①

NAME: Maqsood Khan

ID: 7718

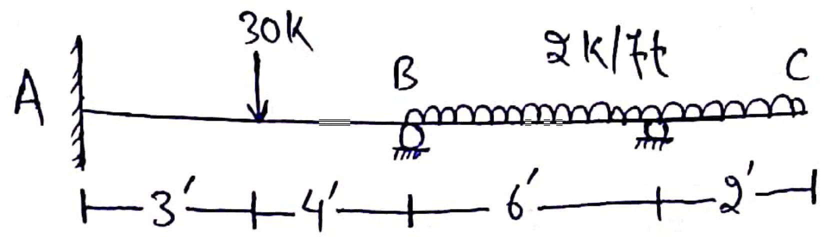
SECTION: B

SUBJECT: Structure Analysis - II

SUBMITTED: Engr. Adeel Khan

Date: 25/9/2020

Q1

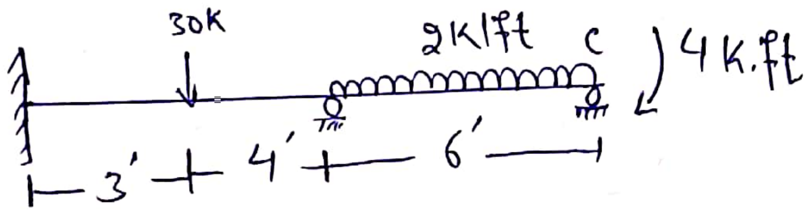


Sol.

Step: 1 Determining Kinematic Inderminancy

K.I = 5°

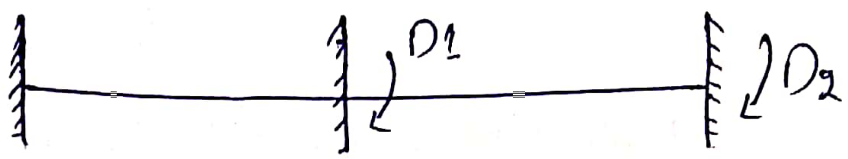
So we have to reduce the external portion



$$\Rightarrow \frac{2(2)}{1} = 4 \text{ k.ft}$$

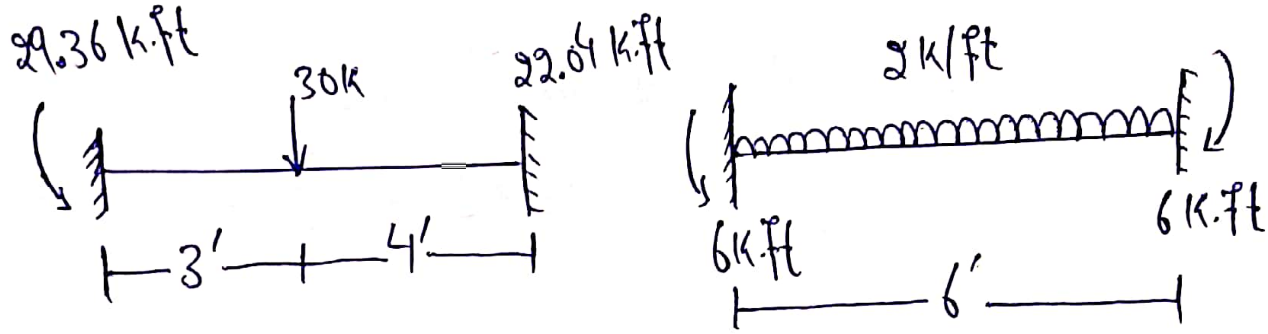
Now K.I = 2°

Step: 2 Determine unknown Joint Displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step: 3 Compute [ADL] Matrix



⟹ For Pointed Load (not at mid)

