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Example 4.1:-

Value of E at $P(\rho = 2, \phi = 40^\circ, z = 3)$ is given as $E = 100a_\rho - 200a_\phi + 300a_z$ V/m. Determine the incremental work required to move $20\mu C$ charge a distance of $6\mu m$.

a) in the direction of a_ρ: Incremental work is given by $dW = -qE \cdot dL$, where in this case $dL = d\rho a_\rho = 6 \times 10^{-6} a_\rho$. Thus

$$dW = -(20 \times 10^{-6} C)(100 V/m)(6 \times 10^{-6} m) = -12 \times 10^{-9} J$$
$$= -12 nJ$$

b) in the direction of a_ϕ: In this case $dL = 2d\phi a_\phi = 6 \times 10^{-6} a_\phi$, and so

$$dW = -(20 \times 10^{-6})(-200)(6 \times 10^{-6}) = 2.4 \times 10^{-8} J = 24 nJ$$

c) in the direction of a_z: Here, $dL = dz\,a_z$

$= 6 \times 10^{-6}) = -3.6 \times 10^{-8}\,J = -36\,nJ$

d) in the direction of E: Here $dL = 6 \times 10^{-6}\,a_E$,

$$a_E = \frac{100a_\rho - 200a_\phi + 300a_z}{[100^2 + 200^2 + 300^2]^{1/2}}$$

$= 0.267a_\rho - 0.535a_\phi + 0.802a_z$

Thus

$dW = -(20 \times 10^{-6})[100a_\rho - 200a_\phi + 300a_z]$

$\cdot [0.267a_\rho - 0.535a_\phi + 0.802a_z](6 \times 10^{-6})$

$= -44.9\,nJ$

e) In the direction of $G = 2a_x - 3a_x - 3a_y + 4a_z$

In this case $dL = 6 \times 10^{-6}a_G$; where

$$a_G = \frac{2a_x - 3a_y + 4a_z}{[2^2 + 3^2 + 4^2]^{1/2}} = 0.371a_x - 0.557a_y + 0.743a_z$$

$dW = -(20 \times 10^{-6})[100a_\rho - 200a_\phi + 300a_z] \cdot [0.371a_x$

$- 0.557a_y + 0.743a_z][6 \times 10^{-6})$

$= -(20 \times 10^{-6})[37.1(a_\rho \cdot a_x) - 55.7(a_\rho \cdot a_y)$

$- 74.2(a_\phi \cdot a_x) + 111.4(a_\phi \cdot a_y)$

$+ 222.9](6 \times 10^{-6})$

Where, at P, $(a_\rho \cdot a_x) = (a_\phi \cdot a_y) = \cos 40^\circ = 0.766$, $(a_\rho \cdot a_y) = \sin(40^\circ) = 0.643$ and $(a_\phi \cdot a_x) = -\sin(40^\circ) = -0.643$

$$dW = -(20\times10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6\times10^{-6}) = -41.8\,nJ$$

x ——— x ——— x

Example 4.2 :-

a) $a_x + a_y + a_z$

$$dW = qE\cdot dL = -4(400a_x - 300a_y + 500a_z) \times \frac{(a_x + a_y + a_z)}{\sqrt{3}} \cdot (10^{-3})$$

$$= -\frac{(4\times10^{-3})}{\sqrt{3}}(400 - 300 + 500) = -1.39\,J$$

b) $-2a_x + 3a_y - a_z$: The computation is similar to that of part a, but we change the direction

$$dW = -qE\cdot dL = -4(400a_x - 300a_y + 500a_z) \times \frac{(-2a_x + 3a_y - a_z)}{\sqrt{14}}(10^{-3})$$

$$= -\frac{(4\times10^{-3})}{\sqrt{14}}(-800-900-500) = 2.35\,J$$

x———x———x

Example 4.3 :-

a) $P(1,2,3)$ toward $Q(2,1,4)$: The vector along this direction will be $Q-P=(1,-1,1)$ from which $a_{PQ} = [a_x - a_y + a_z]/\sqrt{3}$

$$dW = -qE\cdot dL = -(50\times10^{-6})\left[120a_\rho \cdot \frac{(a_x-a_y+a_z)}{\sqrt{3}}\right] \times (2\times10^{-3})$$

$$= -(50\times10^{-6})(120)\left[(a_\rho\cdot a_x)-(a_\rho\cdot a_y)\right]\frac{1}{\sqrt{3}}(2\times10^{-3})$$

At P, $\phi = \tan^{-1}(2/1) = 63.4^\circ$. Thus $(a_\rho\cdot a_x) = \cos(63.4) = 0.447$ and $(a_\rho\cdot a_y) = \sin(63.4) = 0.894$ Substituting these we obtain $dW = 3.1\,\mu J$.

x———x———x

Example 4.5

a) Straight line segments $A(1,-1,2)$ to $B(1,1,2)$ to $P(2,1,2)$ using the path:

$$\int_A^P G \cdot dL = \int_A^P 2y\,dx$$

The change in x occurs when moving b/w B & P during which $y=1$

$$\int_A^P G \cdot dL = \int_B^P 2y\,dx = \int_1^2 2(1)\,dx = \underline{2}$$

b) Straight line segments $A(1,-2,2)$ to $C(2,-1,2)$ to $P(2,1,2)$: x occurs when moving from A to C during which $y=-1$

$$\int_A^P G \cdot dL = \int_A^C 2y\,dx = \int_1^2 2(-1)\,dx = \underline{-2}$$

Example 4.7 :-

a) Straight line : $y = x - 1$, $z = 1$; We obtain

$$\int G \cdot dL = \int_2^4 3xy^2\,dx + \int_1^3 2z\,dy = \int_2^4 3x(x-1)^2\,dx$$

$$+ \int_1^3 2(1)\,dy = \underline{90}$$

b) Parabola : $6y = x^2 + 2$, $z = 1$:

$$\int G \cdot dL = \int_2^4 3xy^2\,dx + \int_1^3 2z\,dy = \int_2^4 \frac{1}{12} x(x^2+2)^2\,dx$$

$$+ \int_1^3 2(1)\,dy = 82$$