11743 Anees Sher ①

Q1 Part a

Determine the response $y(n)$, $n \geq 0$, of the System describe by The second order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

To the input $x(n) = (-1)^n u(n)$. And the intial Conditions are $y(-1) = y(-2) = 0$.

Sol.

The characteristic equation is.

$$\lambda^2 - 4\lambda + 4 = 0$$

$$\lambda = 2, 2.$$

$$y_h(n) = c_1 2^n + c_2 n 2^n.$$

The Paraticular Solution is:—

$$y_p(n) = k(-1)^n u(n).$$

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $k(1 + 4 + 4) \Rightarrow 2 \Rightarrow k = \frac{2}{9}$ Total solution is.

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

~~For~~ From the intial conditions, we obtain $y(0)$

$y(0) = 1$, $y = 2$ Then.

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2.$$

$$\Rightarrow c_2 = \frac{1}{3} //$$

Q1 Part b

Determine the impulse response & unit step response of the system described by the difference equation.

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

Sol:

The characteristics equation is.

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$ we have.

$$y(0) = 2,$$

$$y(1) = 0.7y(0) = 0 \Rightarrow (1) = 1.4,$$

Hence $c_1 + c_2 = 2$ &

$$\frac{1}{2}c_1 + \frac{1}{5} = 1.4 = \frac{7}{5}$$

$$\Rightarrow c_1 + \frac{2}{5}c_2 = \frac{14}{5}.$$

The equation yield.

$$c_1 = \frac{10}{3}, \; c_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$$

The step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}.$$

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$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}^n\left(2^{n+1}-1\right)u(n) - \frac{1}{3}\left(\frac{1}{8}\left(5^{n+1}-1\right)u(n)\right.\right.$$

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Q2 part a

Determine the causal signal $x(n)$ having the z-transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol:-

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4, \; B = -3, \; C = -1$

Hence, $x(n) = \{4(2)^n - 3 - n\} u(n)$.

Q2 b

Evaluate the inverse z-transform of.

$$X(z) = \frac{1}{1-az^{-1}} \quad , |z| > |a|$$

Sol:

using the complex inversion integral.

Sol: we have

$$x(n) = \frac{1}{2\pi j}\oint_{C} \frac{z^{n-1}}{1-az^{-1}}\, dz = \frac{1}{2\pi j}\oint_{C} \frac{z^{n}\, dz}{z-a}$$

C is a circle at radius greater than |a|. we shall evaluate this integral using with $f(z) = z^n$.

① $x(n) = f(z_0) = a^n \quad n \geq 0$

② If $n<0$ $f(z) = z^n$ has an n^{th}-order pole at $z=0$.

$$x(-1) = \frac{1}{2\pi j}\oint_{C} \frac{1}{z(z-a)}\, dz = \frac{d}{dz} = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a}$$

If $n=-2$, we have.

$$x(-2) = \frac{1}{2\pi j}\oint_{C} \frac{1}{z^2(z-a)}\, dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=0} = 0$$

In the same way we can show that $x(n) = 0$ for $n<0$.

$x(n) = a^n u(n)$ //

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Q3 Part A

A two - pole low pass filter has the system response

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

Determine the values of b_0 and p such that the frequency response $H(\omega)$ satisfies the condition

$$H(0) = 1 = |H(\tfrac{\pi}{4})|^2 = \tfrac{1}{2}$$

Sol.

At $\omega = 0$ we have.

$$H(0) \Rightarrow \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$

At $\omega = \pi/4$

$$H(\tfrac{\pi}{4}) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{1-p/\sqrt{2}+jp/\sqrt{2})^2}$$

$$\frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]^2} = \frac{1}{2}$$

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $p = 0.32$

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Q3 part b

Design a two - pole band pass filter-------
response $1/\sqrt{2}$ at $\omega = 4\pi/p$.

