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Q1:- (A) Express the equation of plane passing through the points $A(2,-2,1)$, $B(-1,0,3)$, $C(5,-3,4)$.

Ans:-

Solution:-

The non parallel vectors:-

$$\overline{P_1P_2} = (-3,2,3)$$
$$\overline{P_1P_2} = (3,-1,3)$$

$$(-1,0,3)-(2\ -2\ 1)$$
$$(-1,0,3)\ -2,+2,-1$$

The perpendicular vector is:-

$$m = \overline{P_1P_2} \times \overrightarrow{AB}$$

$$n = \begin{vmatrix} i & j & k \\ -3 & 2 & 2 \\ 3 & -1 & 3 \end{vmatrix}$$

$$\overrightarrow{P_1P_2} = \sqrt{(x_2-x_1)+(y_2-y_1)^2}$$
$$\overrightarrow{P_1P_2} = \sqrt{(-1-2)^2}$$

$$m = i(6+2) - j(-9-6) + k(3-6)$$

$$n = 8i + 15j - 3k$$

02

$m = (2, 15, -3)$

Now

$P_1(x_0, y_0, z_0) = (2, -3, 1)$

$m(a, b, c) = (8, 15, -3)$

So equation of plane is

$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

$8(x - 2) + 15(y + 2) - 3(z - 1) = 0$

$8x + 15y - 3z - 16 + 30 + 3 = 0$

$8x + 15y - 3z + 17 = 0$ Ans

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03

Q1:-
(B) Express a pair of planes whose intersection is the given line, $x = 2 - 3t,\ y = 3 + t,\ z = 2 - 4t$

Ans

Solution:-

$x - 2 = -3t \Rightarrow t = \frac{x-2}{-3}$

$y - 3 = t \Rightarrow t = \frac{y-3}{1}$

$z - 2 = -4t \Rightarrow t = \frac{z-2}{-4}$

So

$$\frac{x-2}{-3} = \frac{y-3}{1} = \frac{z-2}{-4}$$

So for first plane take 1st and 2nd

$$\frac{x-2}{-3} = \frac{z-2}{-4}$$

$\underline{\underline{04}}$

$$-4x + 8 = -3z + 6$$

$$-4x + 3z + 2 = 0$$

or $\boxed{4x - 3z - 2 = 0}$

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05

Q2:-

$L(x,y) = (x+1, y, x+y)$ illustrate that L is linear transformation?

Ans:-

Solution.

$$L(x,y) = (x+1, y, x+y)$$

Let $u = (x_1, y_1)$ $v = (x_2, y_2)$

$$u+v = (x_1, y_1) + (x_2, y_2)$$

$$u+v = (x_1 + x_2, y_1 + y_2)$$

$$L(u+v) = L(x_1 + x_2, y_1 + y_2)$$

$$L(u+v) = (x_1 + x_2 + 1, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$$

So that $u = (x+y)$

$$L(u) = L(x_1, y_1) = (x_1 + 1 + y_1, x_1 + y - 1)$$

$$L(u) = L(x_2, y_2) = (x_2 + 1, y_2, x_2 + y_2)$$

$$L(u) + L(v) = (x_1 + x_2 + 2, y_1 + y_2, x_1 + x_2 + y_1 + y_2)$$

Since $1 \neq 2$ so not LT

06

Q3 :- Using the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ then interpret to decode the message

77 54 38 71 49 29 68 51 33 76 48 40 86 53 52.

Ans :-

a) code ~~the~~ the message "send him money"

b) Decode the message 67 44 41 49 39 113 76 62 104 69 55

① Send him money

19 5 14 4 89 13 13 15 14 5 25.

② Decode the message

77 54 38 71 49 29 68 51 33 76 48 40 86 53 52

$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ For decoding the A^{-1} is ~~[illegible]~~ must

