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Subject : probability and Statistics.

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Q: A man throws two fair dice. What is the conditional probability that the sum of the two dice will be 7 given that.
(1) the sum is even
(2) the sum is greater than 8
(3) The two dice had the same outcome

Ans:
The sample space S for this experiment is.

$S = \{(1,1)(1,2),(1,3)(1,4),(1,5),(1,6)(2,1)(2,2)(2,3)(2,4)$
$(2,5),(2,6),(3,1),(3,2),(3,3)(3,4)(3,5),(3,6)(4,1)$
$(4,2),(4,3),(4,4),(4,5)(4,6),(5,1),(5,2),(5,3)(5,4)$
$(5,5),(5,6),(6,1),(6,2)(6,3)(6,4),(6,5),(6,6)\}$

let
$A = \{$the sum is $7\}$
$B = \{$the sum is odd$\}$
$C = \{$the sum is greater than $8\}$
and $D = \{$the two dice had the same outcomes$\}$

$A = \{(1,6),(2,5),(3,4)(4,3)(5,2),(6,1)\}$
$B = \{(1,2),(1,4)(1,6)(2,1)(2,3)(2,5)(3,2)(3,4) \ldots (6,5)$
$C = \{(1,6),(2,5)(2,6),(3,4)(3,5),(3.6),(4,3)(4,4) \ldots (6,6)$
$D = \{(1,1)(2,2),(3,3)(4,4)(5,5)(6,6)\}$
$A \cap B = \{(1,6),(2,5),(3,4)(4,3),(5,2)(6,1)\}$
$A \cap C = \{(1,6),(2,5),(3,4)(4,3),(5,2),(6,1)\}$
$A \cap D = \emptyset$

$P(A) = \frac{6}{36}$, $P(B) = \frac{18}{36}$, $P(C) = \frac{21}{36}$

$P(D) = \frac{6}{36}$

$P(A \cap B) = \frac{6}{36}$, $P(A \cap C) = \frac{6}{36}$ and

$P(A \cap D) = 0$

Hence

$P(A|B) = P(A \cap B) = \frac{6}{36} \times \frac{36}{18} = \frac{1}{3}$

$P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{6}{36} \times \frac{36}{21} = \frac{2}{7}$

$P(A|D) = \frac{P(A \cap D)}{P(D)} = \frac{0 \times 36}{6} = 0$

Q2 Sum of 2 has 1 way 1,1
Sum of 3 has 2 ways 1,2 and 2,1
Sum of 4 has 3 ways 1,3; 2,2; 3,1.
5 has 4 ways
6 has 5 ways
8 has 5 ways (symmetry)
9 has 4 ways
10 has 3 ways
11 has 2 ways
12 has 1 way
Those are 15/36 for each side
With a sum of 3 0/36
that leaves a 6/36 = 1/6 probability
for a sum o f 7.

Q3 A and B play a game in which A's probability of winning 2/3 In a series of 8 games what is the probability that A will win

1. Exactly 4 games
2. At least 4 games
3. From 3 to 6 games.

Sol: Given that $p = \frac{2}{3}$ $n = 8$

$q = 1 - p$

$= 1 - \frac{2}{3}$

$q = \frac{1}{3}$

Let "x" denotes the number of games won by A, then

i $P(X=4)$

$= \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4$

$= \frac{1120}{6561}$

$= 0.1707$

(ii) $P(x \geq 4)$

$1 - P(X < 4)$

$= 1 - \sum_{x=0}^{3} \binom{8}{x} \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{8-x}$

$= 1 - \left[\left(\frac{1}{3}\right)^8 + 8\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^7 + 28\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^6 + 56\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^5\right]$

$= 1 - \frac{1}{6561}\left[1 + 16 + 112 + 448\right]$

$= 1 - \frac{577}{6561}$

$= \frac{6561 - 577}{6561} \Rightarrow \frac{5984}{6561}$

$= 0.9121$

Q3

(8)

(iii) $P(3 \leq x \leq 6)$

$$\sum_{x=3}^{6} \binom{8}{x} \left(\frac{2}{3}\right)^{x} \left(\frac{1}{3}\right)^{8-x}$$

$$= \binom{8}{3} \left(\frac{2}{3}\right)^{3} \left(\frac{1}{3}\right)^{5} + \binom{8}{4} \left(\frac{2}{3}\right)^{4} \left(\frac{1}{3}\right)^{4} + \binom{8}{5} \left(\frac{2}{3}\right)^{5} \left(\frac{1}{3}\right)^{3} + \binom{8}{6} \left(\frac{2}{3}\right)^{6} \left(\frac{1}{3}\right)^{2}$$

$$= \frac{8}{(3)^8} \left[56 + 140 + 224 + 224 \right]$$

$$= \frac{8 \times 644}{6561}$$

$$= \frac{5152}{6561} = 0.7852$$

Q5 Derive the binomial distribution and find its mean and variance.

