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Section: A

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Q NO 1 Part - (a)

(i) $W = \sin(x+ct) + \cos(2x+2ct)$

Given
$$\frac{\partial^2 w}{\partial t} = c^2 \frac{\partial^2 w}{\partial x^2} \longrightarrow (1)$$

Now
$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial t}\left[\sin(x+ct) + \cos(2x+2ct)\right]$$

$$= \frac{\partial}{\partial t}(\sin(x+ct) + \frac{\partial}{\partial t}(\cos(2x+2ct))$$

$$\frac{\partial w}{\partial t} = c\cos(x+ct) - 2c\sin(2x+2ct)$$

Now
$$\frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial t}\left[c\cos(x+ct) - 2c\sin(2x+2ct)\right]$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)}$$

Now $\frac{\partial w}{\partial x} = \frac{\partial}{\partial x}\left[\sin(x+ct)+\cos(2x+2ct)\right]$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - 2\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial x}\left[\cos(x+ct) - 2\sin(2x+2ct)\right]$$

$$\frac{\partial w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)$$

① $\Rightarrow$ $-c^2\sin(x+ct) - 4c^2\cos(2x+2ct) =$

$c^2\left[-\sin(x+2ct) - 4\cos(2x+2ct)\right]$

$-\cancel{c^2\sin(x+ct)} - \cancel{4c^2\cos(2x+2ct)} = -\cancel{c^2\sin(x+ct)}$
$-\cancel{4c^2\cos(2x+ct)}$

$0 = 0$ (satisfied)

(Part - b)

$w = \tan(2x + ct)$

Now $\frac{\partial w}{\partial t} = c\sec^2(2x+ct)$

& $\frac{\partial^2 w}{\partial t} = \frac{\partial}{\partial t}(c\sec^2(2x+ct))$

$= c_2^2 \cdot 2\sec^2(2x+ct)\tan(2x+ct)$

Now $\frac{\partial w}{\partial x} = 2\sec^2(2x+ct)$

$\frac{\partial^2 w}{\partial x^2} = 4\sec^2(2x+ct)\tan(2x+ct)$

① $\Rightarrow 4c^2\sec^2(2x+ct)\tan(2x+ct) = 4c^2\sec^2(2x+ct)\tan(2x+ct)$

$0 = 0$ (satisfied)

Q NO2

Expened the following function in a Fourier Series.

$$f(x) = x \quad \pi < x \leq 0$$
$$= 2x \quad 0 < x \leq \pi$$

Solu: Fourier Series is written as

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left(\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} x\,dx + \int_{0}^{\pi} 2x\,dx\right.$$

$$= \frac{1}{\pi}\left(\frac{x^2}{2}\Big|_{-\pi}^{0} + x^2\Big|_{0}^{\pi}\right.$$

$$= \frac{1}{\pi}\left(-\frac{\pi^2}{2} + \pi^2\right)$$

$$a_0 = \frac{1}{\pi}\left(\frac{\pi}{2}\right)^2 \neq \frac{\pi}{2}$$

(5)

$$b_n = \frac{1}{\pi} \int_{a}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^{0} x \sin n dx + \int_{0}^{\pi} 2x \sin n dx \right]$$

$$= \frac{1}{\pi} \left[\left(\frac{\sin nx}{n} \Big|_{-\pi}^{0} - x \frac{\cos(nx)}{n} \Big|_{-\pi}^{0} \right) \right] +$$

$$\frac{1}{\pi} \left[\frac{2\sin(nx)}{n} \Big|_{0}^{\pi} - \frac{2x \cos(nx)}{n} \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{\pi} \left[-\frac{x\cos(nx)}{n} \Big|_{-\pi}^{0} \right] + \frac{1}{\pi} \left[\frac{2x \cos(nx)}{n} \Big|_{0}^{x} \right]$$

$$= \frac{1}{\pi} \left[-\frac{0 \cos(n0)}{n} - \left(-\frac{\pi \cos n(\pi)}{n} \right) \right] + \cancel{\frac{1}{\pi} \left[2\pi \right.}$$

$$+ \frac{1}{\pi} \left[\frac{2\pi \cos(n^2)}{n} - \cancel{\frac{2(0) \cos(0)x}{n}} \right]$$

$$= \frac{1}{\pi} \left[\frac{\pi \cos(n\pi)}{n} + \frac{2\pi \cos(n\pi)}{n} \right]$$

$$= \frac{1}{\pi}\left[\pi \frac{\cos(nx)}{n} + 2\pi \frac{\cos(nx)}{n}\right]$$

$$= \frac{1}{\pi}\left[\pi + 2\pi\right] \frac{\cos(n\pi)}{n}$$

$$b_n = \frac{1}{\cancel{\pi}} 3\cancel{\pi} \frac{\cos(n\pi)}{n} = \frac{3(-1)^n}{n}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} x\cos nx\, dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\cos nx\, dx$$

$$= \frac{1}{\pi}\left[x\int_{-\pi}^{0}\cos nx\, dx - \int_{-2}^{0}\cos nx \frac{d}{dx}(x)\, dx\right] + \frac{1}{\pi}$$

$$\left[2x\int_{0}^{\pi}\cos nx - \int_{0}^{\pi}\cos nx \frac{d}{dx}(2x)\right] dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\sin nx}{n}\right)_{-\pi}^{0} - \int_{-\pi}^{0}\cos nx\, dx\right] + \frac{1}{\pi}\left(2x \cdot \frac{\sin nx}{n}\right)_{0}^{2}$$

$$\int \cos nx \cdot 2dx$$

P.T.O

Q NO3 $y'' - 4y' + 13y = 8\sin 3x \rightarrow$ ①

$y(0) = 1$

$y'(0) = 2$

Solu:-

Associated Homogenous Eq of ①

is

$$y'' - 4y' + 13y = 0 \rightarrow ②$$

change ② into Auxiliary Equation.

put $y = m$ in ②

$$m^2 - 4m + 13 = 0$$

use Quadratic formula.

