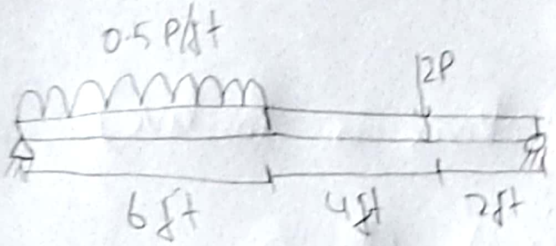
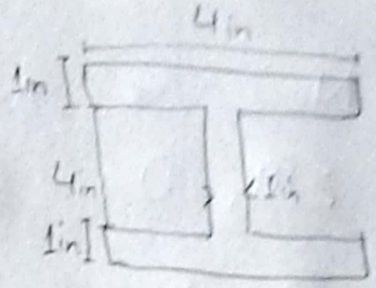




Given Data:

~~Point~~ $P = 94$

Point load $= 2(94) = 188$ lbs.

UDL $= 0.5(94) = 47$ lb/ft.

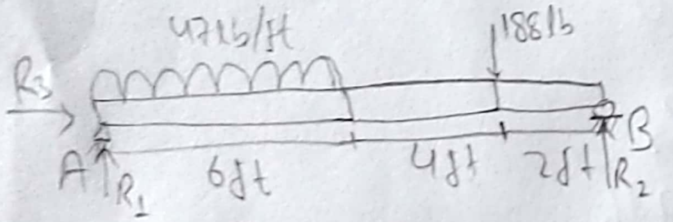
Required:

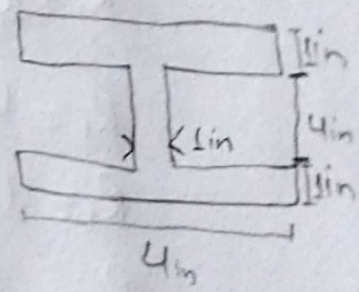
Principle Stress = ?

Stress state of Point C = ?

Mohr's Circle = ?

Solution:





Now We will find the reactions at A and B

$\Sigma F_x = 0$; $R_3 = 0$

$\Sigma F_y = 0$; ↑(+ve) ↓(−ve)

$R_1 + R_2 - (47 \times 6) - 188 = 0$

$R_1 + R_2 - 282 - 188 = 0$

$R_1 + R_2 = 470$ lbs.

Now the moments.

$\sum M_A = 0$ (Anticlockwise is Positive)

$(R_2 \times 12) - (188 \times 10) - (47 \times 6) \times 6/2 = 0$

$R_2 \times 12 - 1880 - 846 = 0$

$R_2 \times 12 = 2726$

$R_2 = \frac{2726}{12}$

$R_2 = 227.16 \, lbs$

As we know

$R_1 + R_2 = 470$

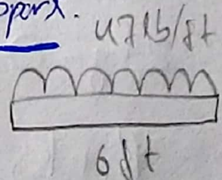
$R_1 = 470 - R_2$

$R_1 = 470 - 227.16$

$R_1 = 242.84 \, lbs.$

Shear force at Beam change Point.

Shear force at 6ft from left Support.



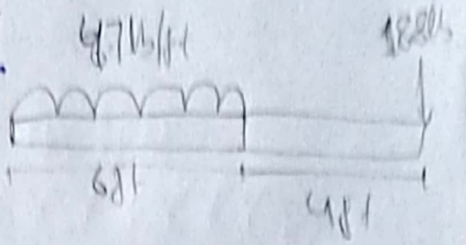
$\sum F_y = 0$ $\uparrow(+ve) \downarrow(-ve)$

$-V_{6ft} + 242.84 - (47 \times 6) = 0$

$V_{6ft} = 242.84 - 282$

$V_{6ft} = -39.16 \, lbs.$

Shear force at 10ft from left Support

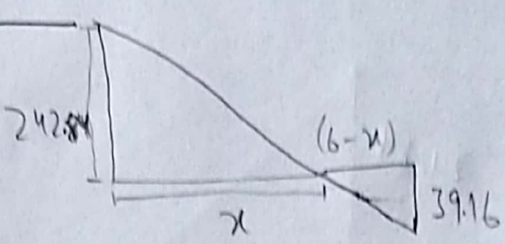


$-V_{right} + 242.84 - 282 - 188 = 0$

$V_{right} = 242.84 - 470 = 0$

$V_{right} = -227.16$ lb

Moments at Zero Shear Point:



