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Section → BS (SE) B

Paper → Operation research

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Q: There are total of 5 machines and five employments are to be relegated and the related cost network is as per the following locate the best possible task.

	A	B	C	D	E
1	6	12	3	11	15
2	4	2	7	1	10
3	8	11	10	7	11
4	16	19	122	23	21
5	9	5	7	6	10

Phas: 1: →

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Ans: steps 1: Subtract the minimum value of each row from the entries of that row.

	Machines					Row Minimum
	A	B	C	D	E	
1	6	12	3	11	15	3
2	4	2	7	1	10	1
3	8	11	10	7	11	7
4	16	19	122	23	21	16
5	9	5	7	6	10	5

Row Reduction:

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step 2: subtract the minimum value of each column from the entries of that column.

	A	Machines			D	E
		B	C	D		
1	3	9	0	8	12	
2	3	1	6	0	9	
3	1	4	3	0	4	
4	0	3	10	6	7	5
5	4	0	2	1	5	

columns min 0 0 0 0 4

Phase 2: optimization of the problem:

$\frac{3}{5} \times \frac{7}{5}$

step III: Draw a minimum no of lines to cover all the zeros of the matrix

(Row scanning:) starting from the first row, are the following equation: is there exactly one zero in that row? If

Yes, mark a square around that zero entry and draw a vertical line passing through that zero, otherwise skip row.

column Reduction

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	A	B	C	D	E
Jobs 1	3	9	0	8	8
2	3	4	6	0	5
3		4	3	0	0
4	0	3	10	7	1
5	4	0	2	1	4

Jobs	Machines	Time
1	3	3
2	4	1
3	5	11
4	1	16
5	2	5
		36

total processing time = 36 chr:

Machines timing

	A	B	C	D	E
Jobs 1	6	12	3	11	15
2	4	2	7	1	10
3	8	11	10	7	11
4	16	19	12	23	21
5	9	5	7	6	10

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Solve the following Linear programming problem:-

$$\min z = 2x_1 + 3x_2$$

$$\text{subject to } \frac{1}{2}x_1 + \frac{1}{4}x_2 \leq 4$$

$$x_1 + 3x_2 \geq 20$$

$$x_1 + x_2 = 10$$

$$x_1, x_2 \geq 0$$

Ans \rightarrow Step 1: Add surplus variable and artificial variable.

$$\frac{1}{2}x_1 + \frac{1}{4}x_2 + s_1 = 4$$

$$x_1 + 3x_2 - s_2 + a_1 = 20$$

$$x_1 + x_2 + a_2 = 10$$

Step 2: Add slack variable and set the objective function equal to 0.

$$\min z = 2x_1 + 3x_2 + M a_1 + M a_2$$

$$z - 2x_1 - 3x_2 - M a_1 - M a_2 = 0$$

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step 3:: crect simple table::

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	-2	-3	0	0	-M	-M	0
0	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

In the above table " a_1 " and " a_2 " both are basic variable and having coefficient of " M " in the objective function. but according to the definition of basic variable -

(MR3 + R1)

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	$M-2$	$3M-3$	0	-M	0	-M	$20M$
0	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	4
0	1	3	0	-1	-1	0	20
0	1	1	0	0	0	1	10

(MR4 + R1)

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	$2M-2$	$4M-3$	0	-M	0	0	$30M$
0	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	4
0	1	3	0	-1	1	0	20
0	1	1	0	0	0	1	10

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Now step 4: to find pivot column, steps
to find pivot Rows, step 6: to
find pivot value.

Z	x_1	x_2	S_1	S_2	a_1	a_2	
1	$2M-2$	$4M-3$	0	-M	0	0	$30M$
0	1	$\frac{1}{4}$	1	0	0	0	$4: \frac{4}{\frac{1}{4}} = 8$
0	1	3	0	-1	1	0	$20: \frac{20}{3}$
0	1	1	0	0	0	1	$10: \frac{10}{1} = 10$

step 4 →

Pivot
column

step 6:
pivot value

step 5
pivot Row

Step 7: Perform row operations to make
pivot equal to 1 and the $(\frac{1}{3} R_3)$
remaining elements in the
pivot column equal to zero

Z	x_1	x_2	S_1	S_2	a_1	a_2	
1	$2M-2$	$4M-3$	0	-M	-M	0	$30M$
0	$\frac{1}{3}$	$\frac{1}{4}$	1	0	0	0	4
0	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$-\frac{1}{3}$	0	$20 \times$
0	1	0	0	0	0	1	10

(R₃ → 7)

(-1R₃ + R₄)

Z	x ₁	x ₂	S ₁	S ₂	a ₁	a ₂	
1	2M-2	4M-3	0	-M	0	0	30M
0	1/2	1/4	1	0	0	0	4
0	1/3	1	0	-1/3	1/3	0	20/3
0	2/3	0	0	1/3	-1/3	1	10/3

