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Sec :- A

Semester :- 6th

Subject :- Numerical Analysis

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to

"MID TERM EXAM"

Q1) Find the root of the equation

$$x^3 + 3.993 \times 10^{-4} = 0.165x^2$$

Use newton raphson method with $x_0 = 0.02$

Sol:- Rearranging the equation

$$x^3 - 0.165x^2 + 0.00039 = 0$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(x) = 3x^2 - 0.33x = 0$$

$$\rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.02 - \frac{0.00033}{-0.0054}$$

$$\boxed{x_1 = 0.081}$$

$f(x_0) = 0.00033$

$f'(x_0) = -0.0054$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.081 - \frac{(-0.00016)}{-0.0070}$$

$f(x_1) = -0.00016$

$f'(x_1) = -0.0070$

$x_2 = 0.058$

$f(x_2) = 0.000030$

$f'(x_2) = -0.0090$

$$\rightarrow x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.058 - \frac{0.000030}{(-0.0090)}$$

$x_3 = 0.061$

Q2) Use the number $x_0 = 2$, $x_1 = 2.75$, $x_2 = 4$ to find the lagrange interpolation polynomial for $f(x) = \frac{1}{x}$ at $x = 3$.

Sol:- putting value in the function.

$y_0 = 0.5$, $y_1 = 0.364$, $y_2 = 0.25$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2$$

$$= \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)}(0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} \times 0.364$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)}(0.25)$$

$$= -0.083 + 0.388 + 0.025$$

$$\Rightarrow \boxed{y = 0.33}$$

(Q3) Complete the divided difference table for the given and Construct the interpolating polynomial that uses all this data.

x = 1.0 1.3 1.6 1.9 2.2

y = 0.7651977 0.6200860 0.4554022 0.2818186 0.1103623

x_i	$f(x_i)$	$f(x_{i-1}, x_i)$	$f(x_{i-2}, x_{i-1}, x_i)$	$f(x_{i-3}, \ldots, x_i)$	$f(x_{i-4}, \ldots, x_i)$
x_0 1	0.7651977				
		-0.4837056			
x_1 1.3	0.6200860		-0.108734		
		-0.548946		0.0658785	
x_2 1.6	0.4554022		-0.0494433		-0.0028049
		-0.578612			
x_3 1.9	0.2818186		0.006818	0.0625255	
		-0.571521			
x_4 2.2	0.1103623				

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i) $f(x_0, x_1) = \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$= \dfrac{0.6200860 - 0.7651977}{1.3 - 1}$

$\boxed{f(x_0, x_1) = -0.4837056}$

2) $f(x_1, x_2) = \dfrac{f(x_2) - f(x_1)}{x_2 - x_1}$

$= \dfrac{0.4554022 - 0.6200860}{1.6 - 1.3}$

$\boxed{f(x_1, x_2) = -0.548946}$

3) $f(x_2, x_3) = \dfrac{f(x_3) - f(x_2)}{x_3 - x_2}$

$= \dfrac{0.2818186 - 0.4554022}{1.9 - 1.6}$

$\boxed{f(x_2, x_3) = -0.578612}$

4) $f(x_3, x_4) = \dfrac{f(x_4) - f(x_3)}{x_4 - x_3}$

$= \dfrac{0.1103623 - 0.2818186}{2.2 - 1.9}$

$\boxed{f(x_3, x_4) = -0.571521}$

Second divided difference.

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$$

$$= \frac{-0.548946 - (-0.4837056)}{1.6 - 1}$$

$$= -0.108734$$

$$f(x_1, x_2, x_3) = \frac{f(x_2, x_3) - f(x_1, x_2)}{x_3 - x_1}$$

$$= \frac{-0.578612 - (-0.548946)}{1.9 - 1.3}$$

$$f(x_1, x_2, x_3) = -0.0494433$$

$$f(x_2, x_3, x_4) = \frac{f(x_3, x_4) - f(x_2, x_3)}{x_4 - x_2}$$

$$= \frac{-0.571521 - (-0.578612)}{2.2 - 1.6}$$

$$= 0.006818$$

Third divided difference.

$$f(x_0, x_1, x_2, x_3) = \frac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$$

$$= \frac{-0.0494433 - (-0.108734)}{1.9 - 1}$$

$$= 0.0658785$$

$$f(x_1, x_2, x_3, x_4) = \frac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$$

$$= \frac{0.0068818 - (-0.049443)}{2.2 - 1.3}$$

$$= 0.06251255$$

4th divided difference.

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$= -0.0028049$$