Name Zia Ur Rehman

ID 11473

Course Digital Signal Processing:

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Q1 Determine the response $y(n)$, $n \geq 0$ of the system described by the second order difference equation

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$ and the initial conditions are $y(-1) = y(-2) = 0$

Sol: The characteristic equation

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2 \quad \text{Hence}$$

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

There partial solution

$$y_p(n) = k(-1)^n u(n)$$

Substituting the solution into the difference equation we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$$

$$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

for $n = 2$. $k(1+4+4) = 2 \Rightarrow k = 2/9$

The total solution is

$$y(n) = \left(c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right) u(n)$$

from the initial condition we obtain

$y(0) = 1$, $y(1) = 2$ then

$$c_1 + 1/4 = 1$$

$$c_1 = 7/4$$

$$2c_1 + 2c_2 = 3/4 = 2$$

$$c_2 = 3/3$$

Q 1 B Sol: The characteristic equation

$$x^2 - 0.7x + 0.1 = 0$$

$$x = 1/2 , 1/5$$

$$y^h(n) = c_1 \frac{1^x}{2} + c_2 1/5^x$$

with $x(n) = f(x)$ we have

$$y(0) = 2$$

$$y(1) = 0.7\,x(0) = 0 \Rightarrow (1) = 1.4$$

Hence $c_1 + c_2 = 2$ &

$$1/2\, c_1 + 1/5 = 1.4 = 7/5$$

$$= c_1 + 2/5\, c_2 = 14/5$$

The equation yield

$$c_1 = 10/3 , c_2 = -4/3$$

$$h(n) = \left[\frac{10}{3}(1/2)^n - 4/3(1/5)^n\right] u(n)$$

The step response as

$$s(n) = \sum_{n=0}^{n} h = (n-k).$$

$$= \frac{10}{3} \sum_{k=0}^{n} (1/2)^{n-k} - 4/3 \sum_{k=0}^{n} (1/5)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - 4/3 (1/5)^{n} \sum_{k=0}^{n} 5^{-k}$$

$$= \frac{10}{3} \left(\frac{1^{n}}{2} (2^{n+1} - 1) \right) u(n) - 1/3 \left(\frac{1}{5} 5^{n+1} - 1 \right) u(n)$$

" ——————————————— "

Q2 (a) Determine the Causal signal $x(n)$.

$$X(z) = 1/1 - 2z^{-1} (1 - z^{-1})^2$$

Sol:

$$X(z) = 1/(1 - 2z^{-1})(1 - z^{-1})^2$$

we know that

$$= \frac{A}{(1 - 2z^{-1})} + \frac{B}{(1 - z^{-1})} + \frac{C}{(1 - z^{-1})^2}$$

$$A = 4, \quad B = -3, \quad C = -1$$

Hence $x(n) = [4(2)^n - 3 - n]\, u(n)$

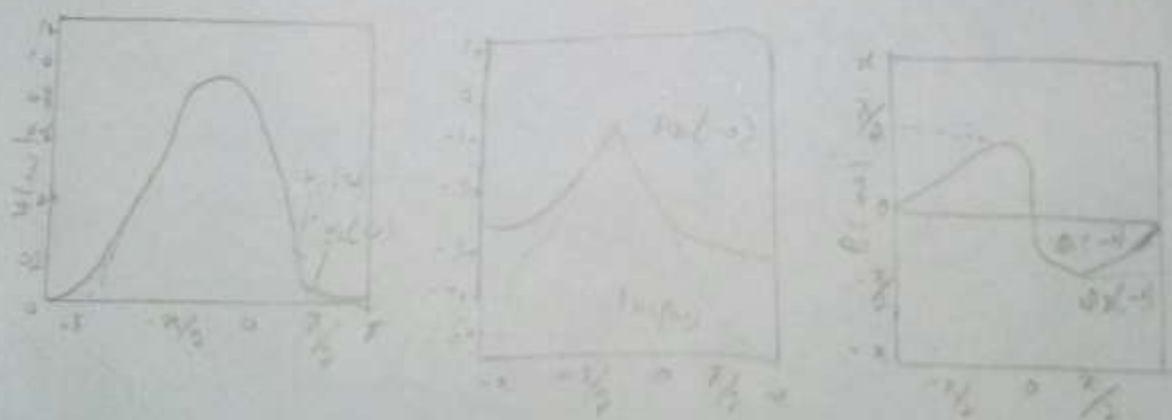
" ——————————————— "

