ASSIGNMENT NO. 1

NAME : SAIFULLAH JAVED

ID NO : 7954

DEPARMENT : BE(CIVIL)

SECTION : "B"

SUBJECT : DIFFERENTIAL EQUATION

SUBMITTED TO : MAAM SHUMAILA

QUESTION No.1

GIVEN:

$$x^3y''' + 2x^2y' + 2y = 10x + 10/x$$

REQIURED:

Find the general solution by Cauchy Euler theorem.

SOLUTION:

$$x^3 y''' + 2x^2 y' + 2y = 10x + 10/x$$

$$\Rightarrow x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{Dy}{dx} + 2y = 10x + 10/x$$

$$\Rightarrow x^3 D^3y + 2x^2 Dy + 2y = 10x + 10/x$$

$$\Rightarrow (x^3 D^3 + 2x^2 D + 2) y = 10x + 10x^{-1} \quad (1)$$

Let $x = e^t \Rightarrow t = \ln x$

$$xD = \Delta$$

$$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$$

$$x^3D^3 = \Delta(\Delta-1)(\Delta-2)$$

Subtituting in equ (1)

(2)

$$(D^3-3D^2+2D+2(D^2-D)+2)y = 10x+10x^{-1}$$

$$\Rightarrow (D^3-D^2+2)y = 10e^t + \frac{10}{e^t}$$

$$\Rightarrow (m^3-m^2+2)y = 10e^t + \frac{10}{e^t}$$

Using Synthetic devision.

$$\begin{array}{r|rrrr} & 1 & -1 & 0 & 2 \\ -1 & & -1 & +2 & -2 \\ \hline & 1 & -2 & 2 & 0 \end{array}$$

$\boxed{m = -1}$

$m^2 - 2m + 2 = 0$

~~$\Rightarrow m^2 - m - m + 2 = 0$~~

Now here $a = 1$, $b = -2$ and $c = 2$

Usig quadratic formula.

$$\Rightarrow m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow m = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(2)}}{2(1)}$$

$$\Rightarrow m = \frac{2 \pm \sqrt{-4}}{2}$$

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$$\Rightarrow m = \frac{2 \pm 2i}{2}$$

$$\Rightarrow m = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$\Rightarrow m = 1 \pm i$$

Since roots are complex

$$y_i = e^{-x}(C_1 \cos t + C_2 \sin t)$$

Now Particular integration

$$y_P = \frac{1}{D^3 - D^2 + 2} \cdot 10e^{t} + \frac{1}{D^3 - D^2 + 2} \cdot 10/e^{t}$$

$$\Rightarrow y_P = \frac{1}{(1)^3 - (1)^2 + 2} \, 10e^{t} + \frac{1}{(1)^3 - (1)^2 + 2} \cdot 10/e^{t}$$

$$\Rightarrow \quad y_P = \frac{\overset{5}{\cancel{10}} e^{t}}{\cancel{2}} + \frac{\overset{5}{\cancel{10}} e^{-t}}{\cancel{2}}$$

$$\Rightarrow y_P = 5e^{t} + 5e^{-t}$$

General Solution

$$y = y_c + y_P$$

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$\Rightarrow y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^{t} + 5e^{-t}$

Put $e^t = x$ and

$t = \ln x$

$$\Rightarrow \boxed{y = e^{-x}(c_1 \cos \ln x + c_2 \sin \ln x) + 5x + 5x^{-1}}$$

QUESTION No.2

GIVEN:

$$x^3 \frac{d^3y}{dx^3} + 4x^2 \frac{d^2y}{dx^2} - 5x\frac{dy}{dx} - 15y = x^4$$

REQUIRED:

To find the general Solution by Cauchy euler Theorem.