$a = 1$, $b = -4$, $c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$m = \frac{4 \pm \sqrt{-36}}{2}$$

P.T.O

$$= \frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$m = 2 \pm 3i$$

$$\boxed{\begin{aligned} m_1 &= 2+3i \\ m_2 &= 2-3i \end{aligned}}$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \longrightarrow \text{(A)}$$

let

$$y_p = A\cos 3x + B\sin 3x \longrightarrow (*)$$

Diff w.r.t. x

$$y_p' = -3A\sin 3x + 3B\cos 3x$$

Again Diff w.r.t "x"

put in (i)

$$\Rightarrow (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos 3x) +$$

$$13(A\cos 3x + B\sin 3x) = 8\sin 3x$$

$\Rightarrow -9A\cos 3x - 12B\cos 3x + 13A\cos 3x$

$-9B\sin 3x + 12A\sin 3x + 13B\sin 3x = 8\sin 3x$

$\Rightarrow (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x =$

$8\sin 3x$

$\Rightarrow (4A - 12B)\cos 3x + (4B + 12A)\sin 3x = 8\sin 3x$

Comparing Co-efficients

$\sin 3x \Rightarrow$ $4B + 12A = 8 \rightarrow$ ⓐ

$4A - 12B = 0 \Rightarrow 4A = 12B$

$\Rightarrow \boxed{A = 3B} \rightarrow$ ⓑ

put b in ⓐ

$4B + 12(3B) = 8$

$4B + 36B = 8$

$40B = 8$

$\boxed{B = 1/5} \rightarrow$ ①

put (c) in (b)

$\Rightarrow$ $A = 3/5$ $\rightarrow$ (d)

put (c) and (d) in (*)

$$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \longrightarrow (B)$$

The G.Sol is

$$y = y_c + y_p$$

$$y = e^{2x}(c_1\cos 3x + c_2\sin 3x) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \longrightarrow (C)$$

Now we need to find the values of c_1 & c_2 for this

put $x = 0$ & $y = 1$ in (C)

$$1 = e^{2(0)}(c_1\cos 3(0) + c_2\sin 3(0)) + \frac{3}{5}\cos 3(0) + \frac{1}{5}\sin 3(0)$$

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$$1 = (C_1(1) + C_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$$

$$1 = C_1 + \frac{3}{5}$$

$$C_1 = 1 - \frac{3}{5}$$

$$\boxed{C_1 = \frac{2}{5}} \longrightarrow (xx)$$

Diff: (C) w.r.to "x"

$$y' = C_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + C_2(2e^{2x}\sin 3x + 3e^{2x}\cos 2x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \longrightarrow (D)$$

put $y' = 2$, $x = 0$ in (D)

$$y' = C_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + C_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x) - \frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$$

put $y' = 2$, $x = 0$

$$2 = C_1\left(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0)\right) +$$

$$C_2\left(2e^{2(0)}\sin 3(0) + 3e^{2(0)}\cos 3(0)\right) - \frac{6}{5}\sin 3(0)$$

$$+ \frac{3}{5}\cos 3(0)$$

$$2 = C_1(2) + C_2(3) - 0 + 3/5$$

$$2 = 2C_1 + 3C_2 + 3/5$$

put $\quad C_1 = 2/5$

$$2 = 4/5 + 3C_2 + 3/5$$

$$2 = 7/5 + 3C_2$$

$$3C_2 = 2 - 7/5$$

$$3C_2 = 3/5$$

$$\boxed{C_2 = 3/15} \longrightarrow (xxx)$$

put (xx) & (xxx) in (C)

$$y = e^{2x}\left(\frac{2}{5}\cos 3x + \frac{3}{15}\sin 3x\right) + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x$$

$$\boxed{y = \frac{2}{5}e^{2x}\cos 3x + \frac{3}{15}e^{2x}\sin 3x + \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x}$$

Q No 4

Solve.
↳ $(D^2 - DD')_z = \cos x \cos 2y$

Solution: we have

$$(D^2 - DD')_z = \cos x \cos 2y \longrightarrow (1)$$

The Associated Homogenous Eq:

$$(D^2 - DD')_z = 0$$

put $D_z = m$

$$((m^2) - (m)(1)) = 0$$

$$m^2 - m = 0$$

$$m(m-1) = 0$$

$$m = 0 \quad , \quad m = 1$$

Thus $Z_c = f_1(y) + f_2(y+x) \longrightarrow (2)$

for particular Solution

let

$$Z_p = \frac{1}{D^2 - DD'} \cdot \cos x \cos 2y$$

$$Z_p = \frac{1}{2} \cdot \frac{1}{D^2 - DD'} [\cos(x-2y) + \cos(x-2y)]$$

$$Z_p = \frac{1}{2}\left[\frac{1}{D^2 - DD'}[\cos(x-2y) + \frac{1}{D^2 - DD'}(\cos(x-2y)\right]$$

using the Integral / Diff:

$$Z_p = \frac{1}{2}\left[\frac{1}{-1+2}\cos(x+2y) + \frac{1}{-1-2}\cos(x-2y)\right]$$

$$= \frac{1}{2}\left[1 \cos(x+2y) - \frac{1}{3}\cos(x-2y)\right]$$

$$Z_p = \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

Now The General Solution is.

$$Z = Z_c + Z_p$$

$$Z = f_1(y) + f_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)$$

$$\boxed{Z = f_1(y) + f_2(y+x) + \frac{1}{2}\cos(x+2y) - \frac{1}{6}\cos(x-2y)}$$

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