Date:

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Section - A
Paper = Calculas

Q no 1

Sol

We know that

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

position vectors of P and Q are:

$r_1 = \vec{OP} = 4\hat{i} + \hat{j} + 3\hat{k}$

$r_2 = \vec{OQ} = \hat{i} + 2\hat{j} + 4\hat{k}$

Now,

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= (\hat{i} + 2\hat{j} + 4\hat{k}) - (4\hat{i} + \hat{j} + 3\hat{k})$$

$$= (1-4)\hat{i} + (2-1)\hat{j} + (4-3)\hat{k}$$

$$\vec{PQ} = -3\hat{i} + \hat{j} + \hat{k}$$

Now, we find distance b/w P and Q

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we know that

$$\text{distance } (d) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$d = \sqrt{(1-4)^2+(2-1)^2+(4-3)^2}$$

$$= \sqrt{(-3)^2+(1)^2+(1)^2}$$

$$d = \sqrt{9+1+1}$$

$$\boxed{d = \sqrt{11}}$$

Position vector of point dividing P and Q in ratio

$m_1 : m_2$

$1 : 3$

$$\frac{m_1 r_2 + m_2 r_1}{m_1 + m_2} = \frac{1(i+2\hat{j}+4\hat{k}) + 3(4\hat{i}+\hat{j}+3\hat{k})}{1+3}$$

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$$= \frac{\hat{i} + 2\hat{j} + 4\hat{k} + 12\hat{i} + 3\hat{j}}{4} + 9\hat{k}$$

$$= \boxed{\frac{13\hat{i} + 5\hat{j} + 13\hat{k}}{4}}$$

$$\Rightarrow \boxed{\frac{13}{4}\hat{i} + \frac{5}{4}\hat{j} + \frac{13}{4}\hat{k}}$$

This is the position vector of point which divides $\vec{PQ}$ in the ratio 1:3

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Q_2

Sol

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

= Rewrite the fraction using partial fraction decomposition.

$$\Rightarrow \int 2x-1+\frac{4}{x}+\frac{3}{2x+1}\,dx$$

using property of integral

$$\Rightarrow \int 2x\,dx - \int 1\,dx + \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$\Rightarrow x^2 - x + 4\ln(|x|) + \frac{3}{2} \times \ln(|2x+1|$$

$$\Rightarrow x^2 - x + 4\ln(|x|) + \frac{3}{2} \times \ln(|2x+1|) + C \quad \text{Ans}$$

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Q3 a

$$\int_0^2 x^2 e^x \, dx$$

Sol

$$\int_0^2 x^2 e^x \, dx$$

$$= x^2 e^x - \int e^x \times 2x \, dx$$

using property of integrals

$$x^2 e^x - 2x \int e^x x \, dx$$

$$= x^2 e^x - 2x \int x e^x \, dx$$

$$= x^2 e^x - 2\left(x e^x - \int e^x \, dx\right)$$

$$= x^2 e^x - 2\left(x e^x - e^x\right)$$

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$x^2e^x - 2xe^x + 2e^x$

= By using limit

$= \left(x^2e^x - 2xe^x + 2e^x\right)\Big|_0^2$

$= 2^2e^2 - 2\times 2e^2 + 2e^2 - (0^2e^0 - 2\times 0e^0 + 2e^0)$

$= 4e^2 - 4e^2 + 2e^2 - (0\times 1 - 2\times 0\times 1 + 2\times 1)$

$= 2e^2 - (0 - 0 + 2)$

$= 2e^2 - 2$

$\Rightarrow 12.778$ Ans

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6. $$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

Sol

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

$$= \int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$$

By Substitution $= t = \sqrt{x}$ $- ①$

$$= \int 2\sin(t)\,dt$$

Using property of integrals

$$= 2 \times \int \sin(t)\,dt$$

$$= 2 \times (-\cos(t)) \quad -②$$

Substitute the value of t in eq. ②

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$2 \times (-\cos(\sqrt{x}))$

