Department of Electrical Engineering
Assignment
Date: 25/06/2020

Course Details

Course Title: Signals & Systems **Module:** 04
Instructor: **Total Marks:** 50

Student Details

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Q1.	(a)	Show with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain.	Marks 06+08
			CLO 3
	(b)	If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$ $h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$ Produce Y(z) and y[n]	
Q2.		$f(x) = \begin{cases} -\pi/2 & -\pi \leq x \leq 0 \\ \pi/2 & 0 \leq x \leq \pi \end{cases}$ Retrieve the Fourier series for the given function.	Marks 10
			CLO 3
Q3.		If $X(z) = 2z^2 + 2z/(z^2 + 2z - 3)$ Retrieve x[n] using inverse Z-transform method.	Marks 10
			CLO 3
Q4.		Express the transfer function using the given data. $A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $D = [0]$	Marks 09
			CLO 3
Q5.		Apply Fourier transform on the signal, $x(t) = e^{-a\|t\|} u(t)$ where u (t) is a unit step function.	Marks 07
			CLO 3

①

Q1:

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Exam: Final Exam
25/6/2020

Q1: ⓐ

Let $x(t)$ be continuse time Signal Then

$$x(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} x(jw) e^{jwt} dw$$

Diff w.r.t t

$$\frac{d}{dt} x(t) = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} x(jw) dw \frac{d}{dt} e^{jwt}$$

$$= \frac{1}{2\pi} \int_{-\alpha}^{\alpha} jw\, x(jw) e^{jwt} dw$$

$$jw \frac{1}{2\pi} \int_{-\alpha}^{\alpha} x(jw) e^{jwt} dw$$

$$\frac{d}{dt} x(t) = jw\, x(t)$$

$$f\left\{\frac{d}{dt}x(t)\right\} = Jwx(Jw)$$

The above show the differentiation of a function in Time Domain will Result in multiplication of it frequency domain

Q1: (b) (3)

$x[n] = 5\delta[n] - 4\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$

Soll:

$y[n] = H(z) * X(z)$

$H(z) = 2 - 4z^{2} + 2z^{-3}$

$h(z) = 3 + z^{-1} + 2z^{-2}$

$= (2 - 4z^{-2} + 2z^{-3}) * (3 + z^{-1} + 2z^{-2})$

$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4}$
$+ 6z^{-3} + 2z^{-4} + 4z^{-5}$

$= 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

$Y[z] = 6 + 2z^{-1} - 8z^{-2} + 2z^{-3} - 6z^{-4} + 4z^{-5}$

- to find $y[n]$, use delay property

$y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta(n-2] + 2\delta[n-3]$
$- 6\delta[n-4] + 4\delta[n-5]$

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Q9

Retrive $x[n]$ using Fourier Series for given function.

$$f(x) = \begin{cases} -\pi/2 & -\pi < x < 0 \\ \pi/2 & 0 < x < \pi \end{cases}$$

Soll:

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{2\pi} \left(\int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{0} -\pi/2\, dx + \int_{0}^{\pi} \pi/2\, dx$$

$$= \frac{1}{2\pi} \left[\frac{-\pi}{2} \int_{-\pi}^{0} 1\, dx + \pi/2 \int_{0}^{\pi} 1\, dx \right]$$

$$= \frac{1}{2\pi} \left[\frac{-\pi}{2} x \Big|_{-\pi}^{0} + \frac{\pi}{2} x \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\pi/2 \left[0 - (-\pi) \right] + \pi/2 \left[\pi - 0 \right] \right]$$

$$= \frac{1}{2\pi} \left[\pi/2 (\pi) + \frac{\pi}{2} (\pi) \right]$$

$$= \frac{1}{2\pi} \left[\frac{+\pi^2}{2} + \frac{\pi^2}{2} \right]$$

P.T.O

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$$= \frac{1}{2\pi}\left[\frac{0}{2}\right] = 0$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\,\cancel{\cos nx}\,\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} -\frac{\pi}{2}\cos nx\,dx + \int_{0}^{\pi} \frac{\pi}{2}\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{2}\frac{\sin nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2}\sin nx\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{n\pi}\left[\frac{-\pi}{2}\left[\sin n(0) - \sin n(-\pi)\right] + \frac{\pi}{2}\left[\sin n(\pi) - \sin(0)\right]\right]$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(0) + \frac{\pi}{2}(0)\right]$$

