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Section = "B"

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Question No 2 :

$Y(x) = x^2 + \sin x$

Sol:

We have

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \dots$$

Now

$f(x) = Y(x) = x^2 + \sin x$

$f'(x) = 2x + \cos x$

$f''(x) = 2 - \sin x$

$f'''(x) = -\cos x$

Thus

$f(0) = (0)^2 + \sin(0)$

$= 0$

$f'(0) = 2(0) + \cos(0)$

$= 1$

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$f''(0) = 2 - \sin(0)$

$= 2$

$f'''(0) = -\cos(0)$

$= -1$

Hence by Maclaurin's Expansion

$$f(x) = y(x) = 0 + x(1) + \frac{x^2(2)}{2!} + \frac{x^3(-1)}{3!} + \cdots$$

$$= 0 + x + x^2 - \frac{x^3}{3!} + \cdots$$

$$= x + x^2 - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots$$

This is the required Maclaurin's Expansion.

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Question No 3 [Part a]

$1 + xy = x^2 + y^2$

Find $y'' = ?$

Sol: 1

Diff w.r.t x

$$0 + x\frac{dy}{dx} + y(1) = 2x + 2y\frac{dy}{dx}$$

$$x\frac{dy}{dx} - 2y\frac{dy}{dx} = 2x - y$$

$$(x-2y)\frac{dy}{dx} = 2x - y$$

$$\frac{dy}{dx} = \frac{2x-y}{x-2y} \quad \text{--- (A)}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\frac{d}{dx}(x-2y)}{(x-2y)^2}$$

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$$= \frac{(x-2y)\left(2(1)-\frac{dy}{dx}\right)-(2x-y)\left(1-2\frac{dy}{dx}\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(2-\frac{2x-y}{x-2y}\right)-(2x-y)\left(1-2\left(\frac{2x-y}{x-2y}\right)\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)\left(\frac{2(x-2y)-(2x-y)}{x-2y}\right)-\left((2x-y)\left(\frac{x-2y-2(2x-y)}{x-2y}\right)\right)}{(x-2y)^2}$$

$$= \frac{(x-2y)(\cancel{2x}-4y-\cancel{2x}+y)-(2x-y)(x-\cancel{2y}-4x+\cancel{2y})}{(x-2y)(x-2y)^2}$$

$$= \frac{(x-2y)(-3y)-(2x-y)(-3x)}{(x-2y)^3}$$

$$= \frac{-3xy+6y^2+6x^2-3xy}{(x-2y)^3}$$

$$= \frac{6x^2+6y^2-6xy}{(x-2y)^3}$$ Ans.

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Question No 3 [Part b]

$y = x^3 (1+x)^9 e^{6x}$

Sol :

$y = x^3 (1+x)^9 e^{6x}$ Take ln b.H.s

$\ln y = \ln(x^3 (1+x)^9 \cdot e^{6x})$

$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$

$\ln y = 3\ln x + 9\ln(1+x) + \ln e^{6x}$

Now diff w.r.t x

$$\frac{1}{y}\frac{dy}{dx} = 3 \times \frac{1}{x} + 9 \times \frac{1}{1+x}\frac{d}{dx}(1+x) + \frac{1}{e^{6x}}\frac{d}{dx}e^{6x}$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x}(0+1) + \frac{1}{\cancel{e^{6x}}} \cdot \cancel{e^{6x}}\frac{d}{dx}6x\right)$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + 6(1)\right)$$

$$\frac{dy}{dx} = x^3(1+x)^9 \cdot e^{6x}\left(\frac{3}{x} + \frac{9}{1+x} + 6\right) \text{ Ans}$$

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Question No 1 [Part a]

Solution:-

To check possibility of dis cont--

First at $t=0$

$$g(t) = t^2$$

$$g(0) = 0^2 = 0$$

For R.H.L:-

$$\lim g(1+h) = \lim_{h \to 0} (1+h)^2$$

$$\lim_{h \to 0} 1 + h^2 + 2h$$

Apply limit:

$$1 + 0^2 + 2(0)$$

$$= 1$$

For: L.H.L

$$\lim_{h \to 0} g(1-h) = 2t + 3$$

$$= \lim_{h \to 0} 2(1-h) + 3$$

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$= \lim_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

R.H.L = L.H.L $= g(t) = 5$

Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

For R.H.L

$\lim_{h \to 0} g(1 + h) = \lim_{h \to 0} 2(1+h) + 3$

$= \lim_{h \to 0} 2 + 2h + 3$

Apply limit

$= 2 + 2(0) + 3$

$= 3$

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for L.H.L

$$\lim_{h\to 0} g(1-h) = 12$$

$g(4) = R.H.L \neq L.H.L$

Point of discontinuity is at 4

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