Sols. The filter must have poles at.
$$P_{1/2} = re^{\pm j\pi/2}.$$

zeros at $z = 1$ & $z = -1$. The system function
$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$
$$= G \frac{z^2 - 1}{z^2 + r^2}$$

evaluating the frequency response $H(\omega)$ of the filter at $(\omega) = \pi/2$.
$$H\left(\frac{\pi}{2}\right) = G \frac{2}{1-r^2} = 1$$
$$G = \frac{1-r^2}{2}.$$

The value of r is determine by evaluating $H(\omega)$ at $\omega = 4\pi/9$
$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4} \frac{2 - 2\cos(8\pi/p)}{1 + r^4 + 2r^2\cos(8\pi/p)} = \frac{1}{2}.$$
$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$
The value of $r^2 = 0.7$ satisfies this equation
$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7z^{-2}}.$$

Q4 a

A finite duration sequence of length L is given:

$$x(n) = \begin{cases} 1, & \leq n \leq L-1 \\ 0, & \text{otherwise.} \end{cases}$$

Determine the N-Point DFT of this sequence for $N \geq L$.

Sol..

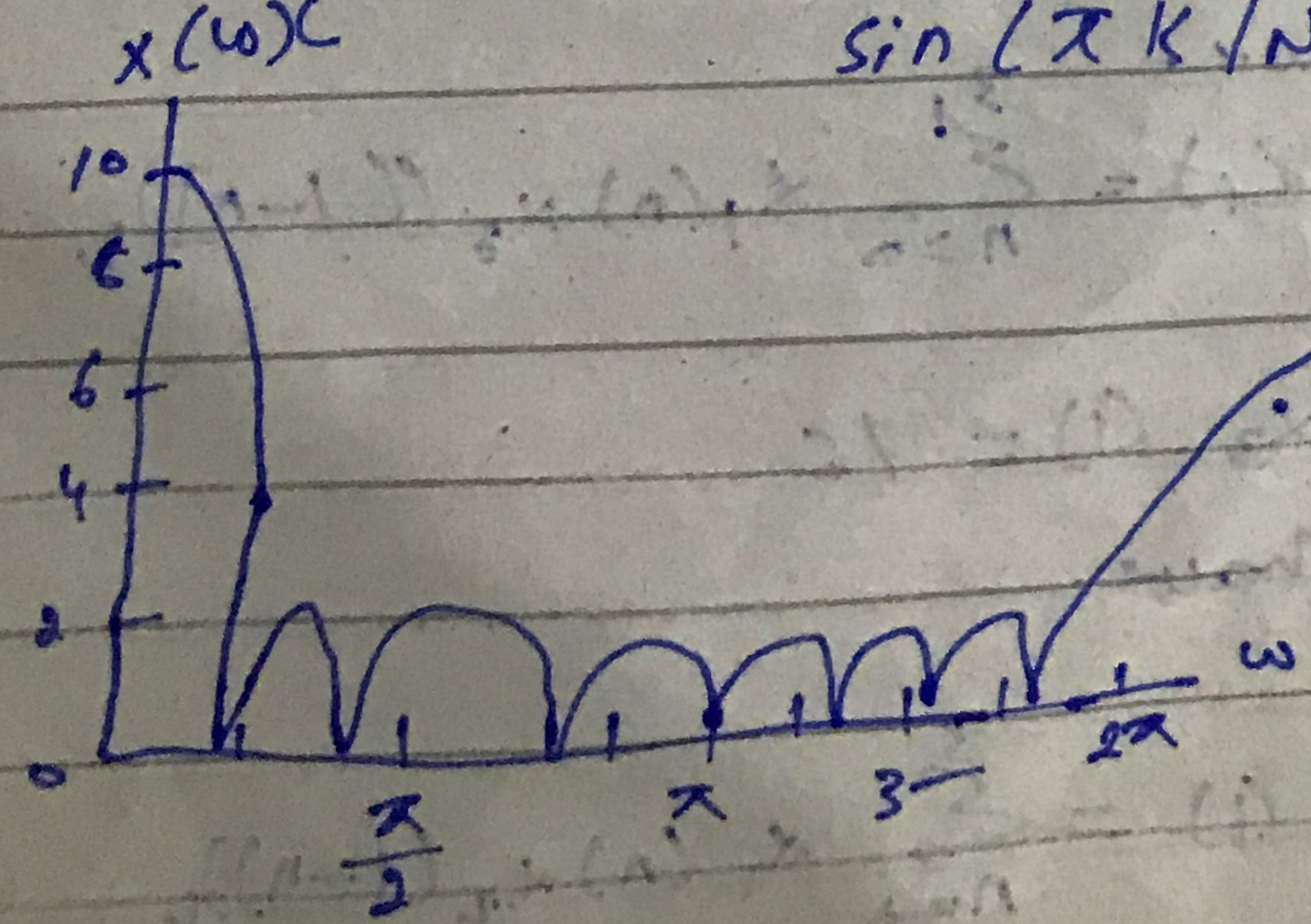
$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega x}$$

