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Paper :- Differential Equation (online)
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Q1 :- Part (a) :-

Estimate the general Solution of

$$y' = (x+2)y^2$$

Solution:-

$$y' = (x+2)y^2$$

$$\Rightarrow \frac{dy}{dx} = (x+2)y^2$$

$$\Rightarrow \int \frac{1}{y^2}\,dy = \int (x+2)\,dx$$

$$\Rightarrow \int y^{-2}\,dy = \int (x+2)\,dx$$

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$$\Rightarrow \int y^{-2}\,dy = \int (x+2)\,dx.$$

$$\Rightarrow \frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + C_1$$

$$\Rightarrow \frac{y^{-1}}{-1} = x^2/2 + 2x + C_1$$

$$\Rightarrow -\frac{1}{y} = \frac{x^2}{2} + 2x + C_1$$

Now Isolate y;

$$-1/y = \left(\frac{x^2}{2} + 2x + C\right)$$

So, $$y = -\left(\frac{2}{x^2+4x+2C_1}\right)$$

$$\boxed{y = -\left(\frac{2}{x^2+4x+C_1}\right)}$$ answer

Q1 Part (b) :- Estimate the general Solution.

$$y' = (y+9x)^2 \quad -(i)$$

Solution :- Let $y + 9x = u$

$$\Rightarrow \frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 9$$

So we get (i) as;

$$\frac{du}{dx} - 9 = u^2$$

$$\Rightarrow \frac{du}{dx} = u^2 + 9$$

$$\Rightarrow \int \frac{1}{u^2+9}\, du = \int dx$$

$$\Rightarrow \int \frac{1}{(u)^2+(3)^2}\, du = \int dx$$

$$\Rightarrow \frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) = x + c_1$$

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$$\Rightarrow \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) = x + c_1$$

$$\Rightarrow \tan^{-1}\left(\frac{u}{3}\right) = 3x + 3c_1$$

$$\Rightarrow \tan^{-1}\left(\frac{u}{3}\right) = 3x + c$$

$$\Rightarrow \left(\frac{u}{3}\right) = \tan(3x + c)$$

$$\Rightarrow u = 3\tan(3x + c)$$

Put $u = y + 9x$.

$$\Rightarrow y + 9x = 3\tan(3x + c)$$

$$\boxed{y = -9x + 3\tan(x + c)}$$ answer

Q2:- Estimate the general Solution.

$$x^3 dx + y^3 dy = 0$$

Solution :-

$$x^3 dx + y^3 dy = 0$$

Let $x^3 = M$, $y^3 = N$

So,

$$M dx + N dy = 0$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial (x^3)}{\partial y} , \quad \frac{\partial N}{\partial x} = \frac{\partial (y^3)}{\partial x}$$

$$\Rightarrow \frac{\partial M}{\partial y} = 0 , \quad \frac{\partial N}{\partial x} = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{So exact.}$$

Now,

$$u = \int M dx + K(y) \quad \text{— putting values in formula}$$

$$\Rightarrow u = \int x^3 dx + K(y)$$

$$u = \frac{x^4}{4} + K(y) \quad \text{— (i)}$$

Now,

$$\frac{\partial u}{\partial y} = 0 + \frac{d}{dy} k(y)$$

$$\Rightarrow \frac{\partial u}{\partial y} = \frac{d}{dy} k(y)$$

So $\frac{\partial u}{\partial y} = N \Rightarrow y^3$

$$\Rightarrow y^3 = \frac{d}{dy} k(y)$$

$$\Rightarrow \int d\,k(y) = \int y^3 dy$$

$k(y) = \frac{y^4}{4} + C_1 \rightarrow$ put in eq (i)

$$C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$\frac{x^4}{4} + \frac{y^4}{4} = C_2 - C_1$$

$\boxed{C = \frac{x^4}{4} + \frac{y^4}{4}}$ answer.

Q3 part a:- Find the general Solution

$$4y'' - 20y' + 25y = 0 \quad - (i)$$

Solution:-

2nd Order of homogeneous DE with Constant Coefficient.

i-e $\quad ay'' + by' + cy = 0$

So, $\quad y = e^{\lambda t} \quad - (ii)$

General Solution:-

$$y = c_1 e^{\lambda t} + c_2 t e^{\lambda t}$$

putting (i) in (ii) we get.

$$\Rightarrow 4((e^{\lambda t}))'' - 20(e^{\lambda t})' + 25(e^{\lambda t}) = 0$$

expand;

$$\Rightarrow 4\frac{d^2}{dt^2}(y) - 20\frac{d}{dt}(y) + 25(y) = 0 \rightarrow eq\ A$$

To put $y = e^{\lambda t}$ we get

$$\Rightarrow 4\frac{d^2}{dt^2}(e^{\lambda t})\ 20\frac{d}{dt}(e^{\lambda t}) + 25(e^{\lambda t}) = 0$$

$$\Rightarrow e^{\lambda t}(4\lambda^2 - 20\lambda + 25) = 0.$$

$\Rightarrow \quad e^{\lambda t}\left(4\lambda^2 - 20\lambda + 25\right) = 0$

$\Rightarrow \quad e^{\lambda t}\left(2\lambda - 5\right)^2 = 0 \quad \therefore \lambda = \frac{5}{2}$

$\Rightarrow$ For General Solution

$$c_1 e^{5/2 t} + c_2 t e^{5/2 t}$$

Refine:

$$\boxed{y = c_1 e^{5/2 t} + c_2 e^{\frac{5t}{2}} t}$$

Q3 Part (b) :- Estimate the General Solution

$$4y'' - 6y' - 7y = 0$$

Solution :-

$$4y'' - 6y' - 7y = 0$$

A Second order linear, homogeneous ODE has the form of

$$ay'' + by' + cy = 0$$

assume the solution of the form

$$\Rightarrow e^{\lambda t}$$

Rewrite the equation with $y = e^{\lambda t}$

$$4(e^{\lambda t})'' - 6(e^{\lambda t})' - 7(e^{\lambda t}) = 0$$

$$\Rightarrow 4\frac{d^2}{dx^2}(e^{\lambda x}) - 6\frac{d}{dx}(e^{\lambda x}) - 7e^{\lambda x} = 0 \quad \text{eq (i)}$$

$$\Rightarrow \frac{d^2}{dx^2}(e^{\lambda x}) = \lambda^2 e^{\lambda x} \quad \text{— (a)}$$

$$\Rightarrow \frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x} \quad \text{— (b)}$$

Put (a) & (b) in (i).

$$\Rightarrow 4\lambda^2 e^{\lambda x} - 6\lambda e^{\lambda x} - 7e^{\lambda x} = 0$$

$$\Rightarrow e^{\lambda x}(4\lambda^2 - 6\lambda - 7) = 0.$$

$$\Rightarrow \lambda_1 = 3/4 - \sqrt{37}/4$$

$$\Rightarrow \lambda_2 = 3/4 + \sqrt{37}/4$$

$$\lambda_1 = \frac{3 - \sqrt{37}}{4}$$

$$\lambda_2 = \frac{3 + \sqrt{37}}{4}$$

For two real roots $\lambda_1 \neq \lambda_2$

the general Sol take the form

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}.$$

$$c_1 e^{\frac{(3+\sqrt{37})x}{4}} + c_2 e^{\frac{(3-\sqrt{37})x}{4}}$$

Requie...

$$\boxed{y = c_1 e^{\frac{(3+\sqrt{37})x}{4}} + c_2 e^{\frac{(3-37)x}{4}}}$$