We know that

$$\frac{242.84}{x} = \frac{39.16}{(6-x)}$$

$$242.84(6-x) = x \times 39.16$$

$$1457.04 - 242.84x = 39.16x$$

$$1457.04 = 282x$$

$$\Rightarrow x = \frac{1457.04}{282} = 5.167 \text{ ft}$$

Now take Section at 5.167ft from left Support

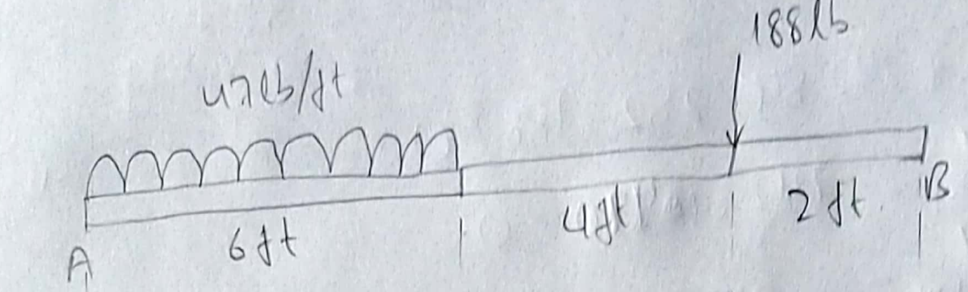
$\Sigma M_{5.167} = 0$ (Anticlockwise is Positive).

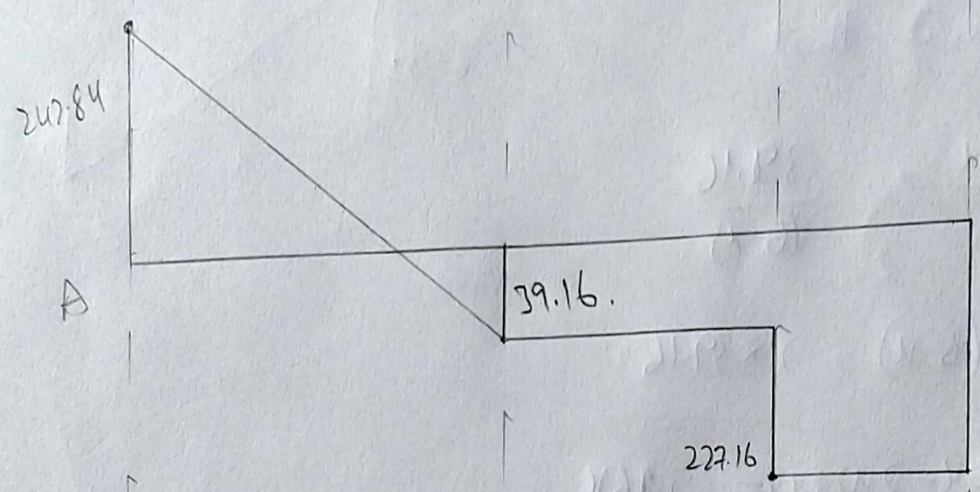
$$M_{5.167} - (242.84 \times 5.167) + (47 \times 6)\frac{5.167}{2} = 0$$

