IQRA NATIONAL UNIVERSITY

Complex and Multivariable Calculas

Final Term Assignment Summer 2020

Name: Kaleem Ullah

ID: 13170

Submitted to: Engr. Mujtaba Ihsan Sir

ID # 13170

⇒ Question No (1)

⇒ Part (A)

⇒ $A = x^2y^4z^3\, i - 3y^2z\, j + 4xz^2 K$

express $\nabla \times (\nabla \times A)$

(Solution):- $\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} K$

Now

$$\nabla \times A = \frac{\partial}{\partial x}(x^2y^4z^3)i - \frac{\partial}{\partial y}(3y^2z)j + \frac{\partial}{\partial z}(4xz^2)K$$

$$\boxed{\nabla \times A = 2xy^4z^3 i - 6yzj + 8xzK}$$

Now

$$\nabla \times (\nabla \times A) = \left(\frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} K\right) \times (2xy^4z^3 i - 6yzj + 8xzK)$$

$$\boxed{\nabla \times (\nabla \times A) = 2y^4z^3 i - 6zj + 8xK}$$

Answer.

⟷

Scanned with CamScanner

⇒ Question No (1)

⇒ Part (B)

⇒ Extend $\iint_1^4 (2x+6x^2y)\,dy\,dx$

(Solution):- First integrating w.r.t dx

$$\Rightarrow \int \left[\int_1^4 2x\,dx + \int_1^4 6x^2y\,dx \right] dy$$

$$\Rightarrow \int \left[2\int_1^4 x\,dx + 6y\int_1^4 x^2\,dx \right] dy$$

$$\Rightarrow \int \left[\frac{\cancel{2}x^2}{\cancel{2}}\Big|_1^4 + 6y\left(\frac{x^3}{3}\Big|_1^4\right) \right] dy$$

$$\Rightarrow \int \left((16-1) + \frac{6y}{3}(64-1) \right) dy$$

$$\Rightarrow \int (15+126y)\,dy$$

$$\Rightarrow \int 15\,dy + \int 126y\,dy$$

$$\Rightarrow \boxed{15y + 63y^2 + c(y)}$$

Answer.

⇒ Question No (2)

⇒ Express the equation of the plane passing through the point $(5,-2,4)$ that is perpendicular to the plane $3x+y-6z+8=0$.

(Solution):-

points $(5,-2,4)$

Plane $(3x+y-6z+8=0)$

$V = 3i + j - 6k$

$1(t) = (5,-2,4) + t(3,1,-6)$

$= (5+3t)i - (2-t)j + (4-6t)k$

$$\boxed{\begin{array}{l} x = 5+3t, \quad y = 2+t \\ z = 4-6t \end{array}}$$

Answer.

⟷

ID# 13170

⇒ Question NO(3)

⇒ Given $a = \langle 2, -1, 6 \rangle$ and $b = \langle -3, 5, 1 \rangle$ express $a \times b$

(Solution):- Given $a = \langle 2, -1, 6 \rangle$ and $b = \langle -3, 5, 1 \rangle$

$$= a \times b = (a_2 b_3 - a_3 b_2)i + (a_3 b_i - a_1 b_3)j + (a_1 b_2 - a_2 b_1)k$$

$$= \big((-1)(1) - (6)(5)\big)i + \big((6)(-3) - (2)(1)i + \big((2)(5) - (-1)(-3)\big)k$$

$$= (-1 - 30)i + (-18 - 2)i + (10 - 3)k$$

$$= -31i - 20j + 7k$$

$$= -31i - 20j + 7k = \langle -31, -20, +7 \rangle$$

⇒ Question No (4)

⇒ Estimate the angle between the plane $4x+2y-6z = 10$ and xz plane.

(Solution):-

xy plane (i.e)

$z=0 \Rightarrow 0x+0y+z=0$

we have

$n_1 = 4\hat{i}+2\hat{j}-6k$

$n_2 = 0\hat{i}+0\hat{j}+k$

we know that

$$\cos\theta = \frac{n_1 \cdot n_2}{|n_1||n_2|}$$

Angle between two planes

$$\cos\theta = \frac{(4\hat{i}+2\hat{j}-6k)\cdot(0\hat{i}+0\hat{j}+1k)}{\sqrt{(4)^2+(2)^2+(-6)^2}\sqrt{(0)^2+(0)^2+(1)^2}}$$

$$\Rightarrow \quad = \frac{(4\times 0)\, i\cdot i + (2\times 0)\, j\cdot j + (-6\times 1)\, k\cdot k}{\sqrt{16+4+36}\cdot\sqrt{0+0+1}}$$

$$= \frac{(0)(1) + (0)(1) + (-6)}{\sqrt{56}\cdot\sqrt{1}}$$

$$\cos\varphi = \frac{-6}{\sqrt{56}}$$

$$\varphi = \cos^{-1} - \frac{6}{\sqrt{56}}$$

$$\boxed{\varphi = 143.3^\circ}$$

Answer.

$\longleftrightarrow$

⇒ **Question No (5)**

⇒ Give all values of $\sin^{-1}\sqrt{5}$

Solution:- $\sin^{-1}\sqrt{5} = i\ln\left[\sqrt{5}i + \left(1-(\sqrt{5})^2\right)^{1/2}\right]$

$$\left(1-(\sqrt{5})^2\right)^{1/2} = (-4)^{1/2} = \pm 2i$$

$$\sin^{-1}\sqrt{5} = -i\ln\left[(\sqrt{5}\pm 2)i\right]$$

$$= -i\left[\log_e(\sqrt{5}\pm 2) + \left(\frac{\pi}{2}+2n\pi\right)i\right],$$

$$n = 0, \pm 1, \pm 2 \ldots\ldots$$

Nothing that

$$\log_e(\sqrt{5}-2) = \log_e \frac{1}{\sqrt{5}+2} = -\log_e(\sqrt{5}+2)$$

Thus for $n = 0, \pm 1, \pm 2, \ldots\ldots$

$$\sin^{-1}\sqrt{5} = \frac{\pi}{2} + 2n\pi \pm \log_e(\sqrt{5}+2)$$

Applying derivatives

if we define $w = \sin-1z$, $z = \sin w$, then

$\Rightarrow \frac{d}{dz} z = \frac{d}{dz} \sin w$ gives $\frac{dw}{dz} = \frac{1}{\cos w}$

using $\cos^2 w + \sin^2 w = 1$, $\cos w = (1-\sin^2 w)^{1/2}$

$= (1-z^2)^{1/2}$, thus

$$\Rightarrow \frac{d}{dz} \sin^{-1} z = \frac{1}{(1-z^2)^{1/2}}$$

$$\Rightarrow \frac{d}{dz} \cos^{-1} z = \frac{-1}{(1-z^2)^{1/2}}$$

$$\Rightarrow \frac{d}{dz} \tan^{-1} z = \frac{1}{1+z^2}$$

Thank You.