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Assignment # 03 Semester 4th

D.E

Aplication of Partial Differential Equation

=> Many engineering problem are governed by different type of PD.E and some of the most important type given below.

Toriconi equation:

$$y \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \begin{cases} y > 0 & \text{: elliptic} \\ y < 0 & \text{hyperbolic} \end{cases}$$

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replace equation (or) = $\partial = \backslash \phi \, \partial x_2 \, \partial_2 \theta \partial y$
$= \Delta_2 \theta = 0$

poissons equat.
$\partial = \theta \partial x^2 + \partial \theta \, 2y^2 = f(x,y)$

Helm holtz eq:
$\partial_2 \theta \partial x_2 + \partial_2 \theta \partial y^2 + C_2 \theta = 0$
plate bending

$\Delta_2 \quad \Delta_2 w = \Delta_4 w = VD$

weave equation = $\frac{\partial^2 u}{\partial t^2} - c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) = 0$

fourier equation $\partial T \partial t = a(\partial_2 T \partial x_2)$

Separable Differential Equation
for those equation which can be expresed in separable form as shown belows the solution can be obtained easily as

$\frac{dy}{dx} = f(x,y) \quad \frac{dy}{dy} = f(x) dx \quad \int \frac{dy}{dx} = \int f(x) dx + c$

$M(x,y)\,dx + N(x,y)\,dy = 0$ $M(x)\,dx = N(y)\,dy$

then

$\int M(x)\,dx = -\int N(y)\,dy + c$

(3) Separable differential Eq-

For equation which can be expressed in separable from is shown below the solution can be obtained easily as

Example: $\frac{dy}{dx} = x^3 + (y^2+1) \Rightarrow \frac{dy}{y^2+1} = x^3\,dx$

$\int \frac{dy}{y^2+1} = \int x^3\,dx + c \Rightarrow \tan^{-1} y = \frac{1}{4}x^4 + c$

$\Rightarrow y = \tan\left(\frac{1}{4}x^4 + c\right)$

Based on the boundary condition $c = 3$ hence $y^2 - 2y = x^3 + 2x^2 + 2x + 3$

This quadratic equation in y^2 can be solved with two solutions by the the quadratic equation as

$y = 1 \sqrt{x^3 + 2x^2 + 2x + y}$ and $y = 1 + \sqrt[4]{x^3 + 2x^2 + 2x + 4}$

Variation of parameter

for the following equation form it is possible to solve it by variation of parameter.

for $\frac{dy}{dx} = P(x)\, y + Q(x)$

put $y = c(x)\, e^{\int P(x)\, dx}$ by ~~differeting~~ differentiating it given

$\frac{dy}{dx} = \frac{dc(x)}{dx} = \int P(x)\, dx + \underline{c(x)P(x)\int P(x)dx}$

$P(x)\, y$

Substitue it To the original ODE

$\frac{dc(x)}{dx} = Q(x) = \int P(x)^{dx}$ compairing

the term it given

$C(x) = \int Q(x) e^{-\int P(x) dx} dx + c$

(5)

The Bernoulli Equation is an important equation type which can be solved in a similarly way by variation of parameter.

Considered the following form of equation

$$\frac{dy}{dx} = P(x)y + Q(x)y^n$$

step # 1 : put $z = y^{1-n}$

step # 2 Then $\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}$

$$\frac{dz}{dx} = (1-n)P(x)z + (1-n)Q(x)$$

The non linear ODE Now becomes linear ODE it can be solved by formula

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(3) Step #3 $n=-1$, z , y^2 Inverting z to get y

$$\frac{dy}{dx} = \frac{y}{2x} + \frac{x^2}{2y}$$

$$\frac{dz}{dx} = \frac{1}{x}z + x^2$$

$$z = c\int \frac{1}{x}dx \left(\int x^2 e^{-\int \frac{1}{x}dx} dx + c \right)$$

$$= cx + \frac{1}{2}x^3$$

Back Substitution of $z = y^2$ of $z = y^2$

$$y^2 = (x) + \frac{1}{2}x^3$$

Homogenous equation
for equation of the following type, where all the cofficient are constant it can be evaluted according to different

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Lal plack equation. Lalplace equation form in important governing condition for many types of problems some of the more common form are given by

There dimensional Laplace equation

$$U_{xx} + U_{yy} + U_{zz} = 0$$

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Two dimensional heat condoction

$a^2 (u_{xx} + u_{yy}) u_t$

Two dimensional seepage problem

$(K_x u_{xx} + K_y u_{yy}) = 0$

There are two major types of boundary condition to this problem.

Direct problem: Boundary Condition prescribed as it.

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