$$M_{5.167} - 1254.75 + 728.547 = 0$$

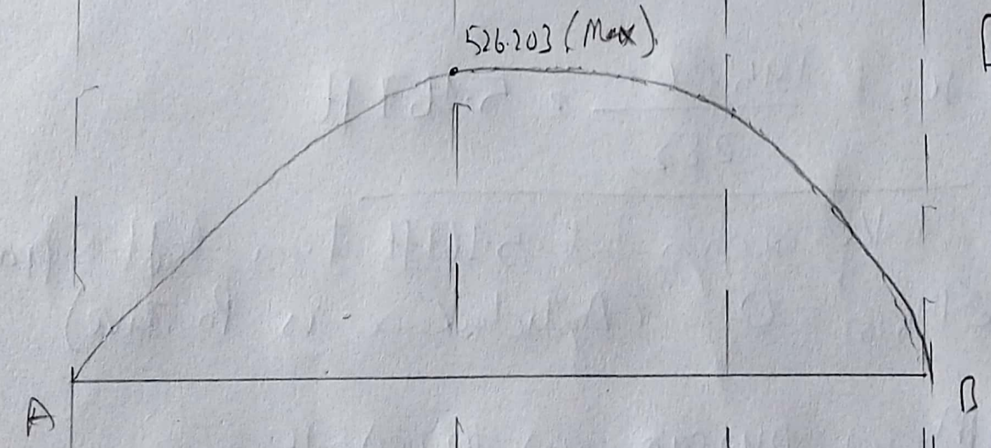
$$M_{5.167} - 526.203 = 0$$

$$M_{5.167} = 526.203$$





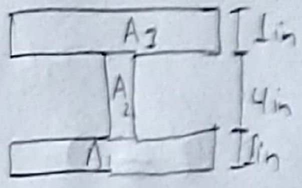
Shear force Diagram



Bending Moment Diagram.

Moment of inertia:-

As we know that Section is Symetrical (Y-axis)
So Centroid will be $h/2 = 6/2 = 3$ in.
So We Know that



$$\bar{y} = \frac{Y_1 A_1 + Y_2 A_2 + Y_3 A_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{(0.5)(4) + (3)(4) + (5.5)(4)}{4+4+4} = \frac{36}{12} = 3 \text{ inches.}$$

Now moment of inertia of whole section is given by

$$I_y = I_1 + I_2 + I_3.$$

$$I_1 = \frac{Bh^3}{12} + Ad_1^2 \qquad \text{Note } d = \bar{y} - y$$

$$I_1 = \frac{4 \times 1^3}{12} + 4(3-0.5)^2 = 0.33 + 25 = 25.33 \text{ in}^4.$$

$$I_2 = \frac{1 \times 4^3}{12} + 4(0)^2 = 5.33 + 0 = 5.33 \text{ in}^4.$$

$$I_3 = \frac{4 \times 1^3}{12} + 4(3-5.5)^2 = 0.33 + 25 = 25.33 \text{ in}^4.$$

$$I_y = I_1 + I_2 + I_3 = 25.33 + 5.33 + 25.33$$

$$I_y = 55.99 \text{ in}^4 = 56 \text{ in}^4.$$

Note.
$V_{max} = 227.16$
$I = 56$
$Q = \bar{Y}A$
$\bar{y} = 3$

Shear stress:

$$\tau = \frac{VQ}{Ib}$$

Now shear stress at top fibre

$$\tau = \frac{227.16 \times 0}{56 \times 4}$$

$$\tau = 0.$$

Shear Stress at Point C.

$$\tau_c = \frac{227.16(3 \times 4)}{56 \times 4} = \frac{2725.92}{224}$$

$$\tau_c = 12.17 \ lb/in^2$$

Shear stress at Centroidal axis (Maximum)

