Just now

Student Details

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Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	Marks 7 CLO 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	Marks 7 CLO 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6 CLO 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	Marks 6 CLO 2
Q3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\lvert H(\frac{\pi}{4})\right\rvert^2 = \frac{1}{2}$.	Marks 6 CLO 3

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	Marks 6 CLO 3
Q4	(a)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \geq L$.	Marks 6 CLO 2
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$ $$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$	Marks 6 CLO 2

Q 1

Part a

$y[n] - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

Soll:

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$ Hence

$$y_n = C_1 2^n + C_2 n 2^n$$

The particular sollution is

$$y_p(n) = k(-1)^n u(n)$$

Substituting this sollution in to different equation we obtain

$$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$$

$$= (-1)^n - (-1)^{n-1} u(n-1)$$

for $n = 2$

$$k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$$

the total sollution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

P.T.O

From the Initial condition we obtain $y(0)=1$, $y(1)=2$ then

$$C_1 + \frac{2}{9} = 1$$

$$\Rightarrow C_1 = 7/9$$

$$9C_1 + 2C_2 - \frac{2}{9} = 2$$

$$\Rightarrow C_2 = \frac{1}{3}$$

Q1

Part B

$$y(n) - 0.7y(n-1) + 0.1y(n-2)$$

$$= 2z(n) - z(n-2)$$

$$= \lambda^2 - 0.7\lambda + 0.1 = 0$$

Note, $\lambda = \frac{1}{2}, \frac{1}{5}$) Here

$$y_h(n) = C_1\left[\frac{1}{2}\right]^n + C_2\left[\frac{1}{5}\right]^n$$

with $z(n) = \delta(n)$ we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hence $C_1 + C_2 = 2$ and $\frac{1}{2}C_1 + \frac{1}{5}$

$$= 1.4 = 7/5$$

P.T.O

$\Rightarrow c_1 + \frac{2}{5} c_2 = \frac{14}{5}$

These equation yield

$$c_1 = \frac{10}{3} , c_2 = \frac{-4}{3}$$

$$h(n) = \left\{ \frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n \right] u(n)$$

The Step response is

$$\Rightarrow S(n) = \sum_{k=0}^{n} h(n-k)$$

$$\frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$= \frac{10}{3}\left(\frac{1}{2}^n\left(2^{n+1} - 1\right)u(n) - \frac{1}{3}\left(\frac{1}{5}^n\right)\left(5^{n+1} - 1\right) - u(n)\right.$$

$$= \frac{10}{3}\left(\frac{1}{2}^n\left(2^{n+1} - 1\right)u(n) - \frac{1}{3}\left(\frac{1}{5}^n\right)\left(5^{n+1} - 1\right)u(n)\right.$$

Q: 9

Part "b" (b)

Sol:

$$X(z) = \frac{1}{1-az^{-1}} \quad |z| > |a|$$

Using The Complex inversion Integral we have

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

Where 'c' is a circal at radius greather Than $|a|$ we Shall evalute This Integral with $f(z) = z^n$ we distinguish Two cases

~~The ... application~~.

1) If $n \geq 0$, $f(z)$ has only zeros and hence no Pole Inside C The only Pole Inside C is $z = a$ hence

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

2) If $n < 0$, $f(z) = z^n$ has an nth order Pole at $z = 0$ which is also inside There are contribution from both poles for $n = -1$ we have

$$X(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)} dz$$

P.t.o

$$= \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n = -2$ we have

$$x(-2) = \frac{1}{2\pi j}\oint_C \frac{1}{z^2(z-a)}\,dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

By contaning in The Same way we can show That $x(n) = 0$ for $n < 0$ Thus

$$x(n) = a^n u(n)$$

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Q2 Part (a)

Soll:

$$X(z) = \frac{1}{(1+2z^{-1})(1-z^{-1})^2}$$

$$X(z) = \frac{1}{4(1+2z-1)} + \frac{3}{4}\,\frac{1}{1-2z^{-1}} + \frac{1}{2}\cdot\frac{z^{-1}}{(1-z^{-1})^2}$$

By applying inversTransfrom

$$x(n) = \frac{1}{8}(-1)^n u(n) - \frac{3}{8}u(n) + \frac{1}{2}n u(n)$$

$$= \left[\frac{1}{8}(-1)^n + \frac{3}{8} + \frac{n}{2}\right]u(n)$$

Q3 (b)

