Final Term Assignment
Probability Methods in Engineering

Time Allowed: 6 hours ***Marks: 50***

Note: Attempt all questions. Copying from Internet and one another is strictly prohibited. Such answers will be marked zero.

Q1. A man throws two fair dice, what is the conditional probability that the sum of the two dice will be 7, given that

1. The sum is even
2. The sum is greater than 8
3. The two dice had the same outcome

Q2. Show that in a single throw of two dice, the probability of throwing more than 7 is equal to that of throwing less then 7, and hence find the probability of throwing exactly 7. State clearly what assumptions you are making.

Q3. : A and **B** play a game in which **A's** probability of winning is 2/3. In a series of 8 games, what is the probability that **A** will win

1. Exactly 4 games
2. At least 4 games
3. From 3 to 6 games

Q4. Let C1,C2,⋯,CMC1,C2,⋯,CM be a partition of the sample space SS, and AA and BB be two events. Suppose we know that

- A and B are conditionally independent given $\mathbf{C_i}$, for all i ∈{1,2,⋯,M}
- B is independent of all **C_i's.**

  Prove that A and B are independent.

Q5. Derive the binomial distribution and find its mean and variance.

Q6. Differentiate between Bi-nominal frequency distribution and Bi-nominal distribution with the help of formulas?

Q7. Below, you will find the mean and standard deviation of several data sets. You're interested in comparing each data set – however, each data set has a different mean, standard deviation and sample size. Find the coefficient of variation for each data set in the table below. Round to the nearest tenth.

Measure	**Data Set A**	**B**	**C**	**D**
Mean	45	60	50	25
SD	3	11	5	15
Sample Size	1 500	3 200	500	2 700

Student Name: Kiramat Ullah
ID: 13290
Semester: 8th
Department: BE(E)
Subject: Probability Methods In Engineering
Teacher : Sir Dawood

QUESTION NO 1

Answer

Q1 A man throws two fair dice what is the conditional probability that the sum of the two dice will be 7, given that
1) The sum is even
2) The sum is greater than 8
3) The two dice had the same outcome.

Solution:-

let A = { sum is seven }

B = { sum is even }

C = { sum is greater than 8 }

D = { 2 dice same outcome }

A = { (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) }

B = { (2,2), (2,6), (4,2), (6,2) ... }

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$C = \{(6,6), (6,4)\}$
$D = \{(1,1), (2,2), (4,4), (6,6)\}$

$P(A) = \frac{6}{36}$

$P(B) = \frac{4}{36}$

$P(C) = \frac{2}{36}$

$P(D) = \frac{4}{36}$

$P(A \cap B) = \phi/36 = \phi = 0$

$P(A \cap C) = \phi/36 = \phi = 0$

$P(A \cap D) = \frac{\phi}{36} = \phi = 0$

Hence $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{4/36} = 0$

$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{0}{2/36} = 0$

$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0}{4/36} = 0$

The result show that probability is zero in case given.

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QUESTION NO 2

Answer

Q2 Show that in a single Throw of two dice, the Probability of throwing more than 7 is equal to that of throwing less then 7, and hence find the probability of throwing exactly 7. state clearly what assumptions you are making.

Ans// When we are rolling two dice, there are 36 different combination Counting these up. There are 15 possibilities less than 7:

(1,1), (1,2), (1,3), (1,4), (1,5), (2,1) (2,2), (2,3), (2,4), (3,1), (3,2), (3,3) (4,1), (4,2), (5,1). The probability of getting less then 7 is

$$\frac{15}{36} = \frac{5}{12}$$

There are 6 possible combinations

of getting a 7 : (1, 6), (2, 5), (3, 4) (4, 3), (5, 2), (6, 1) which gives a Probability of

$$\frac{6}{36} = \frac{1}{6}$$

This means that 21 possibilities account for getting less than or equal to 7, so there are 15 remaining possibilities of getting more than 7. This is the same as the probability of getting less then 7. So the probability must be $\frac{5}{12}$ as well. In calculating this we must assume that each combination is equally likely to roll as any other and there fore the dice are fair, or else the calculations don't work.

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QUESTION NO 3

Answer

Q3 A and B play a game which A'S probability of winning is $2/3$. In a series of 8 games what is the probability that A will win.

1) Exactly 4 Games
2) At least 4 Games
3) From 3 to 6 Games

Solution:-

Given that $P = 2/3$ $n=8$

$$q = 1 - P$$

$$= 1 - 2/3$$

$$q = \frac{1}{3}$$

Let "x" denotes the number of games won by A, Then

i) $P(x=4)$

$$= \binom{8}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^4$$

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$= \frac{1120}{6561}$

$\boxed{= 0.1707}$

ii) $P(x \geq 4)$

$= 1 - P(x < 4)$

$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$

$= 1 - \left[\left(\frac{1}{3}\right)^{8} + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{7} + 28\left(\frac{2}{3}\right)^{2}\left(\frac{1}{3}\right)^{6} + \right.$

$\left. = 56\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right)^{5}\right]$

