# Department of Electrical Engineering
## Sessional Assignment
**Date: 01/06/2020**

**Course Details**

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| **Course Title:** | Digital Signal Processing | **Module:** | 6th |
| **Instructor:** | Sir pir meher Ali Shah | **Total Marks:** | 20 |

**Student Details**

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| Q1. | (a) | Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$. | Marks 6 |
| | (b) | Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$ | |
| Q2. | (a) | Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method) | Marks 6 |
| | (b) | Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$ | |
| Q.3 | (a) | A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of $b_o$ and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$. | Marks 4 |

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| | (b) | Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$. | |
| Q.4 | (c) | A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \geq L$ | Marks 4 |

Ans ① ⓐ $y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$

$x(n)=4^n u(n)$

Solution:-

First we determine the solution to the homogeneous equation. we assume the solution to be the exponential.

$y_h(n)=\lambda^n$

$y_h(n)=C_1(-1)^n+C_2(4)^n$

$y_p(n)=K(4)^n u(n)$

However we observe that $y_p(n)$ is already contained in the homogeneous solution. So the particular solution is redundant.

we assume that;

$y_p(n)=Kn(4)^n u(n)$

Upon substitution we get:

$Kn(4)^n u(n)-3K(n-1)(4)^{n-1}u(n-1)-4K(n-2)(4)^{n-2}u(n-2)=(4)^n u(n)+2(4)^{n-1}u(n-1)$

To determine K, we calculate this equation for any $n \geq 2$, where none of the unit step term vanish. To simplify the arithmetic we select $n=2$ from which we obtain $K = \frac{6}{3}$

Therefore:

$$y_p(n) = \frac{6}{5} n(4)^n u(n)$$

The total solution to the difference equation is obtained by adding these two equation.

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{5} n(4)^n \quad n \geq 0$$

where $C_1$ and $C_2$ are determined such that the initial conditions are satisfied.

$$y(0) = 3y(-1) + 4y(-2) + 1$$

$$y(1) = 3y(0) + 4y(-1) + 6$$

$$= 13y(-1) + 12y(-2) + 9$$

On the other hand, it is evaluated that at $n=0$ and $n=1$ yields.

$$y(0) = c_1 + c_2$$

$$y(1) = -c_1 + 4c_2 + \frac{24}{5}$$

we can simplify the computations above by setting $y(-1) = y(-2) = 0$

$$c_1 + c_2 = 1$$

$$-c_1 + 4c_2 + \frac{24}{5} = 9$$

Hence $c_1 = \frac{-1}{25}$ and $c_2 = \frac{26}{25}$.

Finally we have the zero-state response to the forcing function $x(n) = (4)^n u(n)$ in the form

$$y_{zs}(n) = \frac{-1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5} n(4)^n \quad n \geq 0$$

Ans (1)(b) $y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$

Solution:-

$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$

$y(n) - 0.6y(n-1) + 0.8y(n-2) = x(n)$

To get the homogeneous equation put $x(n) = 0$

$y(n) - 0.6y(n-1) + 0.8y(n-2) = 0$

Now determine the solution to the homogeneous equation

$y_h(n) = \lambda^n$

$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$

$\lambda^2 - 0.6\lambda + 0.8 = 0$

$(\lambda - 0.2)(\lambda - 0.4) = 0$

$\lambda_1 = 0.2, \lambda_2 = 0.4$

So the general form of the solution to the homogeneous equation will be;

$y_h(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n$

$y(n) = c_1(0.2)^n + c_2(0.4)^n$ — (1)

$\lambda_1 = 0.2$ and $\lambda_2 = 0.4$

$$y_h(n) = c_1 \left(\frac{1}{5}\right)^n + c_2 \left(\frac{2}{5}\right)^n$$

with $x(n) = \delta(n)$ the initial condition are;

$y(0) = 1$

$y(1) - 0.6y(0) = 0$

$y(1) = 0.6$

Hence $c_1 + c_2 = 1$ and

$\frac{1}{5}c_1 + \frac{2}{5} = 0.6$

$c_1 = -1, \quad c_2 = 3$

Therefore $h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right]u(n)$

The step response is

$$\delta(n) = \sum_{k=0}^{n} h(n-k), \quad n > 0$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\frac{(1)}{5}^{n+1} - 1\right]\right\}u(n)$$