⟹ For left end:-

$$= \frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ K.ft}$$

$\Rightarrow$ For Right end:-

$$\Rightarrow \frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ K.ft}$$

$\Rightarrow$ For UDL

$$\frac{WL^2}{12} = \frac{(2)(6)^2}{12} = 6 \text{ K.ft}$$

$$ADL_1 = +22.04 - 6 = 16.04 \text{ K.ft}$$

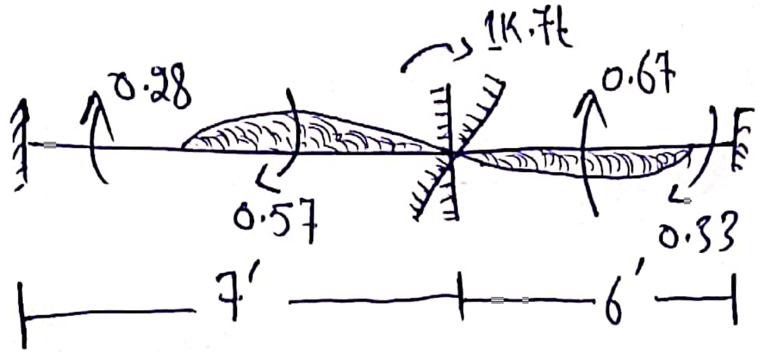
$$ADL_2 = 6 \text{ K.ft}$$

Step: 4 Compute [S] Matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

(a) $D_1 = 1K$ $D_2 = 0$

4



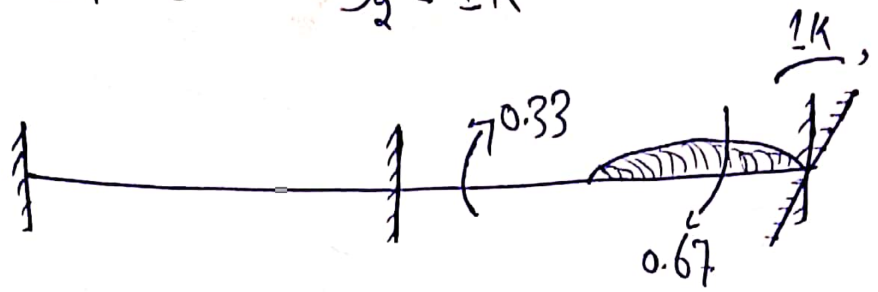
$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$$\delta_{11} = 0.57 + 0.67$$
$$= 1.24\,EA$$

$$\delta_{21} = 0.33\,EA$$

(b) $D_1 = 0 \qquad D_2 = 1K$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$$S_{12} = 0.33$$

$$S_{22} = 0.67$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step: 5 Compute [D] Matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj}A \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$Adj\ A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now

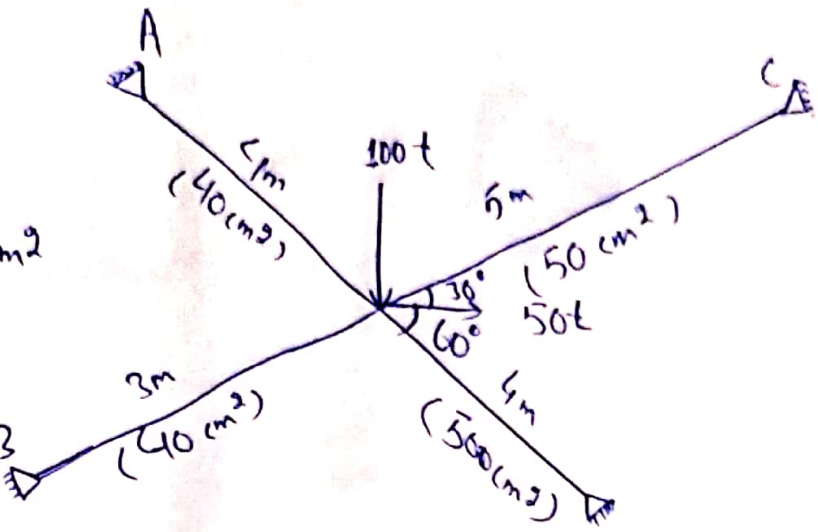
$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ 0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.97 \\ 3.8902 \end{bmatrix}$$

Q2.

