H(s) = \frac{1}{s-3} + \frac{s}{s^2+(6)^2} - \frac{s-3}{s-3^2+(6)^2}$

$\frac{1}{s-3} + \frac{3}{s^2+36} - \frac{s-3}{(s-3)^2+36}$

$\lambda = \frac{6}{8} + \frac{2\sqrt{19}}{8} i \Rightarrow \lambda \frac{3}{4} \pm \frac{\sqrt{19}}{4} i$

$\lambda_1 = \frac{3}{4} + \frac{\sqrt{19}\, i}{4}$, $\lambda_2 = \frac{3}{4} - \frac{\sqrt{19}}{4} i$

so it has The complex conjugate roots

$a(x) = e^{\lambda_1 x} \cos \lambda'(x)$

$a_2(x) = e^{\lambda_2 x}$ is

$y = c_1 e^{\frac{3}{4}x} \cos \frac{2\sqrt{19}}{4}(x) + c e^{\frac{3x}{4}} \sin' \frac{\sqrt{19}}{4}(x_{c2}$

Ans

Q1 b)

ii)

$y'' - 4y' - 12y = 3e^{5x}$

Sol

→ The charactsistics equation and its roots

$y^2 - 4y - 12\ (y-6)(y+2) = 0$

$y' = -2$, $y_2 = 6$

The complentory soultion is Then

$y'' + 14y' + 49y = 0 \qquad y(-4) \ldots$

sol The d/c and its root

we

$y^2 + 14y + 49 = (y+7)^2 = 0 \quad y_1 = 7, \; y = 7$

The gen'al sols and its derivative are

$y(t) = c_1 e^{-7t} + c_2 t e^{-7t}$

$y't = -7c_1 e^{-7t} + c_2 e^{-7t} - 7c_2 t e^{-7}$

Putting in the initial condition

$-1 = y(-4) \qquad -4c_2 e^{28}$

$5 = y(-4) = -7c_1 e^{28} + c_2 e^{28} + 28 c_2 e^{28}$

$= -7c_1 e^{28} + 29 c_2 e^{28}$

It given the following con't by solving

$c_1 = -9e^{-28}$

$c_2 = -2e^{-28}$

The sol for IVP is

$y(t) = -9e^{28} e^{-7t} - 2te^{-28}$

$y(t) = 9e^{-7(t-4)} - 2te^{-7}(t+c)$

Ans

Q2 slove The ~~follw~~ folbwing IVP for The 2nd order liner eguation

i) $16y'' - 40y' + 25y = 0 \quad y(0) = 3,\ y'(0) = -9/4$

Sol

The characteristic eguation and root are given

$16y^2 - 40y + 25 = (4y^{(2)} - 5)^2 = 0 \quad y_1 = 5/4 \quad y_2 = 5/4$

The genral solution and its derivative are

$y(t) = c_1 e^{5t/4} + c_2 e^{5t/4}$

$y'(t) = 5/4\, c_1 e^{5t/4} + c_2 e^{5t/4} + 5/4\, c_2 t e^{5/4}$

Now Put it in The intial Position

$3 = y(0) = c_1$

$-9/4 = y'(0) = 5/4\, c_1 + c_2$

The solution for IVP is Then

$y^t = 3e^{5t/4}$

Ans

$y'' - 8y' + 17y = 0 \quad y(0) = -4 \quad y'(0) = 1$

sol.
The characteric eq and its roots are

$$y^2 - 8y + 17 = 0$$

$$y_1 = 4+i$$

$$y_2 = 4-i$$

The general sol as well as desrivtive is

$$y(t) = c_1 e^{4t} \cos(t) + c_2 e^{4t} \sin(t)$$

$$y'(t) = 4 e^{4t} \cos(t) - \cos^{4t} \cos(t)$$

$$y'(t) = 4c e^{4t} \cos(t) - c_1 e^{4t} \cos(t)$$

$$4 s_2 c^{4t} \sin(t) + c_2 e^{4t} \cos t$$

- By applying the initial codition gives

$$-y = y(0) \; c_1$$

$$-1 = y(0) = 4c_1 + c_2$$

so the sol is

$$y(t) = 4e^{4t} \cos(t) + 15e^{4} \sin(t)$$

Ans

Q2

iii)

~~Solve~~ The $y'' - 4y' + 9y = 0$ $y(0)$. $y'(0) = -8$

solv The charcteristic equation for this DE is

$$y^2 - 4y + 9 = 0$$

The $\sqrt{\ }$ of equation are

$$y_1 = 2 \pm \sqrt{5}$$
$$y_2 = 2 \pm \sqrt{5}$$

The ~~gen~~ general solv to the D.E

$$y(t) = C_1 2^{2t} \cos(\sqrt{5}t) + C_2 e^{2t} \sin(\sqrt{5}t$$

Applying intial contion day with derivative

$$y(t) = C_2 e^{2t} \sin(\sqrt{5}t$$
$$y'(t) = 2_1 2 e^{2t} \sin(\sqrt{5}t + \sqrt{5} c e^{2t} \cos \sqrt{5}$$

$$-8 = y'(0) = \sqrt{5}(2) = C_2 = 8/5$$

~~Solve it~~

$$y(t) = -8/5 \; e^{2t} \sin$$

Ans