Ans (2) (a) $x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$

Solution:-

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

By partial fractional method

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$= \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1}) \quad \text{— (A)}$$

Now put $z = 1$ in equ (A)

$1 = A(1-1)^2 + B(1-2)(1-1) + C(1)(1-2)$

$1 = 0 + 0 - C$

$\{C = -1\}$

Now put $z = 2$ in equ (A)

$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) +$$

$$C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(0)$$

$$1 = \frac{A}{4} + 0 + 0$$

$$\{A = 4\}$$

Now put $x = 3$ in eqn (A)

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$$

$$1 = \frac{4A}{9} + \frac{2B}{9} + \frac{C}{9}$$

put value of A and C

$$1 = \frac{4\times4}{9} + \frac{2B}{9} + \frac{(-1)}{9}$$

$$1 = \frac{16}{9} + \frac{2B}{9} - \frac{1}{9}$$

$$1 + \frac{1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$\frac{9+1}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$\frac{10}{9} - \frac{16}{9} = \frac{2}{9}B$$

$$\frac{2}{9}B = \frac{10-16}{9}$$

$$\frac{2}{9}B = \frac{-6}{9}$$

$$B = \frac{-\cancel{6}^{3}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{2}_{1}}$$

$$\{B = -3\}$$

Hence;

$$x(n) = [4(2)^n - 3 - n]\,u(n)$$

Ans ② ⓑ $X(Z) = \dfrac{1}{1-1.5Z^{-1}+0.5Z^{-2}}$

Solution:-

First we will eliminate the negative powers by multiplying both numerator and denominator by $Z^2$, thus

$$X(Z) = \frac{Z^2}{Z^2-1.5Z+0.5}$$

The poles of $X(Z)$ are $P_1 = 1$ and $P_2 = 0.5$. So the expansion of the form is;

$$\frac{X(Z)}{Z} = \frac{Z}{(Z-1)(Z-0.5)} = \frac{A_1}{Z-1} + \frac{A_2}{Z-0.5}$$

$$Z = (Z-0.5)A_1 + (Z-1)A_2 \quad \text{— (1)}$$

if we set $Z = P_1 = 1$

$$1 = (1-0.5)A_1$$

So we obtain that $A_1 = 2$

Now again put $Z = P_2 = 0.5$ in equ (1)

$$0.5 = (0.5-1)A_2$$

Hence $A_2 = -1$

The result of the partial fraction expansion is;

$$\frac{X(Z)}{Z} = \frac{2}{(Z-1)} - \frac{1}{(Z-0.5)}$$

The general form will be;

$$\frac{(Z-P_K)\,XZ}{Z} = \frac{(Z-P_K)A_1}{Z-P_1} + \cdots\cdots A_K + \cdots\cdots + \frac{(Z-P_K)\,A_N}{Z-P_N}$$

