Paper: Differential EQUACTION Pg# 1

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Q. Solve the following objectives types Question.

Q i- The order of matric A is $m \times P$ & the order of B is $P \times n$ then order of matrix AB is?

Sol:-

order of $A = m \times P$
order of $B = P \times n$

So order of $A \times B = m \times n$

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ii- The number of non-zero rows in an Echelon form?

Sol:-

The number of non zero rows is an Echelon form is called rank of the matrix

for example

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Pg # 2

$$A = \begin{bmatrix} 1 & 0 & -2 & 5 & 3 \\ 0 & 0 & 1 & -4 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since A is an row-reduced form: Since it contains three non-zero rows, its row rank is three.

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iii- if $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matric

then $a = ?$

Sol:

we know that for singuler matric

$$|B| = 0$$

So

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$1 \times a - 2 \times 4 = 0$$

$$a - 8 = 0$$

$$\boxed{a = 8}$$ Ans

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iv- if $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ the $|A| = ?$

Sol:-

$$A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

take modulus of a matrix A

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = 2i \times (-i) - i \times i \qquad (\because i^2 = -1)$$

$$|A| = -2i^2 - i^2$$

$$|A| = -2(-1) - (-1)$$

$$|A| = 2+1$$

$$\boxed{|A| = 3}$$ Ans

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v- The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Sol:-

$$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

Here in matrix A diagnol element is same i.e 9. So it is scaler matrix.

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vi- Solution of $dy/dx + 2xy = y = ?$

Sol:→

$$dy/dx = y - 2xy$$

$$dy/dx = y(1-2x)dx$$

$$dy/y = (1-2x)\,dx$$

$$\int dy/y = \int (1-2x)\,dx$$

$$\ln y = x - 2x^2/2 + c_1$$

$$\ln y = (x - x^2) + c_1$$

$$e^{\ln y} = e^{(x - x^2) + c_1}$$

$$y = e^{x-x^2} \cdot e^{c_1}$$

$$\boxed{y = ce^{x-x^2}}$$

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vii- The order & degree of differential equation

$$(dy/dx)^3 = \sqrt{1 + (dy/dx)^2}$$

Sol:-

Taking square on both sides

$$\left((dy/dx)^3\right)^2 = \left(\left(1 + (dy/dx)^2\right)^{1/2}\right)^2$$

$$(dy/dx)^6 = 1 + (dy/dx)^2$$

Degree = 6

order = 1

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viii- The order & degree of differential equation $\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$ is ?

2 But the degree is undefined because the unknown function y is an argument of transcendental sin function

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(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is ?

Sol: →

Expand w.r.t column number first

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1\begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1\begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$1(bc^2 - cb^2) - 1(ac^2 - ac) + 1(ab^2 - a^2b)$

$(bc^2 - cb^2) - (ac^2 - ac) + (ab^2 - a^2b)$

$bc(c-b) - ac(c-a) - ab(b-a)$

$bc^2 - cb^2 - ac^2 + ac^2 + ab^2 - a^2b$

$(c-b)\{bc - ac - ab - a^2\}$

(ix) $2dy/dx + x^2y = 2x+3, y(0) = 5$

$dy/dx + 1/2\, x^2y = x + 3/2$

$dy/dx = -1/2\, x^3y + 1/2\, x + 3/2$

Integrate

$y = -1/2\, \frac{x^3}{3}y + 1/2\, x^2/2 + 3/2 + c_1$

$y = -1/6\, x^3y + 1/4\, x^2 + 3/2\, x + c_1 \rightarrow$ eq 1

Now use condition

at $x=0$ $y=5$ put in above equation

$5 = 0 + 0 + 0 + c_1$

$c_1 = 5$

$\boxed{y = 1/6\, x^3y + 1/4\, x^2 + 3/2\, x + 5}$ required Particular Sol

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(7)

Q2. Express the determinant

(i) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

as the product of factor which are linear in a, b, c

Sol:-

$$A = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = Adj(A) \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{3}A_{3} \text{ ----- } (1)$$

Now

$$A_{11}\ (-1)^{1+1}\ M_{11} = (-1)^2 \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix}$$

$$A_{12} = (-1)^{1+2}\ M_{12} = (-1)^3 \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix}$$

$$A_{12} = - \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix}$$

$$A_{13} = (-1)^{1+3}\ M_{13} = (-1)^4 \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

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$= a\left[b^2c^3 - c^2b^3\right] - b\left(a^2c^3 - c^2a^3\right) + c\left(a^2b^3 - b^2a^3\right)$

$= ab^2c^3 - ac^2b^3 - ba^2c^3 + bc^2a^3 + ca^2b^3 - cb^2a^3$

$(x^2 + 3y^2)dy - 2xy\,dy = 0$ at $x = 2, y = 6$

$M\,dx + N\,dy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\frac{\partial M}{\partial y} = 6y$, $\frac{\partial N}{\partial x} = -2y$

$\frac{M_y - M_x}{N} = \frac{6y - (-2y)}{-2xy} = \frac{6y + 2y}{-2xy} = \frac{\cancel{8}^{4}\cancel{y}}{\cancel{2}xy}$

$= -4/x$

$I.f = e^{-\int 4/x\,dx} = e^{-4\int 1/x\,dx} = e^{-4\ln x} = e^{\ln x^{-4}}$

$x^{-4}(x^2 + 3y^2)dx - \frac{2xy}{-x^4}dy = 6$

$\left(\frac{1}{x^2} + \frac{3y^2}{x^4}\right)dx - \frac{2y}{x^3}dy = 6$

$\frac{\partial M}{\partial y} = \frac{6y}{x^4}$ $\frac{\partial N}{\partial x} + \frac{6y}{x^4}$

