Muhammad Azam Qazi

7894

Section A.

Assignment NO 02.

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Q(1) $x^3y''' + 2x^2y' + 2y = 10x + \frac{10}{x}$.

Sol:

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3D^3y + 2x^2D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3D^3 + 2x^2D + 2)y = 10x + 10x^{-1} \quad \text{--- (1)}$$

Let $x = e^t \Rightarrow t = \ln x$

$$xD = \Delta.$$

$$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta.$$

$$x^3D^3 = \Delta(\Delta-1)(\Delta-2)$$

Substituting

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10^x + 10x^{-1}$$

$$(\Delta^3 - \Delta^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}.$$

By Synthetic division.

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	0

$R = 0$

$a = 1,\ b = -2,\ c = 2.$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}.$$

$$D = \frac{2 \pm \sqrt{4-8}}{2} \Rightarrow D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$D = \frac{2 \pm 2i}{2} = 1 \pm i$$

$$y_c = e^{-x}(10\ C_1 \cos t + C_1 \sin t)$$

Now Particular integration

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^2 - D^2 + 2} \cdot 10/e^t$$

$$= \frac{10e^t}{1^3 - 1^2 + 2} + \frac{10e^{-t}}{1^3 - 1^2 + 2}$$

$$= 5e^t + 5e^t$$

$$y_p = 5e^t + 5e^t$$

General Solution

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^t$$

Put $e^t = x$ and $t = \ln x$

$$y = e^{-x}(C_1 \ln x + C_2 \sin/x) + 5e^x + 5e^{-x}$$ Answer

Q(2) $x^3y''' + 4x^2y'' - 5xy' - 15y = x^4$.

Solution

let $\frac{d}{dx} = D$

$x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

let $x = e^t \Rightarrow t = \ln x$

$xD = D$

$x^2D^2 = D(D-1) = D^2 - D$

$x^3D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$.

Now substituting.

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$.

$D^3 - 3D^2 + 2D + 4(D^2 - D) - 5(D) - 15)y = e^{4t}$.

$(D^3 + D^2 - 7D - 15)y = e^{4t}$.

By Synthetic division

5	1	+1	-7	-15
		3	12	15
	1	4	5	0

$R = 0$.

$D^2 + 4D + 5 = 0$.

By Quadratic formula.

$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$

$D = \frac{-4 \pm 2i}{2} \Rightarrow -2 \pm i$.

$y_c = e^{3x}(c_1 \cos t + c_2 \sin t)$.

For $y_p = ?$

$y_p = \frac{1}{D^3+D^2-7D-15} \cdot e^{4t}$.

$y_p = \frac{1}{4^3+4^2-7(4)-15} e^{4t}$.

$y_p = \frac{1}{80-43} e^{4t}$.

$y_p = \frac{1}{37} e^{4t}$.

$y = y_c + y_p$.

$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$.

Put $t = \ln x$ and $x = \ln x$

$y = e^{3x}(c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37} e^{4x}$ Answer.

Q(1) $x^2y'' + 2xy' - 6y = 10x^2$; $y(1) = 1$, $y'(1) = 6$.

Solution:

$y(1) = 1$ and $y'(1) = -6$.

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 10x^2$$

$$\Rightarrow x^2\frac{d^2}{dx^2} + 2x\frac{dy}{dx} - 6y = 10x^2$$

Putting $xD = \Delta \Rightarrow x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$.

$x = e^t$ and $\log x = t$.

$(\Delta^2 - \Delta + 2\Delta - 6)y = 10e^{2t}$

$(\Delta^2 + \Delta - 6)y = 10e^{2t}$.

Characteristic equation.

$\Delta^2 + \Delta - 6 = 0$.

$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$.

$\Delta(\Delta+3) - 2(\Delta+3) = 0$.

$(\Delta+3)(\Delta-2) = 0$.

$\Delta = 2$, $\Delta = -3$.

For $y_c = ?$

$y_c = C_1e^{-3t} + C_2e^{2t}$.

$$y_p = \frac{1}{D^2-D-6} \cdot 10^{2t}$$

$$y_p = \frac{10}{D^2-D-6} e^{2t}$$

$$y_p = 10 \frac{1}{0} e^{2t}$$

$$y_p = 10 \frac{1}{d/dD(D^2+D-6)} e^{2t}$$

$$y_p = 10 \frac{t}{2D+1} e^{2t}$$

$$y_p = 2te^{2t}$$

General Solution.