Ans: Derivation of binomial ~~...~~ distribution:

To derive the formula that gives the probability of successes in n trails for a binomial experiment, we proceed as follows.

The experiment has n trials of which may result in S or F. The sample space has 2^n possible sample points or outcome, each outcome consisting of a sentence $(a_1, a_2 \ldots\ldots a_n)$ where each a_i is either S or F. we desire to find the probability of these outcomes according to the number of successes.

First we consider the probability of zero successes i.e $P(X=0)$. In case of zero success every trial results in F and the event consists of a sequence of n Fs, i.e (FF....f).

$$P(FF\ldots f) = P(F)\,P(F)\,P(f) \cdots P(f) \text{ (n times)}$$

$= q^n$

Since there is only one sequence of outcome of n trials resulting in f_s therefore

$P(x=0) = q^n$

Next we consider the probability of one success i.e $P(X=1)$ in this case one results in S and the remaining $(n-1)$, trials results in f_s.

Mean and Variance :

Let x be a random variable with binomial distribution $b(x, n, p)$ Then its mean and variance are given by $\mu = np$ and $\sigma^2 = npq$ respectively

Mean $\mu = E(x)$

$$= \sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x}, \quad \text{where } x = 0,1,2, \ldots n$$

$$= 0 \cdot q^n + 1 \binom{n}{1} q^{n-1} p + 2 \binom{n}{2} q^{n-2} p^2 + \ldots np^n$$

$$= np\left[q^{n-1} + \binom{n-1}{1} q^{n-2} p + \binom{n-1}{2} q^{n-3} p^2 + \ldots + p^{n-1}\right]$$

$$= np(q+p)^{n-1}$$

$= np$, because $q + p = 1$

by definition of variance

$$\sigma^2 = E(x-\mu)^2 = E(x^2) - [E(x)]^2$$

But

$$E(x^2) = E[x(x-1) + x] = E[x(x-1)] + E(x)$$

$$= E(x(x-1)) + np$$

Now

$$E[(x(x-1)] = \sum_{x=0}^{n} x(x-1) \binom{n}{x} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1) \cdot \frac{n(n-1)(n-2)!}{x(x-1)(x-2)!(n-x)!} p^2 p^{x-2} q^{n-x}$$

$$= n(n-1)p^2 \sum_{x=2}^{n} \frac{(n-2)!}{(x-2)!\,(n-x)!} p^{x-2} q^{n-2}$$

(x starts at 2 since $x = 0, 1$ add nothing to the sum)

The term $(n-x)$ may be written as $(n-2)-(x-2)$. Substituting $y = x-2$ and $m = n-2$ in the summation we obtain

$$E[X(X-1)] = n(n-1)p^2 \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y q^{m-y}$$

$$= n(n-1)p^2 \sum_{y=0}^{m} \binom{m}{y} p^y q^{m-y}$$

$$= n(n-1)p^2 \quad \because \text{ Summation is } 1$$

Thus $\sigma^2 = E(X^2) - [E(X)]^2$

$$= E[X(X-1)] + E(X) - [E(X)]^2$$

$$= n(n-1)p^2 + np - (np)^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np - np^2 = np(1-p) = npq \text{ and}$$

$$\sigma = \sqrt{npq}$$

Hence the variance of the number of successes is npq, and the standard deviation is $\sqrt{npq}$.

Q6 Differentiate between Binominal frequency distribution and Binorminal distribution with the help of formula?

Ans 6 The Binomial is denoted and formulated by

$$f(x) = P(X=x) = \binom{n}{x} p^x q^{n-x}$$

Where,

$x = 0, 1, 2 ---- n.$

it shows only the probability of an Individual.

"Binominal frequency"

if the binomial probability distribution is multiplied by N, the number of experiments or sets the resulting distribution is known as the binomial frequency distribution. thus the expected frequency of x successes in N exeriments is $N\binom{n}{x} p^x q^{n-x}$ I, Should be noted that the the n independent trials constitute one experiment or one set.

Q7 Solluation.

Measure	Data Set A	B	C	D
Co-efficient of Veriation.	$CV = \frac{3}{45} \times 100$	$CV = \frac{11}{60} \times 100$	$CV = \frac{5}{50} \times 100$	$CV = \frac{15}{25} \times 100$
	$CV = 6.7$	$CV = 18.3$	$CV = 10$	$CV = 60$

Q4 Proof.

Since the Ci's form a Partition of the sample Space. We can apply the law of total Probability for $A \wedge B$

$P(A \wedge B) = \sum P(A \wedge B / C_i) P(C_i)$

$P(A \wedge B) = \sum P(A/C_i) P(B/C_i) P(C_i)$

(A and B are Conditionally Independent.

$P(A \wedge B) = \sum P(A/C_i) P(B) P(C_i)$

∴ (B is independent of all Ci

$P(A \wedge B) = P(B) \sum P(A/C_i) P(C_i)$

$P(A \wedge B) = P(B) P(A)$

law of total Probability

Hense A and B are Independent.