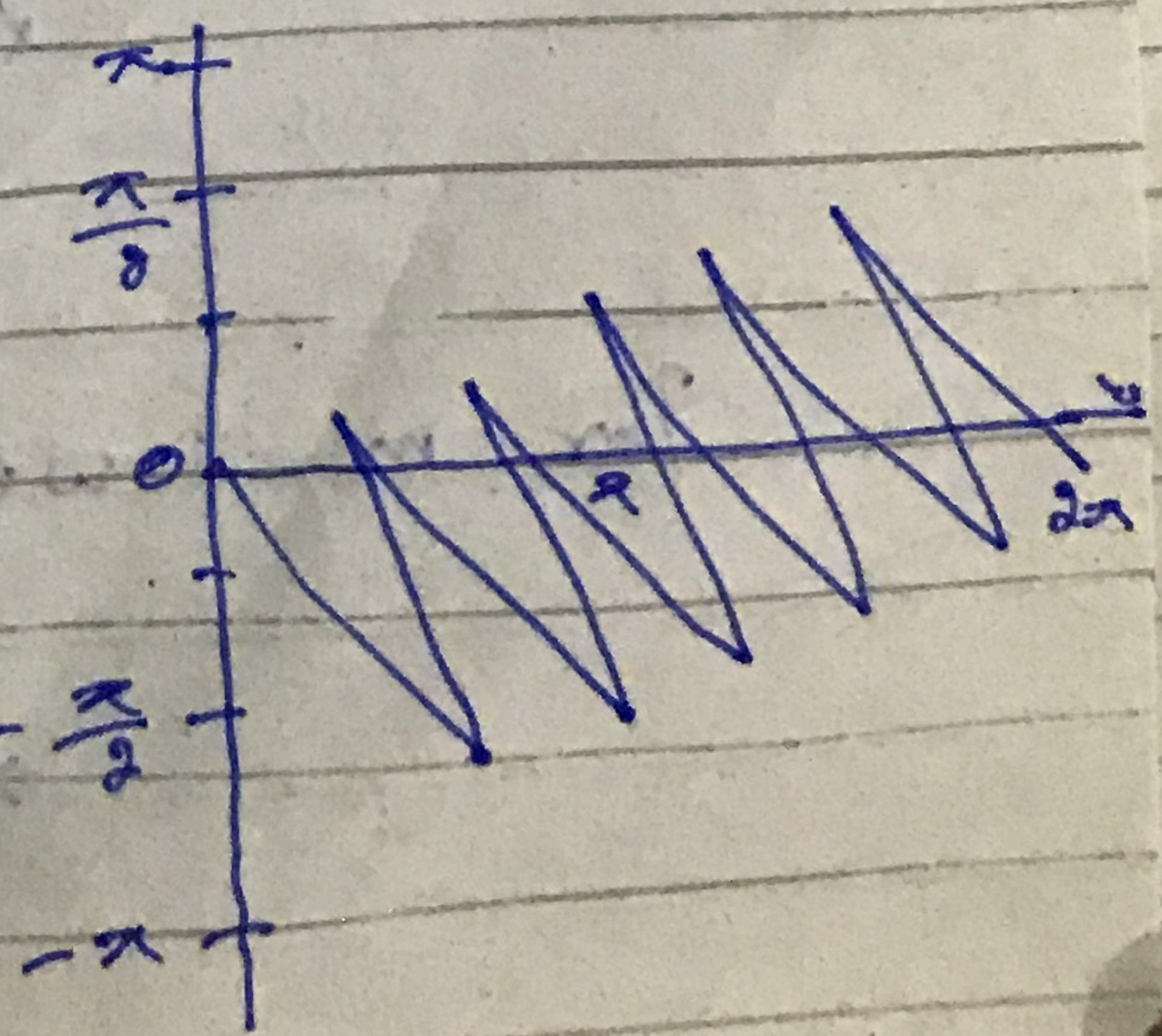
$$= \sum_{n=0}^{L-1} e^{-j\omega x} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}} = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude & Phase of $X(\omega)$ are illustrated 5.5 for $L = 10$. The N-Point DFT of $x(n)$ is ~~is~~ simply $X(\omega)$ evaluated at the set N equally spaced frequencies $\omega_k = 2\pi k/N$, $k = 0, 1, \ldots N$