$$\tau_{\substack{centroidal \\ axis}} = \frac{VQ_T}{Ib}.$$

$$Q_T = Q_1 + Q_2 + Q_3$$

$$Q_1 = y_1 A_1 = (0.5)(4) = 2$$

$$Q_2 = y_2 A_2 = (3)(4) = 12$$

$$Q_3 = y_3 A_3 = (5.5)(4) = 22$$

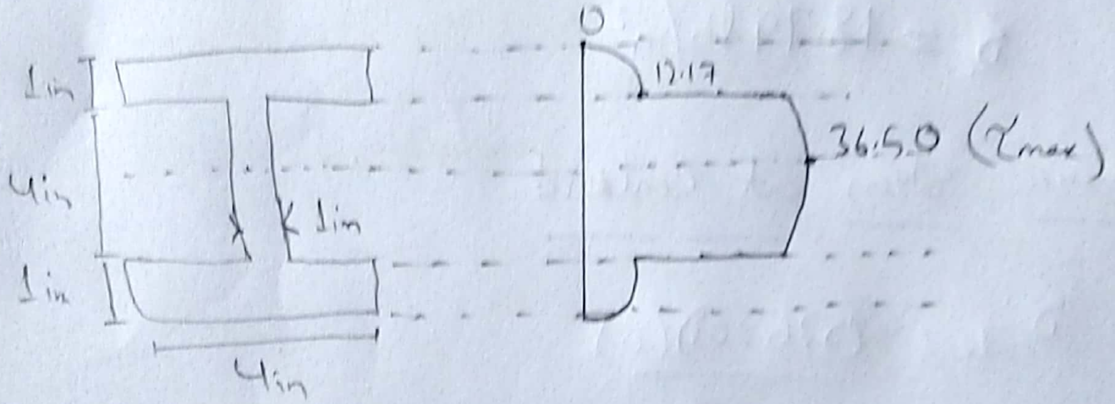
$$Q_T = 2 + 12 + 22 = 36.$$

$$\tau_{centroid} = \frac{227.16 \times 36}{56 \times 4}$$

$$\tau_{centroid} = \frac{8177.78}{224} = 36.507 \ lb/in^2 = \tau_{max}.$$

★ Shear stress is maximum at centroid.

Shear Stress Variation diagram.



Flexure Stresses:-

We know that flexure stress is given by $\sigma = \frac{MY}{I}.$

M → Maximum Bending Moment.

I → Moment of inertia

y → Centroid.

Flexure stress at Top fibre.

$$\delta_{Top} = \frac{(526.203)(3)}{56}$$

$$\delta_{Top} = 28.19\ lb/in^2$$

Flexure Stress at point C

$$\delta_c = \frac{(526.203)(2)}{56}$$

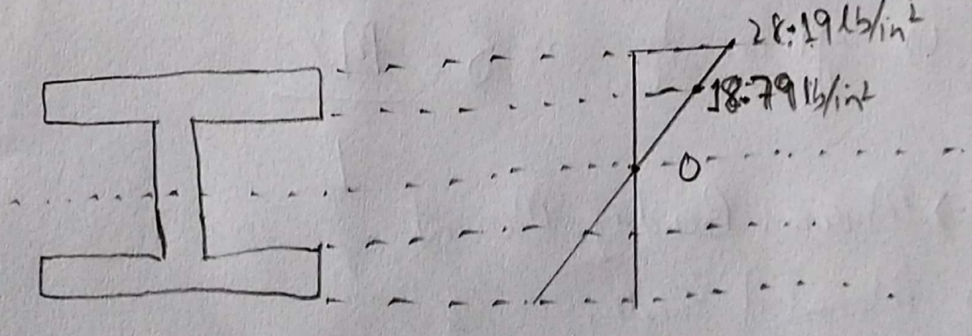
$$\delta_c = 18.79\ lb/in^2$$

Flexure Stress at Centroid

$$\delta_{centroid} = \frac{(526.203)(0)}{56}$$

$$\delta_{centroid} = 0.$$

Flexure stress Variation diagram:-

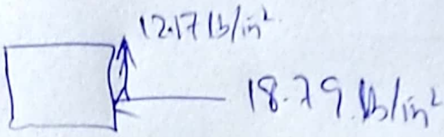


Stress State of Point C:

Shear Stress at Point C = τ_c = 12.17 lb/in²

Flexure Stress at Point C = σ_c = 18.79 lb/in²

Stress is Compressive because point C lies in Compressive zone of beam Cross-section.



Now shear stress Condition at Point C, Assume at 20° clockwise orientation (-ve).

$\sigma_x = -18.79$ lb/in² $\qquad \sigma_x' = ?$

$\sigma_y = 0 \qquad \sigma_y' = ?$

$\tau_{xy} = 12.17 \qquad \tau_{x'y'} = ?$

$\theta = -20°$

So. Stress transformation equations.