$$y = y_c + y_p$$

$$y = C_1 e^{-3t} + C_2 e^{2t} + 2te^{2t}$$

$$y = C_1 x^{-3} + C_2 x^2 + 2(\log x) x^2 \text{ — (A)}$$

$D(1) = 0$ i.e $x = 1$, $y = 1$.

$$1 = C_1 (1)^{-3} + C_2(1)^2 + 2\log(1)$$

$$1 = C_1 + C_2.$$

differentiating w.r.t x.

$$y_1 = -3C_1 x^{-4} + 2C_2 x + \frac{2}{x} x^2 + 4x \log x.$$

$$-6 = -3C_1 + 2C_2 + 2 + 0.$$

2) $-6 = 2 = -3C_1 + 2C_2 + 2$

$-8 = -3C_1 + 2C_2$

$2C_1 + 2C_2 = 2$

$\mp 3C_1 \pm 2C_2 = \ominus 8$

$5C_1 = 10$

$C_1 = 10/5 = 2$

$-8 = -3(2) + 2C_2$

$2C_2 = -8 + 6$

$C_2 = -1$

$y = 2x^{-3} - x^2 + 2\ln x\,(x^2)$

$y = \frac{2}{x^3} - x^2 + 2x^2 \log x$ Answer

Q(4). $x^2y'' + 7xy' + 5y = x^5$; $y(1) = 2$

$y'(1) = 2.$

<u>Solution</u>

$$x^2 \frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow x^2\frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5y = x^5 \quad \text{— (4)}$$

$$xD = D \Rightarrow x^2D^2 = D(D-1) = D^2 - D$$

$x = e^t \Rightarrow \log x = t$ in eq (4)

$$\Rightarrow (D^2 - D + 7D + 5)y = e^{5t}.$$

$$(D^2 + 6D + 5)y = e^{5t}.$$

$$D = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$D = \frac{6 \pm \sqrt{36 - 20}}{2} \Rightarrow \boxed{D = -3 \pm 2.}$$

$$y_c = C_1e^{-5t} + C_2e^{-t}.$$

$$y_p = \frac{1}{D^2 + 6D + 5}e^{5t}.$$

$$y_p = \frac{1}{5^2+6(5)+5} e^{5t}$$

$$y_p = \frac{1}{60} e^{5t}$$

General Solution.

$y = y_c + y_p$.

$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{+5t}$.

$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$.

$x = 0$, $e^0 = 1$

Put $y(0) = 2$ i.e $y = 2 = x = 2$.

$2 = C_1 (2)^{-5} + C_2 (2)^{-1} + \frac{1}{60} (2)^5$.

$2 - \frac{8}{15} = -32C_1 - 2C_2$.

$\frac{22}{15} = -32C_1 - 2C_2$ — (C)

$y' = -5C_1 x^{-6} - C_2 x^{-2} + \frac{1}{12} x^4$.

$y'(1) = 2$; $y' = 2$, $x = 2$.

$2 = 5C_1 (2)^{-6} - C_2 (2)^{-2} + \frac{1}{12} (2)^4$.

$2 = 320C_1 + 4C_2 + 4/3$.

$\Rightarrow 2/3 = 320C_1 + 4C_2$.

$-\frac{44}{15} = 64C_1 + 4C_2$.

$C_1 = \frac{34}{15} \times 256$

$\boxed{C_1 = 580}$

$\frac{2L}{15} = -32(580) - 2C_2$

$C_2 = \frac{18561}{-2} \Rightarrow -9280$

$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$

$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5$

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(11)

⑤ $(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$

Solution:

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow (x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4 \Big) y = x^2$$

$(x+1)D = D \Rightarrow (x+1)^2 D^2 = D(D-1) = D^2 - D$

$x = e^t$

$$\Rightarrow [D^2 - D - 3D + 4] y = e^{2t}$$

$$(D^2 - 4D + 4) y = e^{2t}$$

$$D^2 - 4D + 4)y = e^{2t}$$

For y_c.

$$(D^2 - 4D + 4) = 0$$

$$D^2 - 2D - 2D + 4 = 0$$

$$(D-2) - 2(D-2) = 0$$

$$(D-2) = 0 \; ; D = 2$$

$$(D-2) = 0 \; ; D = 2$$

General Solution.

$$y = (C_1 + C_2 x)^{mx}$$

$$y = (C_1 + C_2 x)^{2x}$$

$$y_p = \frac{1}{D^2 - 4D + 4} \Rightarrow y_p = \frac{2}{2D - 4} e^{2t}$$

$y_p \quad \frac{2}{2} e^{2t}.$

$y = (C_1 + C_2 x)^{2t} e^{2t}$. Answer.

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end.