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7892

Section A.

Numerical Analysis.

①

Q1) find the root of the following
$x^3 + 3.993 \times 10^{-4} = 0.165x^2$
use N.R.M with $x_0 = 0.02$

Sol:- Rearranging the equation.

$x^3 - 0.165x^2 + 0.00039 = 0.$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x) = 3x^2 - 0.33x = 0$

$\Rightarrow x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ | $f(x_0) = 0.00033$
$f'(x_0) = -0.0054$

$= 0.2 - \frac{0.00033}{-0.0054}$

$\boxed{x_1 = 0.081}$

$\Rightarrow x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$ | $f(x_1) = -00016$
$f'(x_1) = -0.0070$

$= 0.081 - \frac{(-0.00016)}{-0.0070}$

$\boxed{x_2 = 0.058}$

②

$\Rightarrow x_3 = x_2 - f(x_2) / f'(x_2)$.

$= 0.058 - \dfrac{0.00003}{(-0.009)}$ $\quad f(x_2) = 0.00003$

$f'(x_2) = -0.009$.

$$\boxed{x_3 = 0.061}$$

Q2) Use the numbers $x_0 = 2, x_1 = 2.75$, $x_2 = 4$ And find the lagrange Interpolation polynomial for $f(x) = \frac{1}{x}$ at $x = 3$.

Sol:- As we know that lagrange interpolation formula.

$$y = \frac{(x - x_1)(x - x_2) \cdots (x - x_n)}{(x_0 - x_1)(x_0 - x_2) \cdots (x_0 - x_n)} y_0.$$

$\Rightarrow x_0 = 2 \Rightarrow y_0 = \frac{1}{2} = 0.5$.

$\Rightarrow x_1 = 2.75 \Rightarrow y_1 = \frac{1}{2.75} = 0.36$

$\Rightarrow x_2 = 4 \Rightarrow y_2 = \frac{1}{4} = 0.25$

(3)

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} y_2 .$$

Putting values.

$$y = \frac{(3-2.75)(3-4)}{(2-2.75)(2-4)} (0.5) + \frac{(3-2)(3-4)}{(2.75-2)(2.75-4)} (0.36)$$

$$+ \frac{(3-2)(3-2.75)}{(4-2)(4-2.75)} (0.25) .$$

$$y = (-0.833) + (0.384) + (0.025)$$

$$\boxed{y = -0.424}$$

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(4)

Q3) Complete the divided difference table for the given and Construct Interpolating polynomial that uses all this data.

x = 1.0 1.3 1.6 1.9 2.2

y = 0.7651977 0.6200860 0.4.554022 0.2818186 0.1103623.

x_i	$f(x_i)$	$f(x_i-1, x_1)$	$f(x_1-2, x_i-1, x_1)$	$f(x_i-3 \cdots x_1)$	$f(x_i-4, \cdots x_1)$
x_0 1	0.7651977				
		-04837056			
x_1 1.3	0.6200860		-0.108734		
		-0.5.48946		0.658785	
x_2 1.6	0.4554022		-0.0494433		-0.0028049.
		-0.578612		0.06251255	
x_3 1.9	0.2818186		0.006818		
		-0.571521			
x_4 2.2	0.1103623				

(5)

⇒ 1st Divided Difference :-

i) $f(x_0, x_1) = f(x_1) - f(x_0) / x_1 - x_0.$

$$= \frac{0.6200860 - 0.7651977}{1.3 - 1}$$

$$f(x_0, x_1) = -0.4837056$$

2) $f(x_1, x_2) = f(x_2) - f(x_1) / x_2 - x_2$

$$= \frac{0.4554022 - 0.6200860}{1.6 - 1.3}$$

3) $f(x_2, x_3) = f(x_3) - f(x_2) / x_3 - x_2$

$$= \frac{0.2818186 - 0.4554022}{1.9 - 1.6}$$

$$f(x_2, x_3) = -0.578612.$$

4) $f(x_3, x_4) = f(x_4) - f(x_3) / x_4 - x_3.$

$$= \frac{0.1103623 - 0.2818186}{2.2 - 1.9.}$$

$$f(x_3, x_4) = -0.571521.$$

(6)

⇒ Second Divided Difference :-

① $f(x_0, x_1, x_2) = f(x_1, x_2) - f(x_0, x_1) / x_2 - x_0$

$= \dfrac{-0.548946 - (-0.4837056)}{1.6 - 1}$

$\boxed{f(x_0, x_1, x_2) = -0.108734.}$

② $f(x_1, x_2, x_3) = f(x_2, x_3) - f(x_1, x_2) / x_3 - x_1.$

$= \dfrac{-0.578612 - (-0.548946)}{1.9 - 1.3}$

$\boxed{f(x_1, x_2, x_3) = -0.0494433}$

③ $f(x_2, x_3, x_4) = f(x_3, x_4) - f(x_2, x_3) / x_4 - x_2$

$= -0.571521 - (-0.578612) / 2.2 - 1.6$

$\boxed{f(x_2, x_3, x_4) = 0.006818.}$

⇒ Third Divided difference :-

① $f(x_0, x_1, x_2, x_3) = \dfrac{f(x_1, x_2, x_3) - f(x_0, x_1, x_2)}{x_3 - x_0}$

$= \dfrac{-0.0494433 - (-0.108734)}{1.9 - 1}$

$f(x_0, x_1, x_2, x_3) = 0.0658785$

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(2) $f(x_1, x_2, x_3, x_4) = \dfrac{f(x_2, x_3, x_4) - f(x_1, x_2, x_3)}{x_4 - x_1}$

$$= \frac{0.006818 - (-0.049443)}{2.2 - 1.3}$$

$$f(x_1, x_2, x_3, x_4) = 0.06251255$$

$\Rightarrow$ 4th Divided difference :-

$$f(x_0, x_1, x_2, x_3, x_4) = \frac{f(x_1, x_2, x_3, x_4) - f(x_0, x_1, x_2, x_3)}{x_4 - x_0}.$$

$$= \frac{0.06251255 - 0.0658785}{2.2 - 1}$$

$$f(x_0, x_1, x_2, x_3, x_4) = 0.0028049.$$

$$f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + (x - x_0)(x - x_1) f(x_0, x_1, x_2)$$
$$+ (x - x_0)(x - x_1)(x - x_2) f(x_0, x_1, x_2, x_3) + (x - x_0)(x - x_1)$$
$$(x - x_2)(x - x_3) f(x_0, x_1, x_2, x_3, x_4).$$

$$= 0.7651977 + (x-1) - 0.4837056 + (x-1)(x-1.3)(-0.108734)$$
$$+ (x-1)(x-1.3)(x-1.6)(0.0658785) + (x-1)(x-1.3)(x-1.6)$$
$$(x-1.9)(-0.0028049).$$