

Question:-

A yarn merchant sells brands A, B, C of yarn each of which is a blend of Pakistani, Egyptian & American Cotton in the ratio 1:2:1, 2:1:1 & 2:0:2 respectively. If one kg of A, B, C costs 40, 50, 60 rupees respectively, find the cost of a kg of cotton of each country.

Solution:

40	
P	E
A	E
A	

50	
P	P
A	E
B	

60	
P	P
A	A
C	

Let x, y, z be the cost/kg of the Pakistani, Egyptian & American cotton respectively. The according to the given condition.

$$\left. \begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}x + \frac{2}{4}z &= 60 \end{aligned} \right\} (S')$$

$$\left. \begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned} \right\} (S)$$

In matrix form, we can write it as:

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \quad \underline{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \underline{b} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}.$$

$$A \underline{x} = \underline{b} \quad A_1 \times B_1.$$

Now Using Cramer's Rule.

$$A x = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix} \quad \text{we just replace by 1st column of } A_1.$$

$$\begin{aligned} |A_1 x| &= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + \begin{vmatrix} 200 & 0 \\ 120 & 0 \end{vmatrix} \\ &= 160(1-0) - 2(200-120) + 1(0-120) \\ &= 160 - 2(80) - 120 = -160 - 160 = -320. \end{aligned}$$

$$|A_1 x| = -320.$$

$$\text{Now } x = \frac{|A_1 x|}{|A|} \rightarrow (4).$$

Now find $|A_{01}|$

$$|A_{01}| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1(1-0) - 2(2-1) + 1(0-1).$$

$$= 1 - 2 - 1 = -2.$$

Now, (4) $\Rightarrow x \frac{|A_{1x}|}{|A_1|} = \frac{-120}{-2}$

$$x = A = 60$$

also, $y = \frac{|A_{1y}|}{|A_1|} \rightarrow (5)$

$A, y \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$ Just replace B in 2nd column of A_1 .

$$|A, y| = 1(200 - 120) - 160(2 - 1) + 1(240 - 200)$$

$$= 80 - 160 + 40$$

$$|A, y| = -40$$

(5) $\Rightarrow y = \frac{|A, y|}{|A_1|} = \frac{40}{-2} = 20$

$$y = B = 20$$

again $z = \frac{|A_{1z}|}{|A_1|} \rightarrow (6)$

$$A, z = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

$$\begin{aligned}
 |A, z| &= 1(120 - 0) - 2(240 - 200) - 120(0) \\
 &= 120 - 80 - 160 \\
 &= -120
 \end{aligned}$$

$$\textcircled{6} \Rightarrow z = \frac{|A, z|}{|A_1|} = \frac{-120}{-20} = 60$$

$$z = c = 60$$

Hence

$$(x, y, z) = (60, 20, 60)$$

OR:

$$(A, B, C) = (60, 20, 60)$$

It's mean that,

Pakastani Blande cost/kg of cotton = 60

Egyptian " " " " = 20

American " " " " = 60