(-1/4 R₂ + R₂)

Z	x ₁	x ₂	S ₁	S ₂	a ₁	a ₂	
1	2M-2	4M-3	0	-M	0	0	30
0	5/12	0	1	1/12	-1/12	0	7/3
0	1/3	1	0	-1/3	1/3	0	20/3
0	2/3	0	0	1/3	-1/3	1	10/3

((-4M+3) R₃ + R₁)

Z	x ₁	x ₂	S ₁	S ₂	a ₁	a ₂	
1	(2/3) ^{M-2}	0	0	(1/3)(M-1)	(1-4/3 M)	0	20 + 10M/3
0	5/12	0	1	1/12	-1/12	0	7/3
0	1/3	1	0	-1/3	1/3	0	20/3
0	2/3	0	0	1/3	-1/3	1	10/3

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Step 8:-

Repeat process again find pivot column pivot Row, and pivot value.

step u.s.b.

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	$(\frac{2}{3})^{M-1}$	0	0	$(\frac{1}{3})^{M-1}$	$1 - \frac{4}{3}M$	0	$20 + \frac{10}{3}M$
0	$\frac{5}{12}$	0	1	$\frac{1}{12}$	$-\frac{1}{12}$	0	$\frac{7}{3} = 5.6$
0	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{20}{3} = 20$
0	$(\frac{2}{3})$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	$(\frac{10}{3}) = 3$

step 7:- operation to make pivot value to 1 and remaining elements in the pivot column equals 0.

C $(\frac{3}{2})$ R 4

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	$\frac{2}{3}M-1$	0	0	$\frac{1}{3}M-1$	$1 - \frac{4}{3}M$	0	$20 + \frac{10M}{3}$
0	$\frac{5}{12}$	0	1	$\frac{1}{12}$	$-\frac{1}{12}$	0	$\frac{7}{3}$
0	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{20}{3}$
0	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	5

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$$\left(-\frac{2}{3}M+1\right) R_4 + R_1$$

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}-M$	$\frac{3}{2}-M$	25
0	$\frac{5}{12}$	0	1	$\frac{1}{12}$	$-\frac{1}{12}$	0	$\frac{7}{3}$
0	$\frac{1}{3}$	1	0	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{20}{3}$
0	1	0	0	$\frac{1}{3}$	$-\frac{1}{2}$	$\frac{3}{2}$	5

$$\left(-\frac{5}{12}\right) R_4 + R_2$$

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	0	0	0	$-\frac{1}{2}$	$\left(\frac{1}{2}-M\right)$	$\frac{3}{2}-M$	25
0	0	0	1	$-\frac{1}{8}$	$\frac{1}{8}$	$-\frac{5}{8}$	$\frac{1}{4}$
0	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{20}{3}$
0	1	0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	5

$$\left(-\frac{1}{3}\right) R_4 + R_3$$

Z	x_1	x_2	s_1	s_2	a_1	a_2	
1	0	0	0	$-\frac{1}{2}$	$\left(\frac{1}{2}-M\right)$	$\frac{3}{2}-M$	25
0	0	1	0	$-\frac{1}{8}$	$\frac{1}{8}$	$-\frac{5}{8}$	$\frac{1}{4}$
0	0	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	5
0	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	5

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Minimum $Z = 25$

	Z	x_1	x_2	s_1	s_2	a_1	a_2	
Z	1	0	0	0	$-\frac{1}{2}$	$(\frac{1}{2}-M)$	$\frac{3}{2}-M$	25
s_1	0	0	0	1	$-\frac{1}{8}$	$\frac{1}{8}$	$-\frac{5}{8}$	$\frac{1}{4}$
s_2	0	0	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	5
x_1	0	1	0	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	5

$$\text{Min } Z = 25 \quad x_1 = 5 \quad s_2 = 0$$

$$s_1 = \frac{1}{4} \quad x_2 = 5 \quad a_1 = 0$$

$$a_2 = 0$$

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Q3 Use Vogel's Approximation Method obtain the initial feasible solution

Solution:

						supply
1	20	22	17	4	120	
2	24	37	9	4	70	
3	32	37	20	7	50	
Demand	60	40	30	110	240	

Demand = supply
Balanced transportation problem

X	40	X	80	80				
20	22	17	4	120	13	13	-	-
10	X	30	30	40				
24	37	9	7	70	2	2	2	17
50	X	X	X	0				
32	37	20	15	50	5	5	5	17
60	40	30	110					
50	0	0	0					
0								

4	(8)	8	3
4	-	8	3
8	-	(11)	8
8	-	-	8

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$$(40 \times 22) + (80 \times 4) + (10 \times 24) + (30 \times 9) + (30 \times 7) + (50 \times 32) = 3520$$

$$820 + 320 + 240 + 270 + 210 + 1600 = \boxed{3520}$$

Ans 3520