$$X(k) = \frac{1-e^{j2\pi kL/N}}{1-e^{-j2\pi k/N}} \quad k = 0, 1, \ldots, N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$





If N is selected such that $N = L$

Then the DFT becomes.

$$X(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2 \cdots L-1. \end{cases}$$ //

Q4b

Perform the circular convolution of the following two sequences. Solve the Problem step by step.

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Sol: Sequence consist of four nonzero point.

$m = 0$ we have,

$$x_3(0) = \sum_{n=0}^{3} x_1(n) x_2((-n))_N.$$

$$x_3(0) = 14$$

For $m = 1$ we have.

$$x_3(1) = \sum_{n=0}^{3} x_1(n) x_2((1-n))_4.$$

$$x_3(1) = 16$$

For $m = 2$ we have.

$$x_3(1) = \sum_{n=0}^{3} x_1(n) x_2((1-n))_4$$

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$m = 2$ we have,

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

$$x_3(2) = 14$$

For $m = 3$ we have

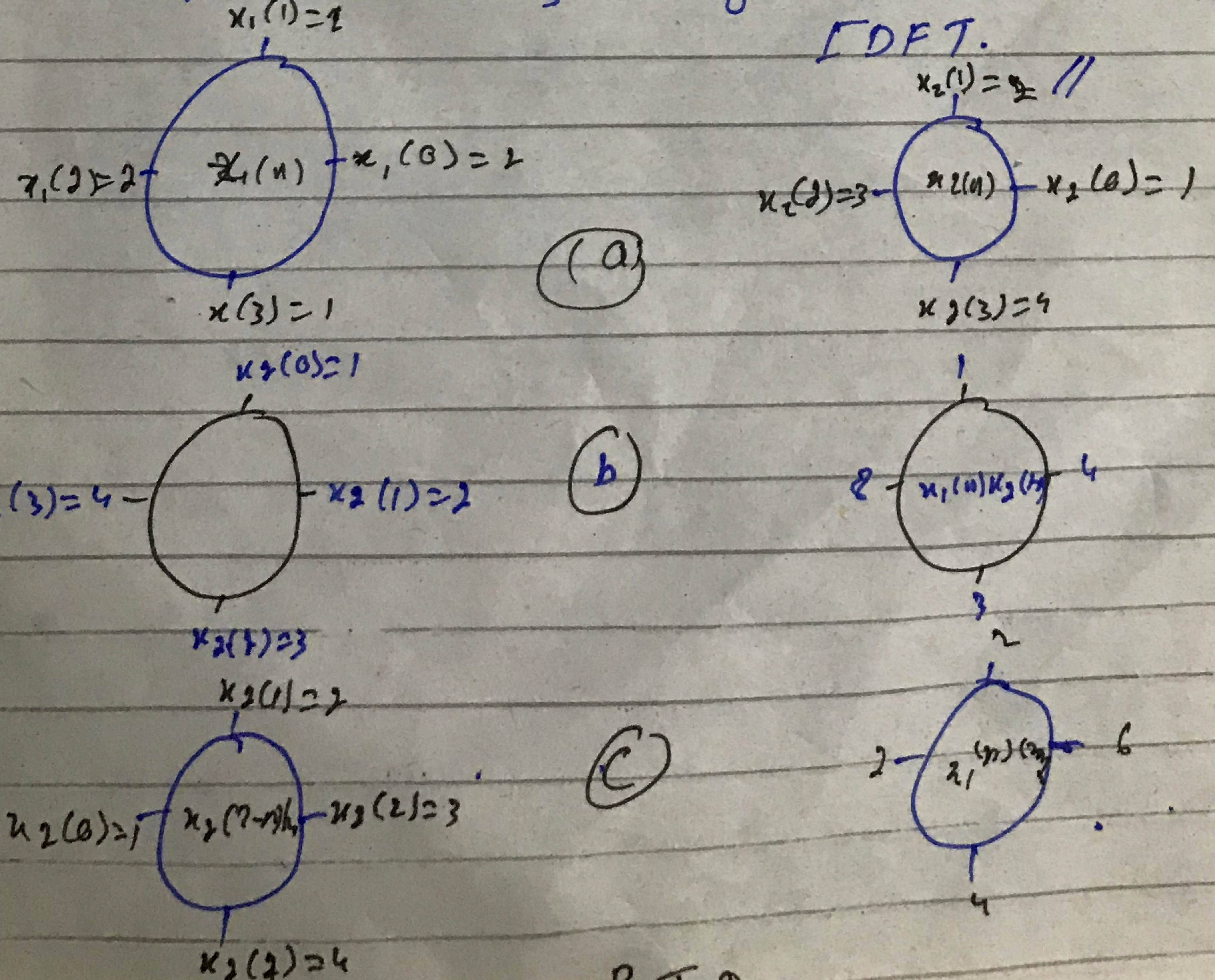
$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4$$

$$x_3(3) = 16$$

$$x_3(n) = \{14, 16, 14, 16)$$

$$x_3(m) = \sum_{n=0}^{N-1} x_2(n)\, x_1((m-n))_N \quad m = 0, 1, \ldots, N-1$$

The following example serves to illustrate the computation of $x_3(n)$ by means of the DFT & IDFT.