Soll: clearly, the Filter must have pole

$p = re^{\pm j\pi/2}$

and zeros at $z = 1$ and $z = -1$

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+1}$$

the gain factor is determind by evaluting the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$ Thus we have

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determind by evaluting $H(\omega)$ at $\omega = 4\pi/9$

thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = \frac{1}{2}$$

P.T.O

$$= \left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

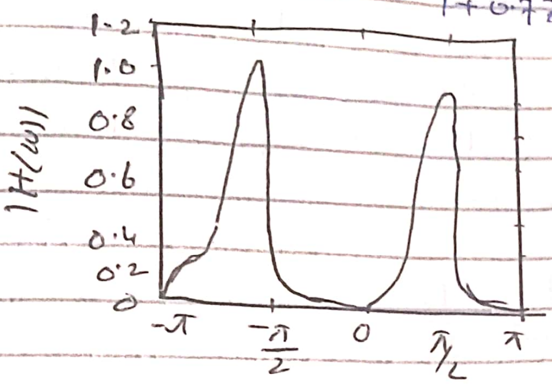
$$= \frac{1}{2}$$

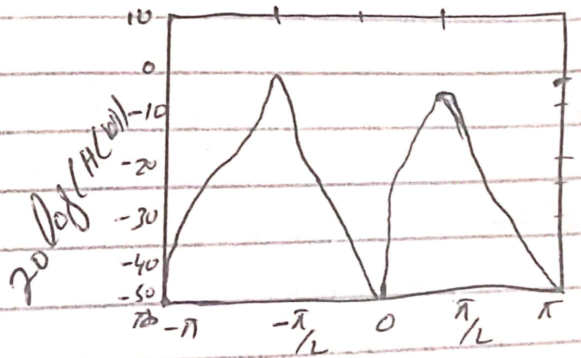
or

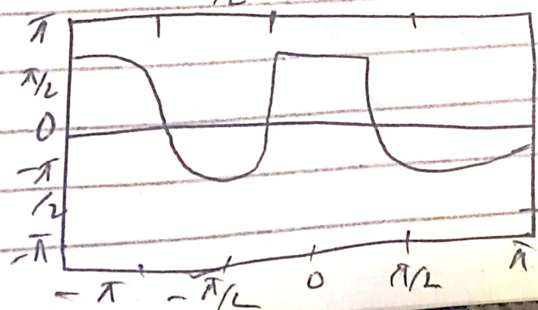
$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. Therefore the system function for the desired filter is

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$







Figur magnitude and phase response of a Sampel filter.

$$H(z) = 0.15(1-z^{-2}) / (1+0.7z^{-2})$$

it Should be emphasized that the main purpose of the foregoing Methodology for designing simpel digital filter by pole-zero palacment to provide in sight in to the effect that poles and zeroes have ona the frequency response charactrastic of system. The Methodology is not Intend as a good method for designing digital filter with well-specified pass band and Stop band charactrastic. Systamatic Method for the design of Sophisticated digital filter for Paractical application are disscused.