$= -2\cos(\sqrt{x})$

$= -2\cos(\sqrt{x})\Big|_1^2$

$= -2\cos(\sqrt{2}) - (-2) \times \cos(\sqrt{1})$

$= -2\cos(\sqrt{2}) + 2\cos(1)$

$= -2\cos(\sqrt{2}) + 2\cos(1)$

$= 0.7687$ Ans

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Q4

$$U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Sol

Laplace equation

$$U_{xx} + U_{yy} + U_{zz} = 0$$

first finding U_{xx}

$$U_x = \frac{\partial}{\partial x}\left[\frac{1}{x^2+y^2+z^2}\right]$$

Now using Quotient rule

$$U_x = \frac{\frac{d}{dx}(1) - \sqrt{x^2+y^2+z^2} - \frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right)}{\left(\sqrt{x^2+y^2+z^2}\right)^2}$$

$$U_x = -\frac{\partial}{\partial x}\frac{\left(\sqrt{x^2+y^2+z^2}\right)}{\left(\sqrt{x^2+y^2+z^2}\right)^2}$$

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$$U_x = \frac{-\frac{\partial}{\partial x}\left(x^2+y^2+z^2\right)}{\left(x^2+y^2+z^2\right)^{-\left(\frac{1}{2}-1\right)} \cdot 2\sqrt{x^2+y^2+z^2}}$$

$$U_x = \frac{-\cancel{2}x}{\left(x^2+y^2+z^2\right) \cdot \cancel{2}\sqrt{x^2+y^2+z^2}}$$

$$U_x = \frac{-x}{\left(x^2+y^2+z^2\right)^{3/2}}$$

$$U_{xx} = \frac{\partial}{\partial x}\left[\frac{-x}{\left(x^2+y^2+z^2\right)^{3/2}}\right]$$

Using Quotient rule

$$U_{xx} = \frac{-\left(x^2+y^2+z^2\right)^{3/2} - \frac{3}{2}\left(x^2+y^2+z^2\right)^{\frac{3}{2}-1} \cdot \frac{\partial}{\partial x}\left(x^2+y^2+z^2\right) \cdot x}{\left(x^2+y^2+z^2\right)^3}$$

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$$U_{xx} = \frac{-(x^2+y^2+z^2)^{3/2} - \frac{3}{2}(x^2+y^2+z^2)^{3/2-1} \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2)}{(x^2+y^2+z^2)^3}$$

$$U_{xx} = \frac{-(x^2+y^2+z^2)^{3/2} - \frac{3}{\cancel{2}} \cdot \sqrt{x^2+y^2+z^2} \cdot (\cancel{2}x^2)}{(x^2+y^2+z^2)^3}$$

$$\Rightarrow \text{So } U_{xx} = -(x^2+y^2+z^2)^{3/2} - 3 \cdot \sqrt{x^2+y^2+z^2} \cdot x^2$$

Now solving for U_{yy}

$$U_y = \frac{\partial}{\partial y}\left[\frac{1}{\sqrt{x^2+y^2+z^2}}\right]$$

using Quotient rule

$$U_y = \frac{-\partial/\partial y\,(\sqrt{x^2+y^2+z^2})}{\sqrt{x^2+y^2+z^2}^{\,2}}$$

$$U_y = \frac{\frac{\partial}{\partial y}(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{-(1/2-1)} \cdot 2\sqrt{x^2+y^2+z^2}^{\,2}}$$

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$$U_y = \frac{-y}{(x^2+y^2+z^2)^{3/2}}$$

Now U_{yy}

$$U_{yy} = \frac{\partial}{\partial y}\left(\frac{-y}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$U_{yy} = \frac{(x^2+y^2+z^2)^{3/2} - \frac{\partial}{\partial y}(x^2+y^2+z^2)^{3/2} \cdot y}{(x^2+y+z^2)^3}$$

$$U_{yy} = \frac{-(x^2+y^2+z^2)^{3/2} - \frac{3}{2}\sqrt{x^2+y^2+z^2}\,(\partial y^2)}{(x^2+y^2+z^2)^3}$$

$$U_{yy} = \frac{-(x^2+y^2+z^2)^{3/2} - 3\sqrt{x^2+y^2+z^2} \cdot y}{(x^2+y^2+\cancel{y}z^2)^3}$$

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Now U_{zz}

$$U_z = \frac{-z}{(x^2+y^2+z^2)^{3/2}}$$

$$U_{zz} = -(x^2+y^2+z^2)^{3/2} - 3\sqrt{x^2+y^2+z^2} \cdot z$$

$$U_{xx} + U_{yy} + U_{zz} = 0$$

Laplace equation is ~~proved~~ Satisfied.