Q3 (a) A two pole low pass filter has the system response.

$$H(z) = \frac{b_0}{(1 - pz^{-1})^2}$$

$$H(0) = 1 \quad \text{and} \quad |H(\pi/4)|^2 = \frac{1}{2}$$

Sol: A two pole filter



Now the value b_0 and ρ ⑥

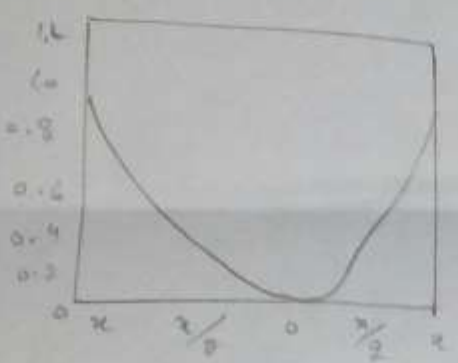
$H(0) = 1$

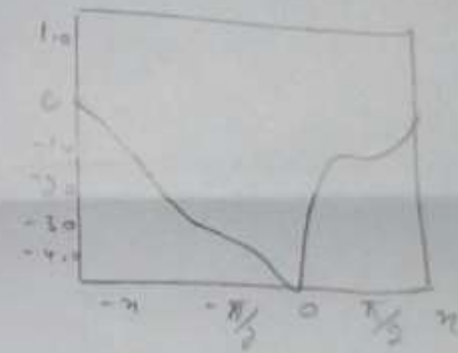
$H(\pi/4)^2 = 1/2$

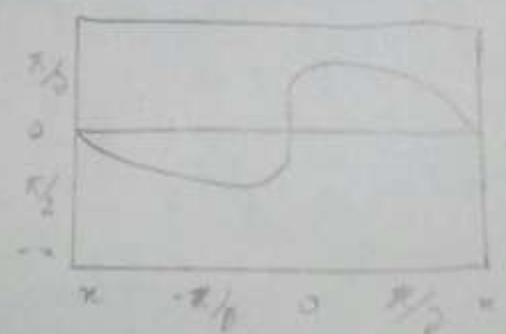
At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-\rho)^2} = 1$$

Hence $b_0 = (1-\rho)^2$







At $= \pi/4$

$$H(\pi/4) = \frac{(1-\rho)^2}{1-\rho e^{j\pi(4)2}}$$

$$= \frac{1-\rho^2}{1-\rho\cos(\pi/4) + j\rho\sin(\pi/4)^2}$$

$$= \frac{(1-\rho)^2}{(1-\rho\sqrt{2}+j\rho/\sqrt{2})^2}$$

Hence $\dfrac{(1-\rho)^4}{[(1-\rho/\sqrt{2})^2+\rho^2/2]^2} = \frac{1}{2}$

or Equivalently.

$$\sqrt{2}\,(1-\rho)^2 = 1+\rho^2-\sqrt{2}\,\rho$$

The value of $\rho = 0.32$ satisfies the equation. Consequently. The system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Q 3 (B) Solution ⑧

Given data

$\omega = \pi/2$

$\omega = 0$ and $\omega = \pi$

$1/\sqrt{2}$ at $\omega = 4\pi/9$

By the filter requirement

Poles $= P_{1,2} = r e^{\pm j\pi/2}$

Zeroes $= Z_{1,2} = \pm 1$

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2 - 1}{z^2 + r^2} \Rightarrow G = \frac{2}{-1+r^2}$$

$$G = \frac{1-r^2}{2}$$

To set r use the $H\ 4\pi/9 = 1/\sqrt{2}$ requirement.

Now
$$\left(H\left(\frac{4\pi}{9}\right)\right)^2 = \frac{(1-r^2)^2}{4} \frac{2 - 2\cos(8\pi/9)}{1 + r^4 + 2r^2\cos(8\pi/9)}$$

$$= 1/2$$

Evaluating gives $r^2 = 0.7$ therefore

$$H(z) = \cancel{0.5}\ 0.15 \frac{1 - z^2}{1 + 0.7 z^{-2}}$$

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