$E = 2000\ t/cm^2$



Sol.

For A:

$$\sin 45^\circ = \frac{P}{H} = \frac{P}{4}$$

$$P = 2.828\ m$$

$$\Rightarrow \cos 45 = \frac{b}{H} = \cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828\ m$$

~~For~~ For B:

$$\sin 45^\circ = \frac{P}{H} \qquad \sin 45^\circ = \frac{P}{3}$$

$$\Rightarrow P = 2.12\ m$$

$$\cos 45^\circ = \frac{b}{H}$$

$\Rightarrow b = 2.12m$ (8)

For C

$$\sin 60 = \frac{P}{H} = \frac{P}{4}$$

$$(\sin 60)(4) = P$$

$$\Rightarrow P = 3.46$$

$$\cos 60 = \frac{b}{H} = \frac{b}{4}$$

$$\cos 60 \times 4 = b$$

$$\Rightarrow b = 2$$

For D

$$\sin 30^\circ = \frac{P}{5}$$

$$P = 2.5\ m$$

$$\cos 30 = \frac{b}{5} \Rightarrow b = 4.33m$$

Now

$$EA_{(A)} = 2000 \times 40 = 80{,}000t$$

$$EA_{(B)} = 2000 \times 40 = 80{,}000t$$

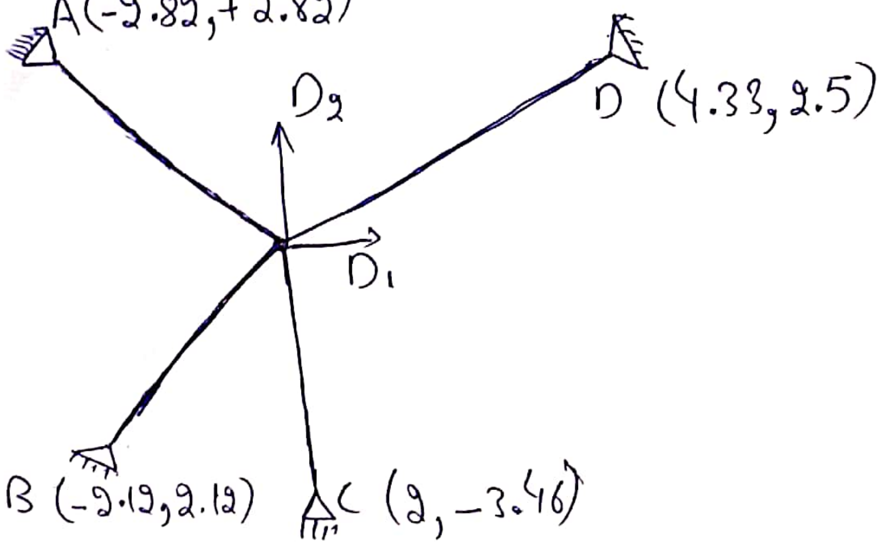
$$EA_{(C)} = 2000 \times 50 = 100{,}000t$$

$$EA_{(D)} = 2000 \times 50 = 100{,}000t$$

Step: 1 KI

$$KI = 2j - r$$
$$= 2(5) - 8$$
$$KI = 2^{\circ}$$

Step: 2 Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Scanned with CamScanner

Step: 3

$$[AMD]_{4\times 2} \quad [S]_{2\times 2}$$

(i) $D_1 = 1k$, $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80{,}000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80{,}000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{100{,}000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100{,}000}{(400)^2} \times (0-200) = -125$$

Now

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_k - X_j)^2$$

$$= \frac{80{,}000}{(400)^3}(282)^2 + \frac{80{,}000}{(300)^3} \times (212)^2 +$$

$$\frac{100{,}000}{(500)^3} \times (-433)^2 + \frac{100{,}000}{(400)^3} \times (-200)^2$$