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta.$$

$$= \frac{-18.79 + 0}{2} + \left(\frac{-18.79 - 0}{2}\right) \cos -2(20) + (12.17) \sin -2(20).$$

$$= -9.395 + 7.198 - 7.822$$

$$\boxed{\sigma_x' = -10.02 \text{ lb/in}^2.}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\sigma_{y'} = \frac{-18.79 + 0}{2} - \left(\frac{-18.79 - 0}{2}\right) \cos -(40) - 12.17 \sin -40$$

$$\sigma_{y'} = -9.395 - 7.196 + 7.822$$

$$\sigma_{y'} = \underline{-8.769 \text{ lb/in}^2}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta.$$

$$\tau_{x'y'} = -\left(\frac{-18.79 - 0}{2}\right) \sin -(49) + 1217 \cos -49.$$

$$= (-9.395)(-0.64) + 12.17(-0.766)$$

$$= -6.0128 - 9.322$$

$$\tau_{x'y'} = \underline{-15.3348}$$

Principle Stresses :-

We know that Principle Stress equation is given by.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{1,2} = \frac{-18.79+0}{2} \pm \sqrt{\left(\frac{-18.79-0}{2}\right) + (12.17)^2}$$

$$\sigma_{1,2} = -9.395 \pm \sqrt{(-9.395)^2 + 148.1089}$$

$$\sigma_{1,2} = -9.395 \pm \sqrt{88.266 + 148.1089}$$

$$\sigma_{1,2} = -9.395 \pm \sqrt{236.3749}$$

$$\sigma_{1,2} = -9.395 \pm 15.37$$

$$\sigma_y = \sigma_1 = -9.395 + 15.37 = \boxed{5.975}$$

$$\sigma_x = \sigma_2 = -9.395 - 15.37 = \boxed{-24.77}$$

Now we have to find $\theta_p = ?$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

(★ Formula Corrected from Internet)

$$\tan 2\theta_p = \frac{2(12.17)}{\left(\frac{\cancel{18.79}}{2}\right)}$$

$$\theta_p = \tan^{-1}\left(\frac{2 \times 12.17}{\cancel{[illegible]}\ 9.395}\right) \Rightarrow \theta_p = \tan^{-1}\left(\frac{24.34}{\cancel{[illegible]}\ 9.395}\right)$$

$$\theta_p = \cancel{\tan^{-1}(0.7916)}\ \tan^{-1}(2.59)$$

$$\theta_p = -68.89^\circ$$

$$\sigma'_{x_{max}} = \frac{-18.79+0}{2} + \frac{-18.79-0}{2}\cos 2(-68.89) + 12.17 \sin 2(\cancel{[illegible]})$$

$$\sigma_{p_{max}} = -9.395 \mp \cancel{[illegible]}\ 13.91 - 8.177$$

~~$\sigma_p = 0.295\ lb/in^2 = 0.295\ lb/in^2$.~~

$$\sigma_{p_{max}} = 31.48$$

Now Maximum in Plane shear stress

$$\tan 2\theta_s = -\frac{\left(\frac{\sigma_x - \sigma_y}{2}\right)}{\tau_{xy}}$$

$$\tan 2\theta_s = \frac{+9.395}{24.34}.$$

$$\theta_s = \tan^{-1}(0.3859)$$

$\theta_s = 21.10^\circ$ (Anti clockwise).

Putting this in general equation for $\tau_{x'y'}$

$$\tau_{x'y'} = -\left[\frac{-18.79-0}{2}\right]\sin 2(21.10) + 12.17 \cos 2(21.10)$$

$$= (9.395)(0.6717) + (12.17)(0.7408)$$

$$= 6.310 + 9.015$$

$\tau_{x'y'} = 15.3256$

Coordinate of Center $(h,k) = \left(\frac{\sigma_x - \sigma_y}{2}, 0\right)$

$$(h,k) = \left(\frac{-18.79-0}{2}, 0\right)$$

$$(h,k) = (9.395, 0)$$

Radius of Mohr's Circle:-

$$r = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - (\tau_{xy})^2}$$

$$r = \sqrt{\left(\frac{-18.79 - 0}{2}\right)^2 + (12.17)^2}$$

$$r = \sqrt{(-9.395)^2 + 148.1089}$$

$$r = \sqrt{236.34929}$$

$$r = 15.37 \ lb/in^2$$

· Scale = 3 Psi = 1 cm.