Q3 Part a

Sollution

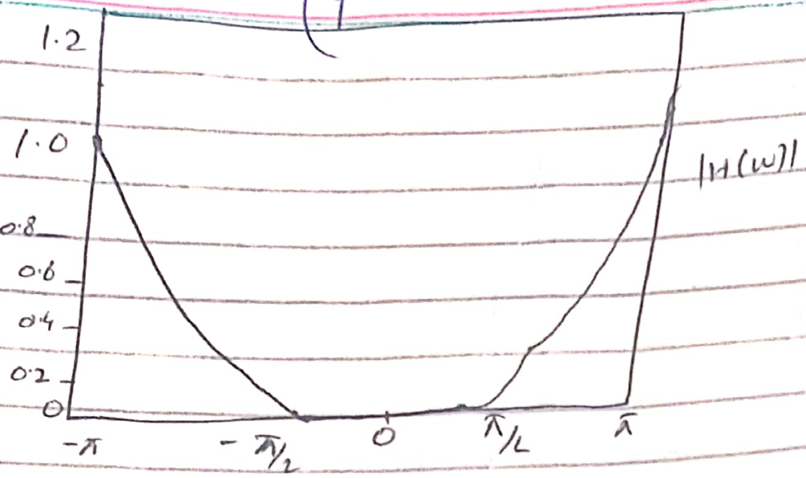
At $w=0$ we have

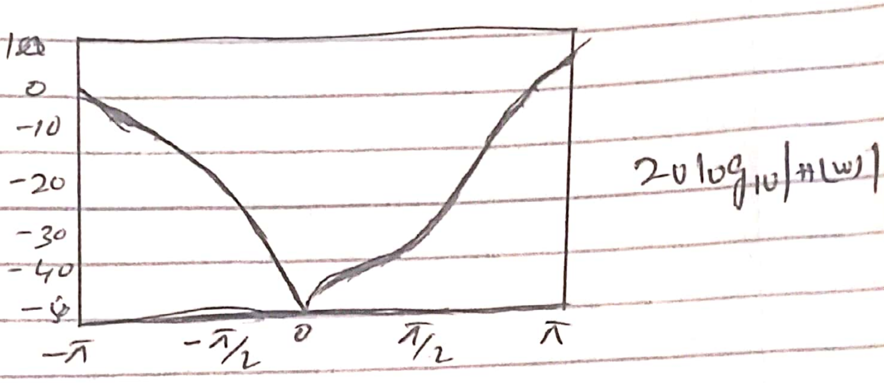
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$

$$P:\neq 0$$


1.2
1.0
0.8
0.6
0.4
0.2
0
−π
−π/2
0
π/2
π
|H(ω)|


0
−10
−20
−30
−40
−π
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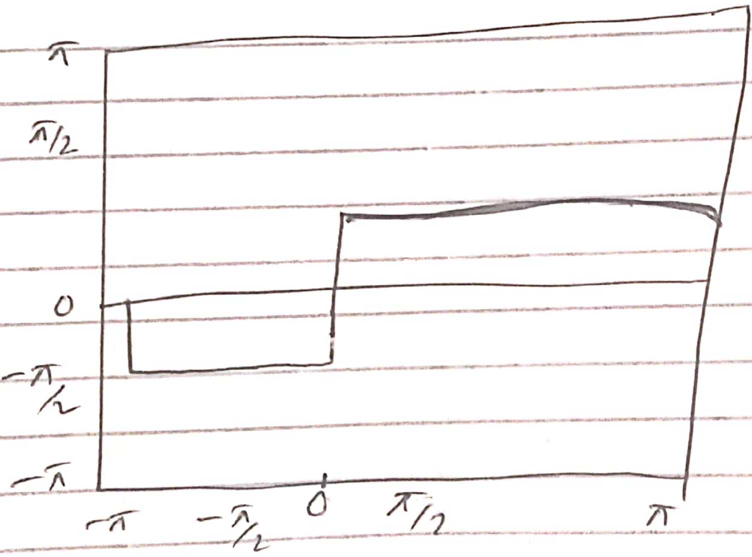
0
π/2
π
20 log10 |H(ω)|


π
π/2
0
−π/2
−π
−π
−π/2
0
π/2
π

p.to

ence

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At $\omega = \pi/4$

$$R = \frac{(1-P)^2}{1 - P\cos(\pi/4) + jP\sin(\pi/4)^2}$$

$$= \frac{(1-P)^2}{1 - P/\sqrt{2} + jP/\sqrt{2})^2}$$

$$= \frac{(1-P)^4}{[(1-P/\sqrt{2})^2} = \frac{1}{2}$$

x x x — x

Q4:

Part (a)

$$X(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determine The N-point DFT of This sequence for $n \geq L$

Sol:

$$X(w) = \sum_{n=0}^{L-1} X(n) e^{-JwL}$$

$$= \sum_{n=0}^{L-1} e^{-Jwn} = \frac{1 - e^{-JwL}}{1 - e^{-Jw}}$$

$$= \frac{\sin(wL/2)}{\sin(w/2)} e^{-jw(L-1)/2}$$

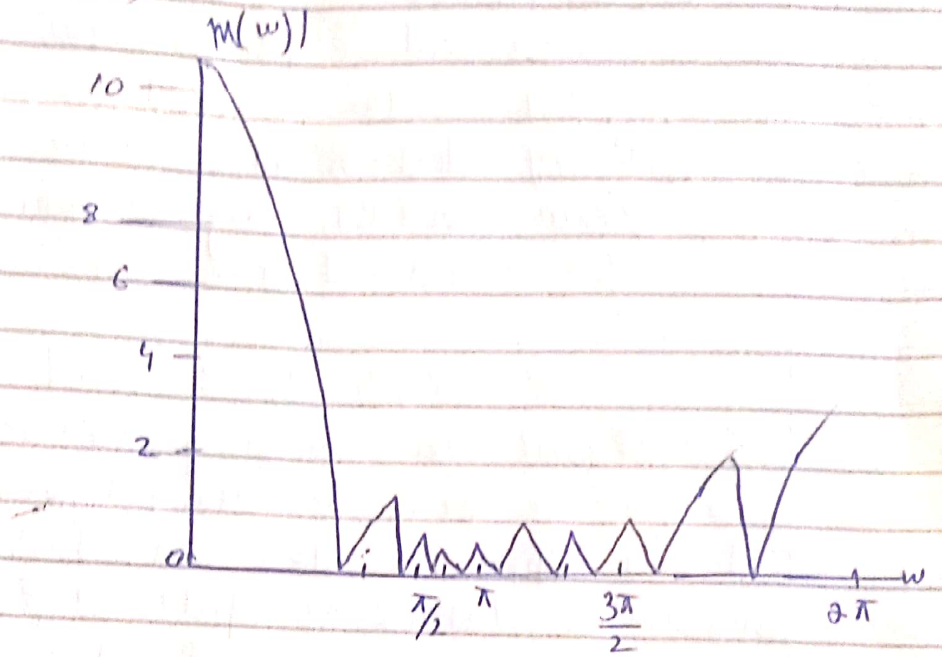
The Magnitude and phase of $X(w)$ are Illustrated for $L = 10$ The N Point DFT of $x(n)$ is Simply $X(w)$ evaluted at the Set of N equality Spaced frequencies

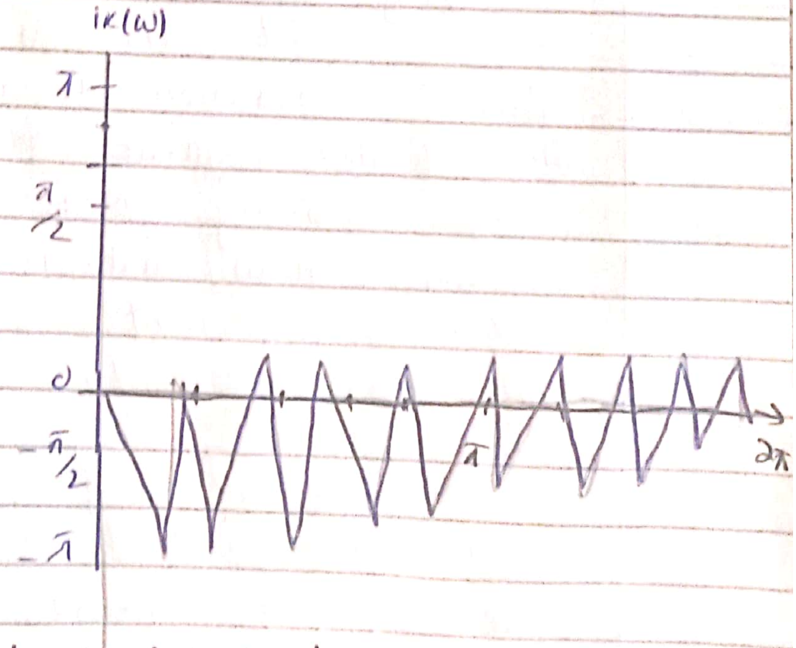
$$W_L = \frac{2\pi K}{N}, K = 0, 1, \ldots N-1$$

$$X(K) = \frac{1 - e^{-2j\pi KL/N}}{1 - e^{-2j\pi K/N}}$$

(12)

$$\frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





If N is selected such that $N = L$

Then DFT became

$$x(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2, \cdots L-1 \end{cases}$$

Thus There is only one non zero Value in The DFT This is appamt From observation of $x(\omega)$ sin $x(\omega)$ $= 0$ at frequencies $\omega_k = 2\pi k/L$ $k \neq 0$ The reader Should verify that $x(n)$ can be recoverd from $x(k)$ by Parforming and L - point IDFT.