$$a_n = \frac{1}{n\pi}(0)$$

$$\boxed{a_n = 0}$$

$$bn = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x) \sin nx \, dx + \int_{0}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2} \sin nx \, dx + \int \frac{\pi}{2} \sin nx \, dx\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0} \sin nx \, dx + \frac{\pi}{2}\int_{0}^{\pi} \sin nx \, dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\left(\frac{-\cos nx}{n}\right]_{-\pi}^{0} + \frac{\pi}{2}\left(\frac{-\cos nx}{n}\right|_{0}^{\pi}\right.$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-\cos(0) + \cos(-\pi)\right] + \frac{\pi}{2}\left\{-\cos n(\pi) + \cos(0)\right.\right.$$

$$= \frac{1}{n\pi}\cdot\frac{\pi}{2}\left[-1(1+1)\right) + \frac{1}{n\pi}\frac{\pi}{2}\left[-1-1\right]$$

$$= \frac{1}{2n}\left[-1(1+1)\right) + \frac{1}{2n}\left(-1-1\right)$$

$$bn = \begin{cases} 0 & \text{if } n \text{ is even} \\ 2/n\pi & \text{if } n \text{ is odd} \end{cases}$$

$$= \boxed{f(x) = \frac{2}{1\pi}\sin 1x + \frac{2}{3\pi}\sin 3x + \frac{2}{5\pi}\sin 5x + \frac{2}{7\pi}\sin 7x}$$

⑧ ⑦

Q5: $x(t) = e^{-a|t|} u(t)$ where $u(t)$

Soll: $x(t) = e^{-a|t|} u(t)$ is an unit step function

$x(t) = e^{-a|t|}$ $a > 0$

$x(j\omega) = ?$

$$x(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= x(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t} dt$$

$$x(j\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t} dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{at-j\omega t} dt + \int_{0}^{\infty} e^{(-at-j\omega t)} dt$$

P.T.O

(8) (8)

$$= \int_{-\alpha}^{0} e^{(at - jw)t}\,dt + \int_{0}^{\alpha} e^{-a(a+jwt)t}\,dt$$

$$= \frac{1}{a-jw}\left| e^{(a-jw)t} \right|_{-\alpha}^{0} - \frac{1}{a+jw}\left| e^{-(a+jw)t} \right|_{0}^{\alpha}$$

$$= \frac{1}{a-jw}\left[e^{(a-jw)(0)} - e^{(a-jw)(\alpha)} \right] - \frac{1}{a+jw}\left[e^{-(a+jw)(\alpha)} - e^{-(a+jw)(0)} \right]$$

$$= \frac{1}{a-jw}\left[e^{0} - e^{-\alpha} \right] - \frac{1}{a+jw}\left[e^{-\alpha} - e^{0} \right]$$

$$= \frac{1}{a-jw}\left[1 - 0 \right] - \frac{1}{a+jw}\left[0 - 1 \right]$$

$$= \frac{1}{a-jw}(1) - \frac{1}{a+jw}(-1)$$

$$= \frac{1}{a-jw} + \frac{1}{a+jw}$$

P.T.O

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$$\frac{1}{a-jw} + \frac{1}{a+jw}$$

$$= \frac{(a+\cancel{jw}) (a-\cancel{jw})}{(a-jw)(a+jw)}$$

$$= \frac{2a}{(a-jw)(a+jw)}$$

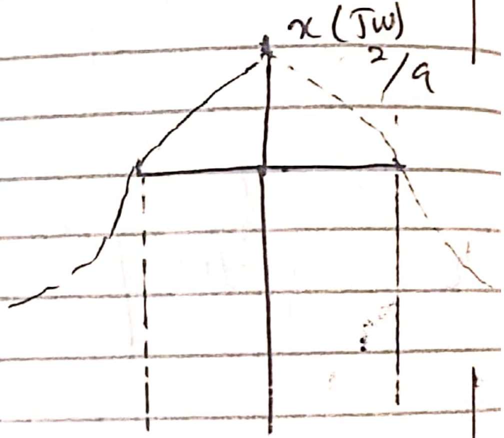
$$= \frac{2a}{(a)^2-(jw)^2}$$

$$= \frac{2a}{a^2-jw^2}$$

$$= \frac{2a}{a^2-(-1)w^2}$$

$$= \frac{2a}{a^2+j^2}$$

$$= x(jw) = \frac{2a}{a^2+jw^2}$$



Q# 3 (10)

$$X(z) = \frac{2z^2+2z}{z^2+2z-3}$$

Soll:

$$X(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$= \frac{2z(z+1)}{z^2+2z-3}$$

$$= \frac{2z(z+1)}{z^2+3z-z-3}$$

z^2-z

$$= \frac{2z(z+1)}{z(z+3)-1(z+3)}$$

$$= \frac{2z(z+1)}{(z+3)(z-1)}$$

$$\frac{x(z)}{2z} = \frac{z+1}{(z+3)(z-1)}$$

$$\frac{z+1}{(z+3)(z-1)} = \frac{A}{z+3} + \frac{B}{z-1} \rightarrow$$ apply partial fraction

