CALCULUS AND ANALYTIC GEOMETRY

Mid Term Assignment

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Q#01:- $\frac{2x^3 - 3x^2 + 5}{x^2 + 1}$ (Quotient Rule)

Sol:- $= \frac{(x^2+1)(6x^2 - 6x) - (2x^3 - 3x^2 + 5)(2x)}{(x^2+1)^2}$

$= \frac{6x^4 - 6x^3 + 6x^2 - 6x - 4x^4 + 6x^3 - 10x}{(x^2+1)^2}$

$= \frac{2x^4 + 6x^2 - 16x}{(x^2+1)^2}$

$= \boxed{\frac{2x(x^3 + 3x - 8)}{(x^2+1)^2}}$

Q#02 :- $\frac{(x^2+1)^2}{(x^2-1)}$ Quotient Rule.

$$= \frac{(x^2-1)\left[2(x^2+1)2x\right] - (x^2+1)^2(2x)}{(x^2-1)^2}$$

$$= \frac{(x^2-1)\left[4x(x^2+1)\right] - 2x(x^2+1)^2}{(x^2-1)^2}$$

$$= \frac{(x^2-1)(4x^3+4x) - 2x(x^2+2x+1)}{(x^2-1)^2}$$

$$= \frac{4x^5 + 4x^3 - 4x^3 - 4x - 2x^3 - 4x^2 - 2x}{(x^2-1)^2}$$

$$= \frac{4x^5 - 2x^3 - 4x^2 - 6x}{(x^2-1)^2}$$

$$= \boxed{\frac{2x(2x^4 - x^3 - 2x - 3)}{(x^2-1)^2}}$$

Q#03

$Y = u^3 \cdot x^{2/3}$ $\quad u = 1 + 2\sqrt{x} \Rightarrow \frac{du}{dx} = \frac{2}{2\sqrt{x}} = \frac{1}{\sqrt{x}}$

$$\frac{dy}{dx} = u^3 \cdot \frac{2}{3} x^{-1/2} + x^{\frac{2}{3}} \cdot 3u \cdot \frac{du}{dx}$$

$$= \frac{2}{3} \frac{u^3}{\sqrt{x}} + x^{2/3} \cdot 3u \cdot \frac{1}{\sqrt{x}}$$

$$= \frac{2u^3 + 6u \cdot x^{2/3}}{3\sqrt{x}}$$

$$= \frac{2u(u^2 + 3x^{2/3})}{3\sqrt{x}}$$

put value of 'u'

$$\boxed{\frac{dy}{dx} = \frac{2(1+2\sqrt{x})\left[(1+2\sqrt{x})^2 + 3x^{2/3}\right]}{3\sqrt{x}}}$$

Q#04. :- $$y = \sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{u}{v}} = \left(\frac{u}{v}\right)^{1/2}.$$

Let $u = 1-x$. and $v = 1+x$.

$\frac{du}{dx} = -1$ $\qquad$ $\frac{dv}{dx} = 1$.

Now

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{u}{v}\right)^{-1/2} \cdot \frac{v \cdot \frac{du}{dx} - u \cdot dv/dx}{v^2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{u}{v}\right)^{-1/2} \cdot \frac{v(-1) - u(1)}{v^2}$$

$$= \frac{1}{2}\left(\frac{u}{v}\right)^{-1/2} \cdot \frac{(-)[(+v) + u]}{v^2} \quad (-) \text{ Common.}$$

$$= -\frac{1}{2\sqrt{(u/v)}} \cdot \frac{v+u}{v^2}$$ putting value of u & v. and solve.

Q#05 $\int \frac{1}{\sqrt{x^3}}\, dx$

$= \int (x^3)^{-1/2}\, dx$

$= \frac{(x^3)^{-1/2+1}}{-1/2+1} = \frac{(x^3)^{1/2}}{1/2}$

$= \boxed{2.\sqrt{x^3}}$

Q#06:- $\int (6x+7)^{-6}\, dx$

let $u = 6x+7$.

$\frac{du}{dx} = 6 \Rightarrow du = 6\,dx$

$\Rightarrow dx = du/6$.

Now put values.

$= \int u^{-6} \cdot \frac{du}{6} = \frac{1}{6}\int u^{-6}\, du.$

$= \frac{1}{6}\frac{u^{-6+1}}{(-6+1)} = \frac{1}{6}\cdot\frac{u^{-5}}{-5} = -\frac{1}{30}\cdot u^{-5}$

$= \frac{-1}{30}(6x+7)^{-5}$ or $\boxed{= \frac{-1}{30(6x+7)^5}}$