Al Though The L - point DFT is Sufficiant to uniquely represent The Sequence $x(n)$ in The frequency Domain it is apperant that it doesnot Provide Sufficiant detail to yield a good picture. We must evalute (Interpolate) $x(\omega)$ at more closly spaced frequency say $\omega_k = 2\pi k/N$ were $N > L$, in effect we can view This Computation as expending The Size of The sequence from 'L' points to 'N' by appending N-L Zeroes to The Sequence $x(n)$ that is zero size Padding Then The N - point DFT Provides Finer Interpulation Then The L - point

$L = 10$, $N = 50$, and $N = 100$
Now Spactral Charactristic of The Sequence are more clearly evedent as one will conclude by Comparing These spectra with the continuse spectrum $x(\omega)$.

Q4: Circular Convalution 14

(b)

$x_1(n) = \{1, 2, 3, 4\}$

and $x_2(n) = \{1, 2, 1, 2\}$

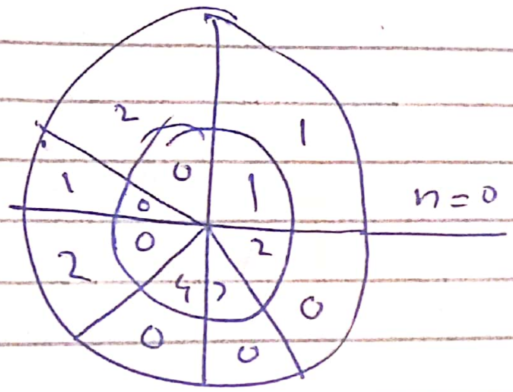
$L = 4$, $m = 4$

lenght of $y[n] = L + M - 1 = 4 + 4 - 1 = 7$

$x_1[n] = \{1, 2, 3, 4, 000\}$

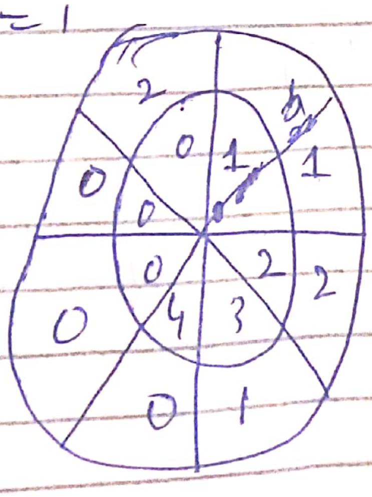
$x_2[n] = \{1, 2, 1, 2\ 000\}$

for $y(0)$



$y(0) = 1 \times 1 = 1$

for $y(1)$.



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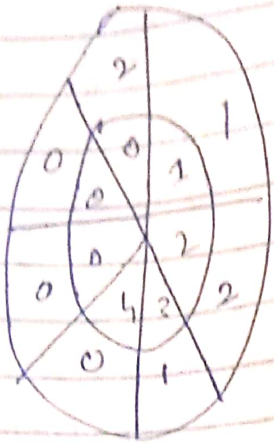
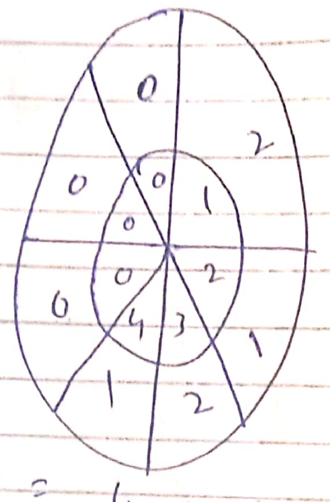
$y(1) = 1\times1\times2\times2+3 = 4$

For $y(2)$,



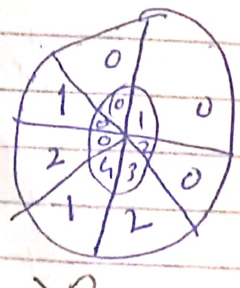
$y(2) = 1\times1\times2\times2+3\times1 = 8$