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10 (a) (b)

$z+1 = A(z-1)+B(z+3) \rightarrow (1)$

Put $z = 1$ in (1)

$1+1 = A(1-1)+B(1+3)$

$2 = A(0)+B(4)$

$2 = B4$

$B = \frac{2}{4} \quad \frac{1}{2}$

$\boxed{B = \frac{1}{2}}$

Put $z = -3$ in eq (1)

$-3+1 = A(-3-1)+B(-3+3)$

$-2 = A(-4)+B(0)$

$= -2 = A(-4)$

$= A = \frac{+2}{+4} = \boxed{\frac{1}{2}}$

Put the value in (x)

$\frac{X(z)}{z} = \frac{\frac{1}{2}}{z-1} + \frac{\frac{1}{2}}{z+3}$ take Z Transform

$x[n] = \frac{1}{2}(1) + \frac{1}{2}(3)^k$

$$z+1 = A(z-1)+B(z+3) \rightarrow (1)$$

Put $z = 1$ in (1)

$$1+1 = A(1-1)+B(1+3)$$

$$2 = A(0)+B(4)$$

$$2 = B4$$

$$B = 2/4 \quad \frac{1}{2}$$

$$\boxed{B = \frac{1}{2}}$$

Put $z = -3$ in eq (1)

$$-3+1 = A(-3-1)+B(-3+3)$$

$$-2 = A(-4)+B(0)$$

$$-2 = A(-4)$$

$$A = +2/+4 = \boxed{\frac{1}{2}}$$

Put the value in (x)

$$\frac{X(z)}{z} = \frac{\frac{1}{2}}{z-1} + \frac{\frac{1}{2}}{z+3}$$

take z Transform

$$x[n] = \frac{1}{2}(1) + \frac{1}{2}(3)^k$$

Q 3 (10 16) (18) (11) (12)

Other Method

$$X(z) = \frac{2z^2+2z}{z^2+2z-3}$$

$$= \frac{2z(\cancel{z+1})}{(\cancel{z+1})(z-3)}$$

$$= \frac{2z}{z-3}$$

$$x(z)\,z^{n-1} = \frac{2z}{z-3}\,z^{n-1}$$

$|z=3$

$$\frac{6}{0}\,z^{n-1}$$

Sollution not

$z=-1$

$$\frac{-2}{-4}\,z^{n-1}$$

$$= \frac{1}{2}\,z^{n-1}$$

$$R(z=-1) = \frac{1}{2}\,z^{n-1}$$

$x(n) = res\,(z=-1) + res\,(z=3$

$= \frac{1}{2} z^{n-1}$ ~~form~~ form $n > 0$

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14   16   13

Q4:   Q: 3

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \end{bmatrix} \quad D = [0]$$

let

$$q = Aq + BU = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} q + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$= \begin{bmatrix} -2q & -q \\ q & 0 \end{bmatrix} + \begin{bmatrix} u \\ 0 \end{bmatrix}$$

$$y = Cq + DU = \begin{bmatrix} 1 & 2 \end{bmatrix} q + [0]\, u$$

$$= \begin{bmatrix} q & 2q \end{bmatrix}$$

$$SI - A = S \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} S+2 & 1 \\ -1 & S \end{bmatrix}$$

$$\Phi = (SI - A)^{-1} = \begin{bmatrix} S & -1 \\ 1 & S+2 \end{bmatrix}$$

$$HS = \frac{y(S)}{u(S)} = C\Phi B + D$$

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$$HS = \frac{\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix}}{s^2+3s+1}$$

$$HS = \frac{\begin{bmatrix} s+1 & 0 \\ s+1 & 0 \end{bmatrix}}{s^2+2s+1}$$ Ans.

2nd Method

$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 \end{bmatrix}$ $D = [0]$

$G(s) = C[sI - A]^{-1} B + D$

$= [1\ 2]\left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$

$= [1\ 2]\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$

$[1\ 2] \begin{bmatrix} s+2 & 1 \\ -1 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 0$

$[1, 2] \dfrac{1}{s(s+4)+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} s \\ 1 \end{bmatrix}$

$G(s) = \dfrac{1}{s^2+2s+1} [s \quad 2]$ Ans.