SOLUTION:

$$x^3 \frac{d^3y}{dx^3} + 4x^2 \frac{d^2y}{dx^2} - 5x\frac{dy}{dx} - 15y = x^4$$

Let $\frac{d}{dx} = D$

$$\Rightarrow x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$$

$$\Rightarrow (x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$$

Let $x = e^t \Rightarrow t = \ln x$

$$xD = \Delta$$
$$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$$
$$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$$

Now substituting.

$$\Rightarrow (x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$$

$$\Rightarrow (\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2 - \Delta) - 5\Delta - 15)y = e^{4t}$$

$$\Rightarrow (\Delta^3 + \Delta^2 - 7\Delta - 15)y = e^{4t}$$

Synthetic devision

3	1	+1	−7	−15
3		3	12	15
	1	4	5	0

$\Delta^2 + 4\Delta + 5 = 0$

Now by Quadratic formula.

$$D = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow D = \frac{-4 \pm \sqrt{(4)^2-4(1)(5)}}{2(1)}$$

$$\Rightarrow D = \frac{-4 \pm \sqrt{16-20}}{2}$$

$$\Rightarrow D = \frac{-4 \pm 2i}{2}$$

$$\Rightarrow D = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

$$\Rightarrow \boxed{D = -2 \pm i}$$

$$y_c = e^{5y}(c_1 \cos t + c_2 \sin t)$$

For y_P

$$y_P = \frac{1}{D^3+D^2-7D-15} \cdot e^{4t}$$

$$\Rightarrow y_P = \frac{1}{(4)^3+(4)^2-7(4)-15} \cdot e^{4t}$$

$$\Rightarrow y_P = \frac{1}{64+16-28-15} e^{4t}$$

$$\Rightarrow y_P = \frac{1}{80-43} \cdot e^{4t}$$

$$\Rightarrow y_P = \frac{1}{37} e^{4t}$$

Hence $y = y_c + y_P$

$$\Rightarrow y = (C_1 \cos t + C_2 \sin t) + \frac{1}{37} e^{4t}$$

Again put $t = \ln x$ and $e^t = x$

$$\Rightarrow \boxed{y = e^{3x} (c_1 \cos \ln x + C_2 \sin \ln x) + 1/_{37} x^4}$$

QUESTION NO. 3

GIVEN DATA:

$$x^2 y'' + 2x^2 y' - 6y = 10x^2$$

REQUIRED:

To find general solution by Cauchy euler theorem.

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Solution:

$$y(1) = 1 \quad \text{and} \quad y'(1) = -6$$

$$x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 10x^2$$

$$\Rightarrow \left(x^2 \frac{d^2}{dx^2} + 2x \frac{d}{dx} - 6\right) y = 10x^2$$

Put $\cancel{x}\frac{d}{dx} = D$

$$\Rightarrow (x^2D^2 + 2xD - 6)y = 10x^2$$

Put $xD = \theta$

$$x^2D^2 = \theta(\theta - 1) = \theta^2 - \theta$$

$$\Rightarrow (\theta^2 - \theta + 2\theta - 6)y = 10x^2$$

$$\Rightarrow (\theta^2 + \theta - 6)y = 10x^2$$

$$\Rightarrow (\theta^2 + 3\theta - 2\theta - 6)y = 10x^2$$

The characteristic equation

$$\theta^2 + 3\theta - 2\theta - 6 = 0$$

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$\Rightarrow D(D+3)-2(D+3)=0$

$\Rightarrow (D+3)(D-2)$

$\boxed{D=-3}, \boxed{D=2}$

Since roots are real and distinct.

For yc =

$$y_c = C_1 e^{-3t} + C_2 e^{2t}$$

For yp

$$y_P = \frac{1}{D^2 - D - 6} \cdot 10e^{2t}$$

$$\Rightarrow y_P = \frac{10}{D^2 + D - 6} e^{2t}$$

$$\Rightarrow y_P = \frac{10}{\frac{d}{dD}(D^2+D-6)} e^{2t}$$

$$\Rightarrow y_P = \frac{10}{2D+1} e^{2t}$$

$$\Rightarrow y_P = \frac{10}{2(2)+1} e^{2t}$$

$\Rightarrow y_p = 5e^{2t}$

Now General Solution

$y = y_c + y_p$

$\Rightarrow y = c_1 e^{-3t} + c_2 e^{2t} + 5e^{2t}$

= Put $x = e^t$ and $t = \ln x$

$\Rightarrow \boxed{y = c_1 x^{-3} + c_2 x^2 + 5x^2}$

QUESTION No. 4

GIVEN:

$x^2 y'' + 7xy' + 5y = x^5$

$y(0) = 2, y'(1) = 2$

REQUIRED DATA:

To find the General Solution by Cauchy euler theorem.