For $y(3)$



$\therefore y(4) = 4\times2+3\times1+2\times2 = 15$

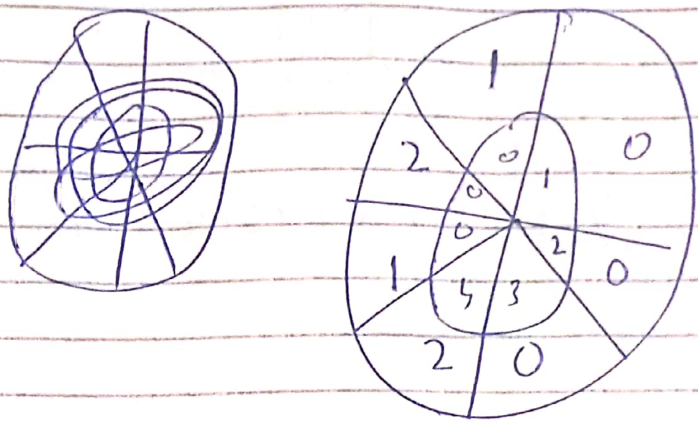
For $y(5)$



D.K.O

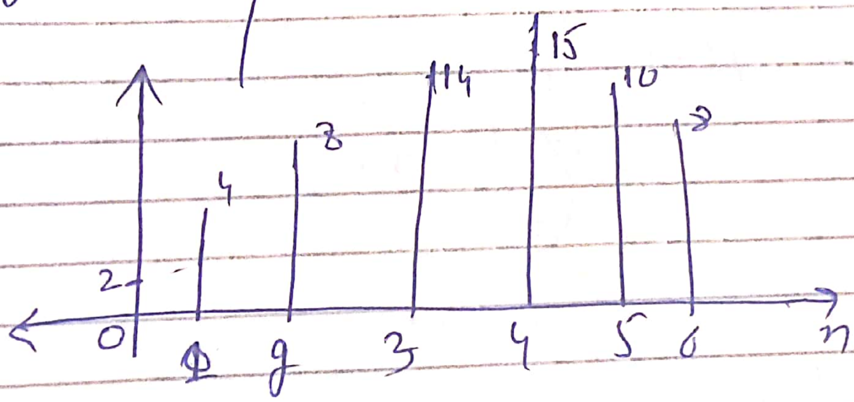
$y(5) = 4\times1 + 3\times2 = 10$

$y(6)$



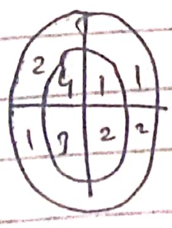
$\therefore y(6) = 4\times2 = 8$

$y[n] = \{1, 4, 8, 14, 15, 10, 8\}$



Linner using circular convolution

For $y=0$

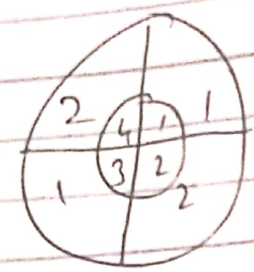


$y(0) = 1+4+3+8 = 16$

for $y(1)$:

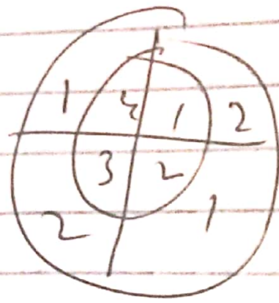
$\therefore y(1) = 2+2+6+4 = 14$

for $y(2)$



$y(2) = 1+4+3+8 = 16$

for $y(3)$

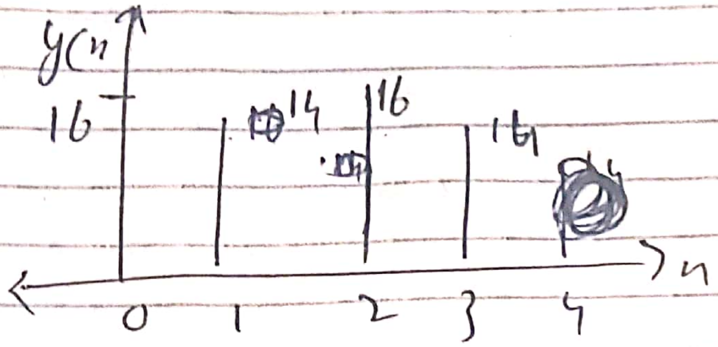


$y(3) = 2+2+6+4 = 14$

$y(n) = 16, 14, 16, 14$

p.t.o

$y(n) = 16, 14, 16, 14$



End.