$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$

$= 1 - \frac{577}{6561}$

$= \frac{6561 - 577}{6561}$

$= \frac{5984}{6561}$

$\boxed{= 0.9121}$

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iii) $P(3 \le x \le 6)$

$$= \sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3}\left(\frac{2}{3}\right)^{3}\left(\frac{1}{3}\right) + \binom{8}{4}\left(\frac{2}{3}\right)^{4} + \binom{8}{5}\left(\frac{2}{3}\right)^{5}$$

$$= \left(\frac{1}{3}\right)^{3} + \binom{8}{6}\left(\frac{2}{3}\right)^{6}\left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^8}\left[56 + 140 + 224 + 224\right]$$

$$= \frac{8 \times 644}{6561} = \frac{5152}{6561} = \boxed{0.7852}$$

Ans

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QUESTION NO 4

Answer

Q4 Let C1, $C_2, \cdots,$ CMC1, C2, $\cdots$ CM be a partition of the sample space SS, and AA and BB be two events, Suppose we know that

1) A and B are conditionally independent given C, for all $i \in (1, 2, \cdots, M)$
2) B is independent of all C_i's

Prove that A and B are independent.

Ans// Proof:-

Sine the C_i's form a partition of the sample space, we can apply the law of total probability for $A \cap B$.

$$P(A \cap B) = \sum_{i=1}^{M} P(A \cap B / C_i) P(C_i)$$

$$P(A \cap B) = \sum_{i=1}^{M} P(A/C_i) P(B/C_i) P(C_i)$$

$\because$ (A and B are conditionally independent)

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$$P(A \cap B) = \sum_{i=1}^{M} P(A/C_i)\, P(B)\, P(C_i)$$

$\because$ (B is independent of all C_i's)

$$P(A \cap B) = P(B) \sum_{i=1}^{M} P(A/C_i)\, P(C_i)$$

$$P(A \cap B) = P(B)\, P(A)$$

$\because$ (Law of total probability)

Hence, A and B are Independent.

Ans

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QUESTION NO 5

Answer

Q5 Derive the binomial distribution and find its mean and variance.

Solution:-

Mean & variance of binomial random variable

The probability function for binomial random variable is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}$$

This is the probability at having x successes in a series of independent trials when the probability of success is any one of the trial is p. if x is a random variable with the probability distribution.

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

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Since $x=0$ term vanishes let
$y = x-1$ & $m = n-1$ subbing $n = y = 1$
& $n = m+1$ into the last sum
$y = x-2$ & $m = n-2$

$$E(x(x-1)) = \sum_{x=0}^{n} x(x-1)\binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=2}^{n} \frac{n!}{(n-2)!\,(n-x)!} p^x (1-p)^{n-x}$$

$$= n(n-1)p^2 \sum_{n=2}^{n} \frac{(n-2)!}{\overline{(n-2)}(n-x)!} p^{x-1} (1-p)^n$$

$$= n(n-1)\ p^2\ (p + (1-p))^m$$

$$\boxed{= n(n-1)p^2}$$

So the variance of x axis
$(x^2) - E(x^4) = E(x(x-1) + E(x)$
$E(x^2) = (n(n-1)\ p^2 + np - (np)$

$$E(x) = \sum_{y=0}^{n} \frac{(m+1)!}{y!\,(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)\ p \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y (1-p)^{m-y}$$

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$$np \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y (1-p)^{m-y}$$

By binomial theorem

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y}$$

Set $a = p$ & $b = 1-p$

$$\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)} p^y (1-p)^{m-y}$$

$$= (a+b)^m$$

$$= (p+1-p)^m$$

$$= 1$$

So that

$$\boxed{E(x) = np}$$

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Question No 6

Answers

Q6 Differentiate between Bi-nominal frequency distribution and Bi-nominal distribution with the help of formulas?

Ans/ a) **Bi-nominal Distribution:-**

A bi-nominal distribution can be though of as simply the probability of a success or Failure outcome in an experiment or survey that is repeated multiple times.

$$P(x-x)\ f(x) = {}^{n}C_{x}\ p^{x}\ q_{n}^{n-x}$$

b) **Bi-nominal frequency Distribution:-**

If the bi-nominal probability distribution is multiplied by N, the number of experiments or sets, the resulting distribution is known as the bi-nominal frequency Distribution.

$$N \binom{n}{x} p^{x}\ q_{n}^{n-x}$$

Ans///

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QUESTION NO 7

Answer

Q7

Ans **Solution:-**

- Cofficient of Variation:-

⇒ **For Data Set A:-**

$$CV = \frac{6}{\mu} \times 100$$

$$CV = \frac{3}{45} \times 100$$

$$\boxed{CV = 6.7}$$

⇒ **For Data Set B:-**

$$CV = \frac{11}{60} \times 100$$

$$\boxed{CV = 18.3}$$

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=> **For DatasetC :-**

$$CV = \frac{5}{50} \times 100$$

$$\boxed{CV = 10}$$

=> **For Dat set D :-**

$$CV = \frac{15}{25} \times 100$$

$$\boxed{CV = 60}$$

Ans

* The END *