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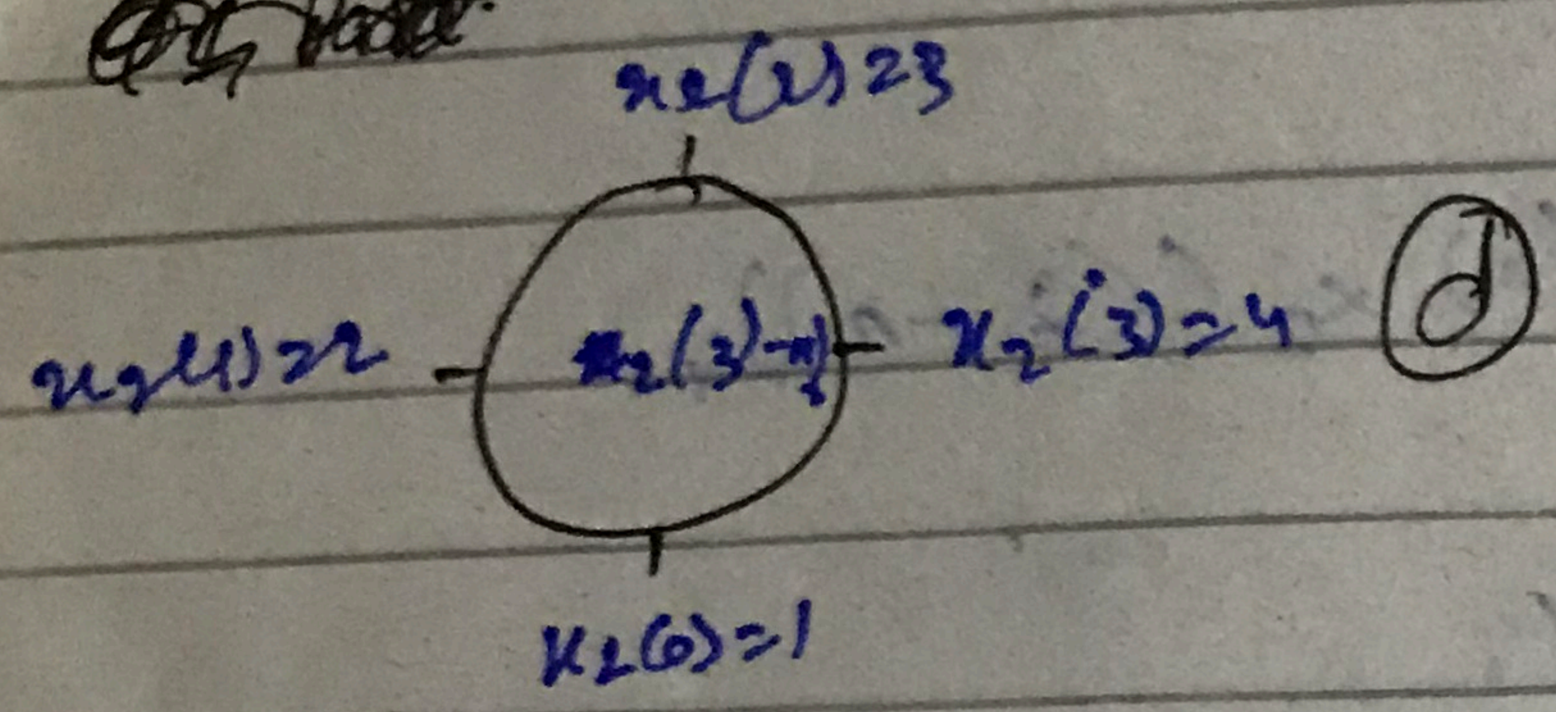