$A^{-1} = \frac{adj\,A}{|A|}$

$\underline{07}$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$x_5 = \begin{bmatrix} 88 \\ 53 \\ 52 \end{bmatrix} \quad x_1 = \begin{bmatrix} 77 \\ 54 \\ 38 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 68 \\ 51 \\ 33 \end{bmatrix}, x_4 = \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix}$$

$$A^{-1} x_1 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 77 \\ 54 \\ 58 \end{bmatrix} = \begin{bmatrix} 16 \\ 8 \\ 15 \end{bmatrix}$$

$$A^{-1} x_2 = \begin{bmatrix} 11 \end{bmatrix} \begin{bmatrix} 71 \\ 49 \\ 29 \end{bmatrix} = \begin{bmatrix} 20 \\ 15 \\ 7 \end{bmatrix}$$

$$A^{-1} x_3 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 62 \\ 51 \\ 73 \end{bmatrix} = \begin{bmatrix} 18 \\ 1 \\ 15 \end{bmatrix}$$

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$$A x_4 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 76 \\ 48 \\ 40 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \\ 12 \end{bmatrix}$$

$$A x_5 = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 86 \\ 53 \\ 52 \end{bmatrix} = \begin{bmatrix} 1 \\ 14 \\ 19 \end{bmatrix}$$

A	B	C	D	E	F....		w	x	y	z.
1	2	3	4	5	6		23	24	25	26

Shayan $\longrightarrow$ Basit

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}.$$

m = 15, e = 5, e = 5, t = 20, t = 20

o = 15, m = 13, o = 15, r = 18, r = 18

o = 15   w = 23.

09

$$x_1 = \begin{bmatrix} 15 \\ 5 \\ 5 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 20 \\ 20 \\ 15 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 13 \\ 15 \\ 18 \end{bmatrix}, \quad x_4 = \begin{bmatrix} 18 \\ 15 \\ 23 \end{bmatrix}$$

$$Ax_1 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 15 \\ 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 38 \\ 28 \\ 15 \end{bmatrix}$$

$$Ax_2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 20 \\ 20 \\ 15 \end{bmatrix} = \begin{bmatrix} 105 \\ 70 \\ 50 \end{bmatrix}$$

$$Ax_3 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 13 \\ 15 \\ 28 \end{bmatrix} = \begin{bmatrix} 97 \\ 64 \\ 51 \end{bmatrix}$$

$$Ax_4 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 18 \\ 15 \\ 23 \end{bmatrix} = \begin{bmatrix} 119 \\ 79 \\ 61 \end{bmatrix}$$

so Shoyon send the message

38 28 15 105 70 50 97 64 51 119 79.

10

Q 4: Find an equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to the vector $n = (0, 1, -3)$

Ans: Solution:

Equation of plane

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

So that

$$P = (x_0, y_0, z_0) = (-1, 3, 2)$$
$$n = (a, b, c) = (0, 1, -3)$$

So

$$0(x - (-1)) + 1(y - 3) - 3(z - 2)$$
$$0(x + 1) + 1(y - 3) - 3(z - 2)$$
$$0 + y - 3 - 3z + 6$$
$$\Rightarrow y - 3z - 3 + 6$$
$$\Rightarrow y - 3z + 3 \text{ Ans}$$

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Q5:- $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$; $A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Eigen values & Eigen Vectors

(1) Eigen values:- $\det(A - \lambda I) = 0$

$$\det \begin{bmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{bmatrix} = (ad-bc) = (1-\lambda)(4-\lambda) - (-2) = 0$$

$$= 4 - \lambda - 4\lambda + \lambda^2 + 2 = \lambda^2 - 5\lambda + 6$$

$$= \lambda^2 - 2\lambda - 3\lambda + 6$$

$$= \lambda(\lambda - 2) - 3(\lambda - 2)$$

$$= (\lambda - 3)(\lambda - 2) = 0$$

$\lambda_1 = 3 \rightarrow$ (1)
$\lambda_2 = 2 \rightarrow$ (2) Eigen values.

(2) Eigen Vectors :=

$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ $\lambda = \boxed{2}, \boxed{3}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1-2 & 1 \\ -2 & 4-2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{aligned} -x + y &= 0 \\ -2x + 2y &= 0 \end{aligned}$$

$$x = y$$

For $\lambda = 2$, Eigen vectors are $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow$ (3)

For $\lambda = 3$

$$A - \lambda I = \begin{bmatrix} 1-3 & 1 \\ -2 & 4-3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-2x + y = 0$$
$$-2x + y = 0$$
$$y = 2x \rightarrow \begin{matrix} x = 1 \\ y = 2 \end{matrix}$$

For $\lambda = 3$, Eigen vectors are $\begin{pmatrix} 1 \\ 2 \end{pmatrix} \rightarrow 4$