$\frac{\partial M_1}{\partial y} = \frac{\partial N_1}{\partial x}$

general solution

$\int \frac{6y}{x^2}dx + \int 0\,dy = c$

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$$\frac{6y}{x} = c$$

$$-6y = cx$$

Now at $x = 2, y = 6$

$$-6(6) = c(2)$$

$$-12 = 2c$$

$$c = -6$$

$$\frac{-6y}{x} = -6$$

$$-6y = -6x$$

$$-6y + 6x = 0.$$

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ii find the eigen value $\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

Sol:-

let $A = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$

let "t" is eigen value of A then

$$\det\left[A - tI\right] = 0$$

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$$\det\left\{\begin{bmatrix}2 & -1 & -1 & 0\\ -1 & 3 & -1 & -1\\ -1 & -1 & 3 & -1\\ 0 & -1 & -1 & 2\end{bmatrix} - \begin{bmatrix}t & 0 & 0 & 0\\ 0 & t & 0 & 0\\ 0 & 0 & t & 0\\ 0 & 0 & 0 & t\end{bmatrix}\right\}$$

$$\det\begin{bmatrix}2-t & -1 & -1 & 0\\ -1 & 3-t & -1 & -1\\ -1 & -1 & 3-t & -1\\ 0 & -1 & -1 & 2-t\end{bmatrix} = 0$$

Expanding along R_1

$$(2-t)\begin{vmatrix}3-t & -1 & -1\\ -1 & 3-t & -1\\ -1 & -1 & 2-t\end{vmatrix} + 1\begin{vmatrix}-1 & -1 & -1\\ -1 & 3-t & -1\\ 0 & -1 & 2-t\end{vmatrix} - \begin{vmatrix}-1 & 3-t & -1\\ -1 & -1 & -1\\ 0 & -1 & 2-t\end{vmatrix} = 0$$

$$(2-t)\{(3-t)((3-t)(2-t)-1)+1(t-2-1)-1(1+3-t)\} + \{-1((3-t)(2-t)-1)+1(t-2-0)-1(1-0)\} - 1\{-1(t-2-1) - (3-t)(t-2-0)-1(1-0)\} = 0$$

$$(2-t)\{(3-t)(6-3t-2t+t^2-1)+(t-3)-(4-t)\}+\{-1(6-3t-2t+t^2-1)+(t-2)-1\} - 1\{-1(t-3)-(3-t)(t-2)-1\} = 0$$

$$(2-t)\{(3-t)(t^2-5t+5)+(t-3)-4+t\}+\{-6+5t-t^2+1+t-2-1\} - 1\{3-t-(3t-6-t^2+2t)-1\} = 0$$

$$(2-t)\{3t^2-15t+15-t^3+5t^2-5t+2t-7\}+(-t^2+6t-8) - (3t-3t+6+t^2-2t) = 0$$

$$(2-t)(-t^3+8t^2-18t+8) - t^2+6t-8+t^2\,6t-8 = 0$$

$$\Rightarrow -2t^3+16t^2-36t+\cancel{16}+14-8t^3+18t^2-8t-2t^2+12t\cancel{-16} = 0$$

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$$t^4 - 10t^3 + 32t^2 - 32t = 0$$

$$\Rightarrow t\,(t^3 - 10t^2 + 32t - 32) = 0$$

$$t = 0 \quad t^3 - 10t^2 + 32t - 32 = 0$$

Now $t^3 - 10t^2 + 32t - 32 = 0$

at $t = 2$

$$L.H.S = 2^3 - 10(2)^2 + 32(2) - 32$$

$$= 8 - 40 + 64 - 32 = 72 - 72 = 0$$

using sythetic division

2	1	-10	32	-32
	↓	2	-16	32
	1	-8	16	0

$$\Rightarrow t^2 - 8t + 16 = 0$$

$$\Rightarrow t^2 - 4t - 4t + 16 = 0$$

$$\Rightarrow t(t-4) - 4(t-4) = 0$$

$$(t-4)(t-4) = 0$$

$$t = 4, 4$$

So the required eigen values of matric A are 0, 2, 4, 4

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Q3- The rate of change in the form of differential equation is given by $(x^2+3y^2)dx - 2xy\,dy = 0$. find the general solution at $x=2$ & $y=6$.

Sol:-

$$(x^2+3y^2)dx - 2xy\,dy\,x$$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} = \frac{1+3(y/x)^2}{2(y/x)}$$

its Homogeneous

Put $y = vx \Rightarrow y/x = v$ Diff w.r.t x

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{1+3v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+3v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1+3v^2-2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\frac{2v}{1+v^2}dv = \frac{dx}{x}$$

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$$\int \frac{2v}{1+v^2}\,dv = \frac{dx}{x}$$

$$\ln(1+v^2) = \ln x + \ln c$$

$$1+v^2 = cx$$

$$1+\frac{y^2}{x^2} = cx$$

$$\frac{x^2+y^2}{x^2} = cx$$

Now put $x=2$, $y=6$ we get

$$\frac{4+36}{4} = 2c$$

$$\frac{40}{4} = 2c$$

$$10 = 2c$$

$$\boxed{c=5}$$

Thus general solution is given by

$$\frac{x^2+y^2}{x^2} = 5x$$

$$\boxed{x^2+y^2 = 5x^3}$$

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