Solution:

$$x^2y'' + 7xy' + 5y = x^5$$

$$x^2 \frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$x^2 D^2y + 7xDy + 5y = x^5$$

$$\Rightarrow (x^2D^2 + 7xD + 5)y = x^5 \quad \text{— ①}$$

Put $x = e^t \Rightarrow t = \ln x$

$$xD = \theta$$
$$x^2D^2 = \theta^2 - \theta$$

So

$$(\theta^2 - \theta + 7\theta + 5)y = \cancel{x}\, e^{5t}$$

$$\Rightarrow (\theta^2 + 6\theta + 5)y = e^{5t}$$

Complemendory Solution

$$m^2 + 6m + 5 = 0$$
$$\Rightarrow m^2 + 5m + 1m + 5 = 0$$
$$\Rightarrow m(m+5) + 1(m+5) = 0$$

$\Rightarrow (m+5)(m+1)=0$

$\Rightarrow \boxed{m_1=-5}, \boxed{m_2=-1}$

So roots are real and distinct

$y_c = c_1 e^{m_1 t} + c_2 e^{m_2 t}$

$\Rightarrow y_c = c_1 e^{-5t} + c_2 e^{-t}$

Now Particular Solution.

$$y_p = \frac{1}{D^2+6D+5} e^{5t}$$

$$\Rightarrow y_p = \frac{1}{(5)^2+6(5)+5} e^{5t}$$

$$\Rightarrow y_p = \frac{1}{25+30+5} e^{5t}$$

$$\Rightarrow y_p = \frac{1}{60} e^{5t}$$

Now general Solution is

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$y = y_c + y_p$

$\Rightarrow y = c_1 e^{-t} + c_2 e^{-5t} + e^{5t}/60$

Put $x = e^t$

$\Rightarrow \boxed{y = c_1 x^{-1} + c_2 x^{-5} + \frac{x^5}{60}}$

QUESTION No. 5

GIVEN:

$(x+1)^2 y''' + 3(x+1) y' + 4y = x^2$

REQUIRED:

To find the general solution by cauchy euler theorem.

SOLUTION:

$(x+1)^2 \frac{d^3y}{dx^3} + 3(x+1)\frac{dy}{dx} + 4y = x^2$

$\Rightarrow (x+1)^2 D^3y + 3(x+1) Dy + 4y = x^2$

$\Rightarrow \{(x+1)^2 D^3 + 3(x+1) D + 4\} y = x^2$

Put $x = e^t$ and $t = \ln x$

$(x+1) D = \Delta$

$(x+1)^2 D^2 = \Delta^2 - \Delta$

So

$(\Delta^2 - \Delta - 3\Delta + 4)y = e^{2t}$

Complementary Solution

$m^2 - 4m + 4 = 0$

$\Rightarrow m^2 - 2m - 2m + 4 = 0$

$\Rightarrow m(m-2) - 2(m-2) = 0$

$\Rightarrow (m-2)(m-2) = 0$

$\Rightarrow \boxed{m=2}, \boxed{m=2}$

Now the roots are real and ~~distinct~~. repeated

$y_c = (C_1 + C_2 x)^{mx}$

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$\Rightarrow y_c = (c_1 + c_2 x)^{2x}$

For Particular integral

$$y_P = \frac{1}{D^2 - 4D + 4} e^{2x}$$

$$\Rightarrow y_P = \frac{1}{(2)^2 - 4(2) + 4}$$

$$\Rightarrow y_P = \frac{1}{0}$$

So particular Solution does not exist. mean not Possible

$y = y_c + y_P$

$$\Rightarrow \boxed{y = (C_1 + C_2 x)^{2x}}$$