(11)

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

$$\Rightarrow \quad S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (X_k - X_j)(Y_k - Y_j)$$

$$= \frac{80{,}000}{(400)^3}(282)(-282) + \frac{80{,}000}{(300)^3}(212)(212)$$

$$+ \frac{100{,}000}{(500)^3}(-433)(0-250) + \frac{100{,}000}{(400)^3}(-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

(ii) $D_1 = 0 \qquad D_2 = 1k$

$$AMD = \frac{EA}{L^2}(Y_k - Y_j)$$

$$AMD_{12} = \frac{80{,}000}{(400)^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{(300)^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{(500)^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{(400)^3}(346) = 216.25$$

Now, $S_{22} = \sum_{j=1}^{m} \frac{EA}{L^3} (Y_k - Y_j)^2$

$$= \frac{80{,}000}{(400)^3}(-282)^2 + \frac{80{,}000}{(300)^3}(212)^2$$

$$+ \frac{100{,}000}{(500)^3}(-250)^2 + \frac{100{,}000}{(400)^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

Step: 4 $[D] = [S]^{-1} \times [AD]$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

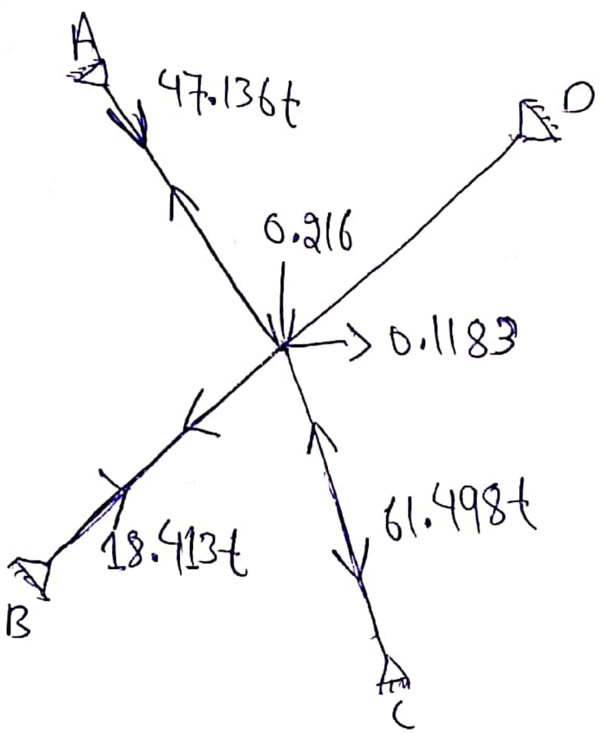
Step: 5 $[AM]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

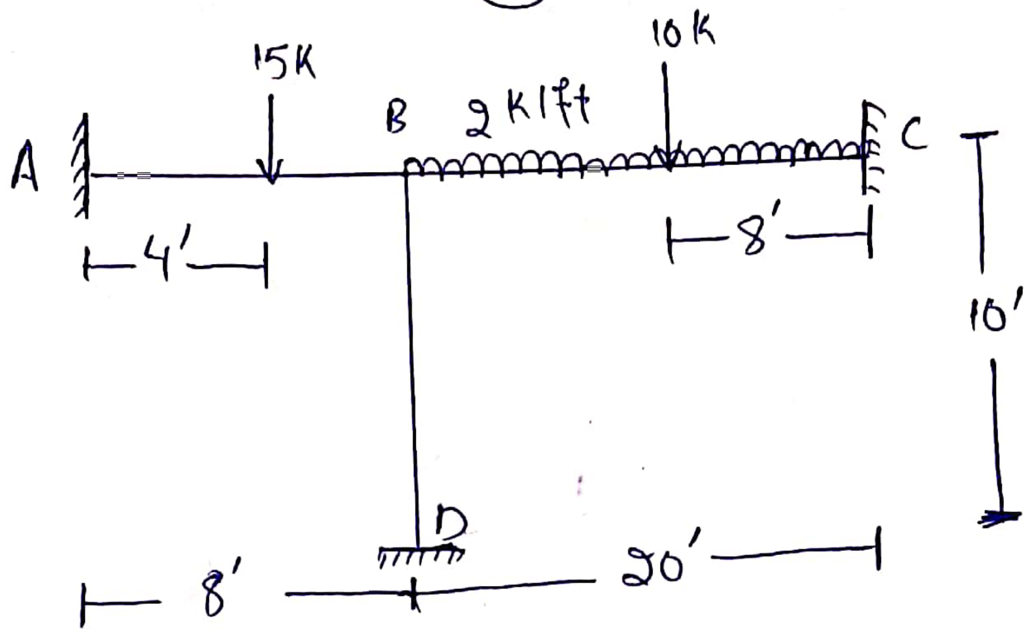
(13)