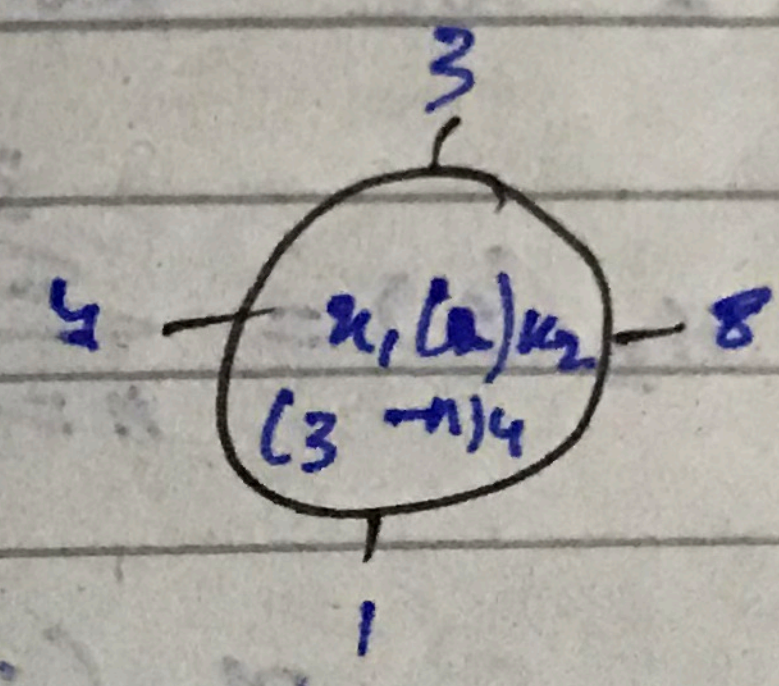


Folded sequences.



Product sequence.