Ans) 3 a $H(z) = \frac{b_0}{(1-pz^{-1})^2}$

$H(0) = 1$

$H(\frac{\pi}{4}) = \frac{1}{2}$

Find $b_0$ and $p$

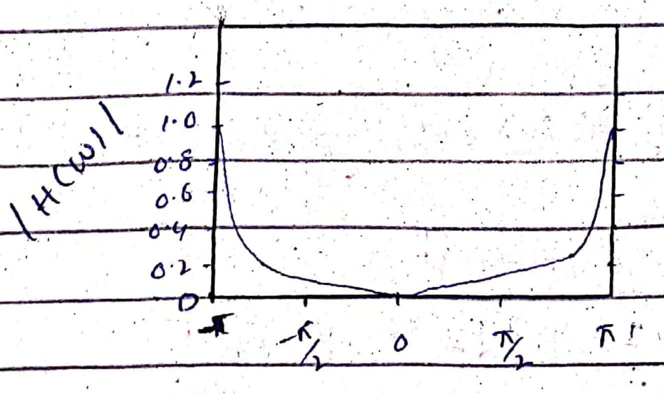
Solution:-

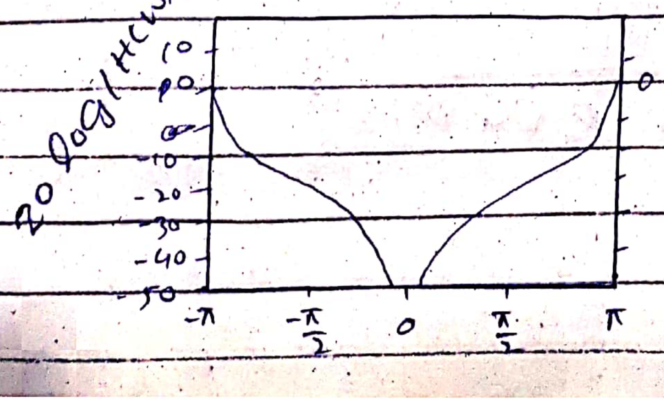
At $\omega = 0$

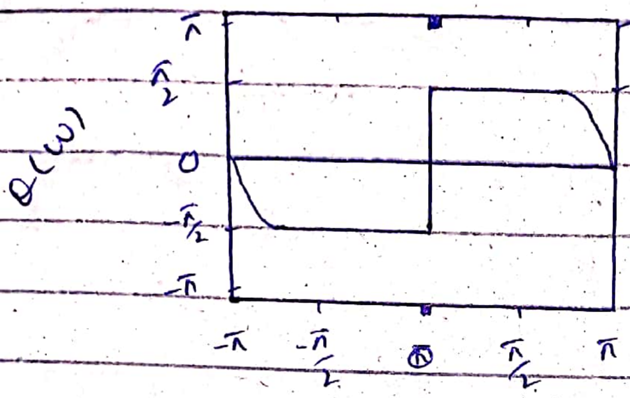
we have

$H(0) = \frac{b_0}{(1-p)^2}$

So $b_0 = (1-p)^2$






Magnitude and phase response of a simple highpass filter;
$H(z) = [(1-a)/2][(1-z^{-1})/(1+az^{-1})]$
with $a = 0.9$

At $\omega = \frac{\pi}{4}$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence

$$= \frac{(1-P)^4}{((1-P/\sqrt{2})^2 + P^2/2]^2}$$

$$= \frac{1}{2}$$

or equivalently;

$$\sqrt{2}\,(1-p)^2 = (1+p)^2 - \sqrt{2}\,p$$

The value of $p = 0.32$ satisfies the equation.

The system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same principles can be applied for the design of bandpass filter.

Ans (3) (b) $\omega = \pi/2$

Zero in its frequency response characteristics at $\omega = 0$. $\omega = \pi$ and magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = \frac{4\pi}{9}$

Solution:-

The filter must have poles at

$$P_{1,2} = re^{\pm j(\pi/2)}$$

and zeros at $z = 1$ and $z = -1$

So the system function is

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$

So we have;

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of $r$ is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$.

Thus we have

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\;\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