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + (188.44) \times (-0.216) \\ -173.2 \times 0.1183 + (-100)(-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



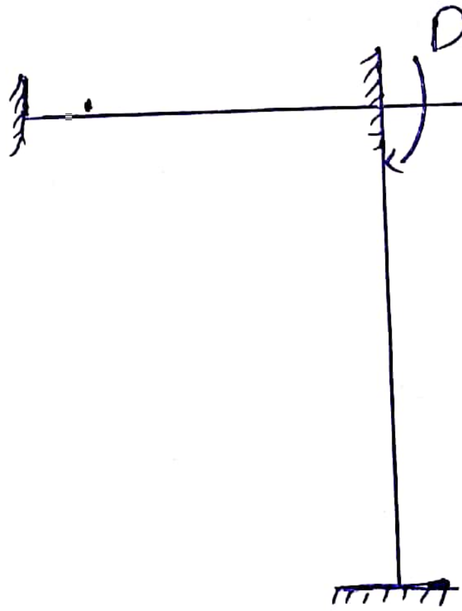
Q3



Sol: Step-1 Determine Kinematic Indeterminancy

K.I = 1°

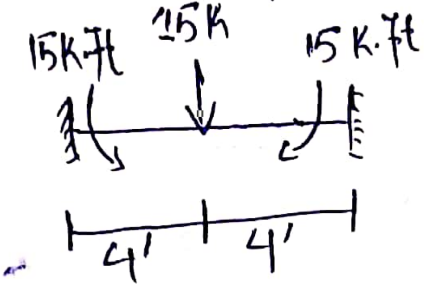
Step: 2 Determine Unknown Joint Displacement

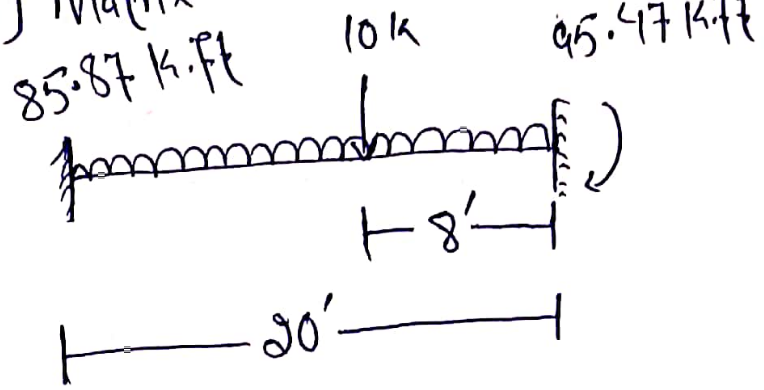


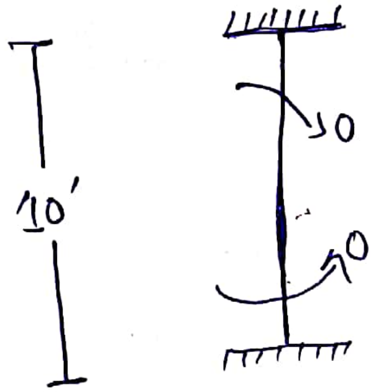
$[D] = [?]$

$[AD] = [0]$

Step: 3 Compute [ADL] Matrix







=> Point Load at center:

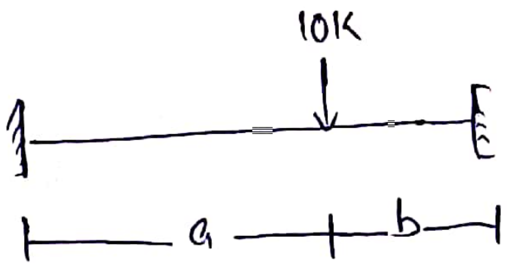
$$\frac{PL}{8} = \frac{(15)(8)}{8} = 15 \text{ Kip.ft}$$

=> Uniformly Distributed Load

$$\frac{WL^2}{12} = \frac{(2)(20)}{12} = 66.67 \text{ K.ft}$$

⇒ Point Load (Not at mid) :-

Suppose



⇒ for Left End

$$\frac{Pab^2}{L^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k ft}$$

⇒ for Right End

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k.ft}$$

So Total Moment at left end

$$19.2 + 66.67 = 85.87 \text{ k.ft}$$

Similarly at right End

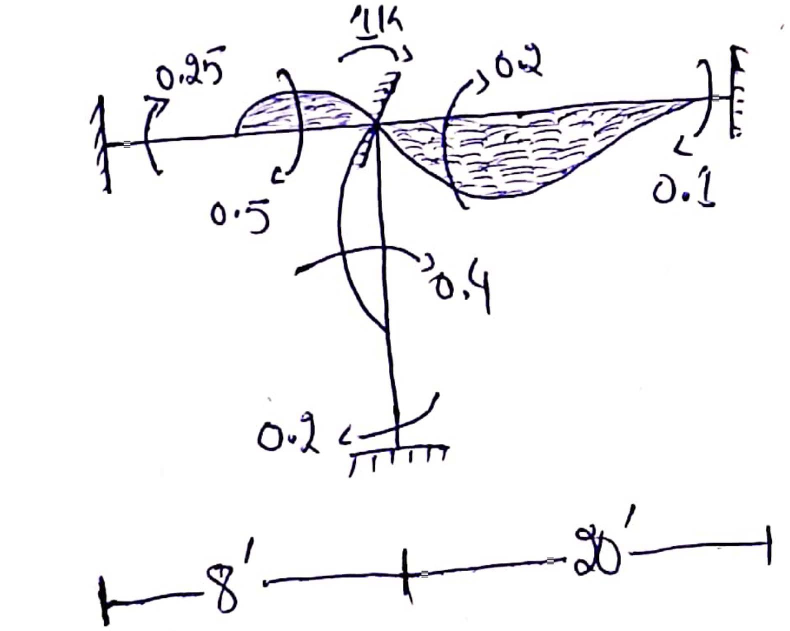
$$28.8 + 66.67 = 95.47 \text{ k.ft}$$

So $[AD] = -85.87 + 15 = -70.87$ K.ft

Step: 4 Determine $[S]$ Matrix

$$[S] = [S_{11}]$$

Now $D = 1K$



$\Rightarrow \dfrac{4EI}{8} = 0.5$ $\qquad \dfrac{2EI}{8} = 0.25$

$\Rightarrow \dfrac{4EI}{20} = 0.2$ $\qquad \dfrac{2EI}{20} = 0.1$

$\Rightarrow \dfrac{4EI}{10}$ $\qquad \dfrac{2EI}{10} = 0.2$

$$[S] = (0.5 + 0.4 + 0.2)EI$$

$$= 1.1 EI$$

$$[S] = 1.1 EI$$

Step : 5

Compute [D] Matrix

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42] \, 1/EI$$