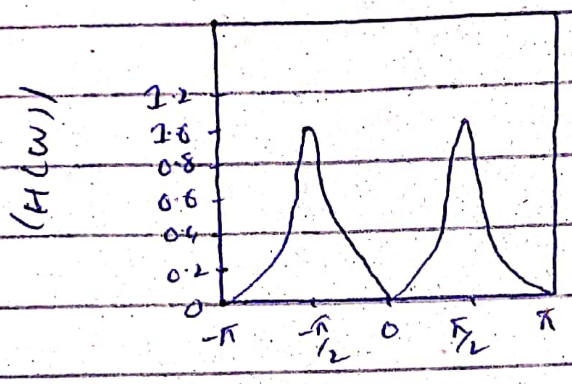
$$= \frac{1}{2}$$

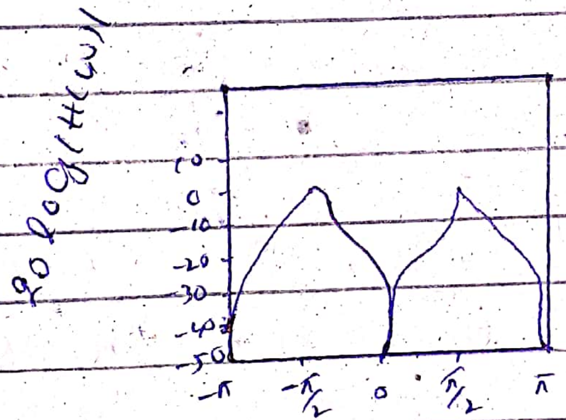
or

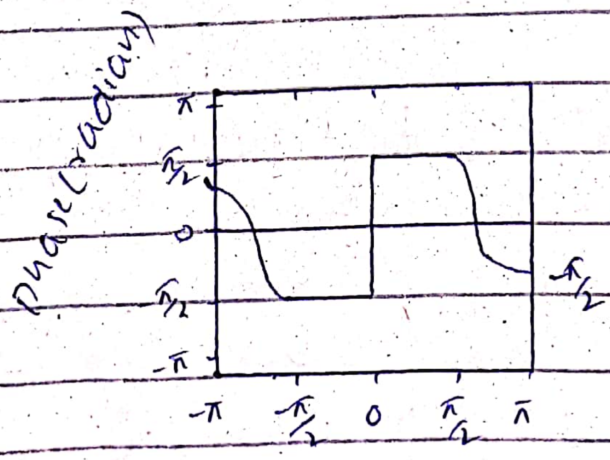
$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfies this equation. So the system function for the desired filter is;

$$H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}$$




Magnitude and phase response of a simple band pass filter.

$H(z) = 0.15[(1 - z^{-2}) / (1 + 0.7 z^{-2})]$

Ans (4) (a) $x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$

Determine the N-point DFT of this sequence for $N \ge L$

Solution:

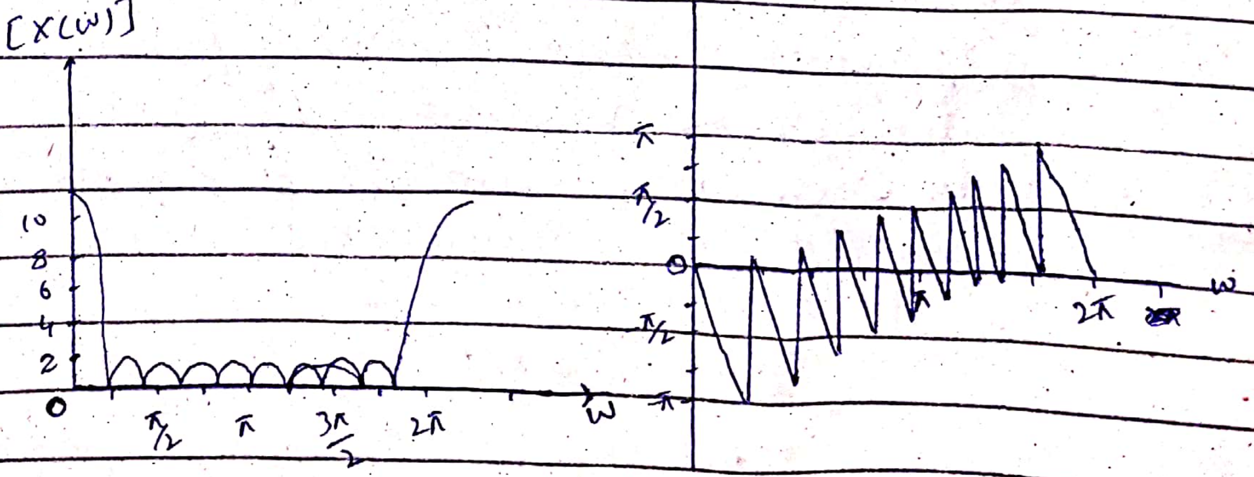
The fourier transform of the Sequence is;

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n}$$

$$= \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} \approx \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of X(w) is Given below for L=10.



$\omega_k = 2\pi k / N$

$k = 0, 1 \ldots N-1$

Hence

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \quad ; k = 0, 1 \ldots N-1$$

$$= \frac{\sin(\pi kL/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$

if $N$ is selected such that $N = L$ then the DFT becomes,

$$X(k) = \begin{cases} L, & k = 0 \\ 0, & k = 1, 2, \ldots L-1 \end{cases}$$

There is only one non-zero value in the DFT. This is apparent from observation of $X(\omega)$ since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi k/L$, $k \neq 0$.

Ans (4)(b) $x(n) = (0 \ 1 \ 2 \ 3)$

Solution:-

The first step is to determine the matrix $W_4$ By exploiting the periodicity property of $W_4$ and the Symmetry property;

$$W_N^{K+N/2} = -W_N^K$$

The matrix may be expressed as;

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then

$$X_4 = W_4 x_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The IDFT of $X_4$ is determined by conjugating the element of $W_4$ to get $W_4^*$ and then put in $x_N = \frac{1}{N} W_N^* X_N$