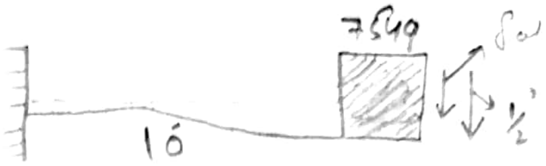
Question 1:

Given Data:

$E = 29000$ ksi
$I = 150 \, in^4$
$\delta_{st} = 7549$ lb



sol:

The general E.O.M for SDOF system

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

system is undamped ($c=0$) undergoing free vibration

$P(t) = 0$

EOM become

$$ku + m\ddot{u} = 0 \quad \text{— (1)}$$

Now

$$k = \frac{3EI}{L^3} = \frac{3 \times 29000 \frac{k}{in^2} \times 150 in^4}{(10 \times 12 in)^3}$$

$k = 7.55$ k/in
$k = 90625$ lb/ft

$$\rightarrow m = \frac{7549 \text{ lb sec}^2}{32.2 f}$$

$$\boxed{m = 234.44 \text{ slug}}$$

$$W_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{96025}{234.44}} = 20.238 \text{ rad/sec}$$

$$T_n = \frac{2\pi}{W_n} = \frac{2 \times 3.14}{20.238} = 0.310 \text{ sec}$$

Put the Value in eq ①

$$Ku + m\ddot{u} = 0$$

$$90625u + 234.44\ddot{u} = 0$$

General solution to the EOM for undamped free vibration is,

$$U(t) = U(0)\cos(W_n t) + \frac{\dot{U}(0)}{W_n}\sin(W_n t)$$

$$\therefore U(0) = 1/2'' = \frac{1}{2} \times \frac{1}{12} = \frac{1}{24} \text{ ft}$$

and

$$\dot{u}(0) = 0$$

$$u(t) = (1/24)(\cos(20.238)) + 0 = (1/24)(\cos(20.238))$$

Equivalent static force at any time 't' is

$$f_s(t) = K \cdot u(t) = 90625 \times \frac{(\cos(20.238))}{24}$$

$f_{(s)}(t) = 3776 \cos(20.238)$ ③

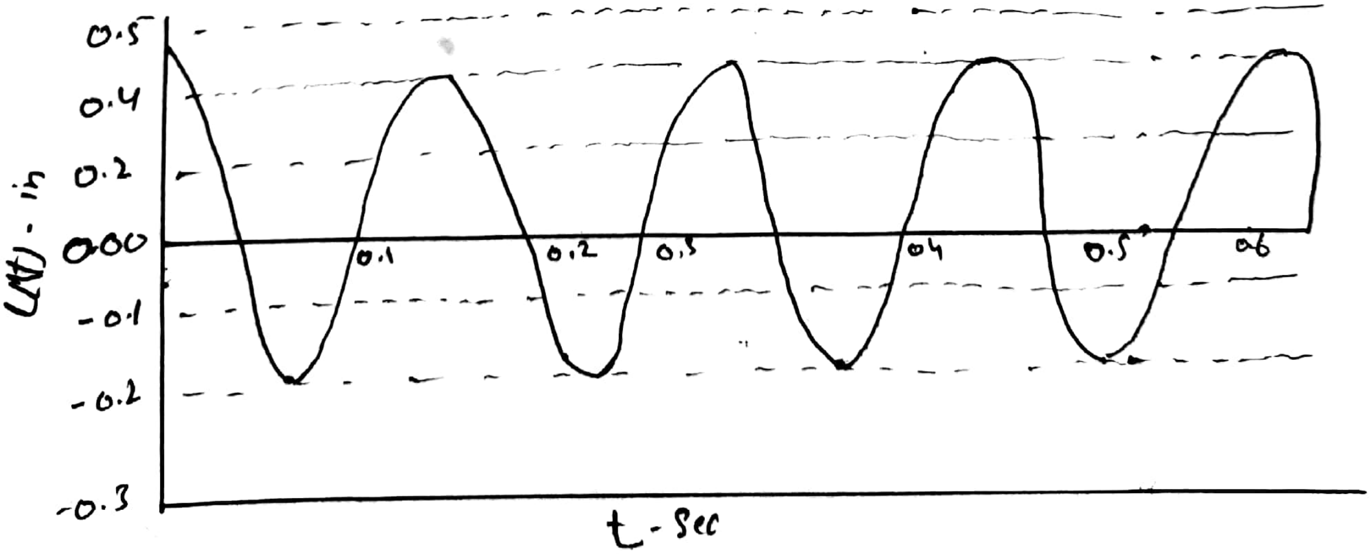
Amplitude of dynamic displacement u_0 for undamped free vibration.

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]}$$

$$u_0 = \sqrt{(1/24)^2 + 0} = 1/24 \text{ ft}$$

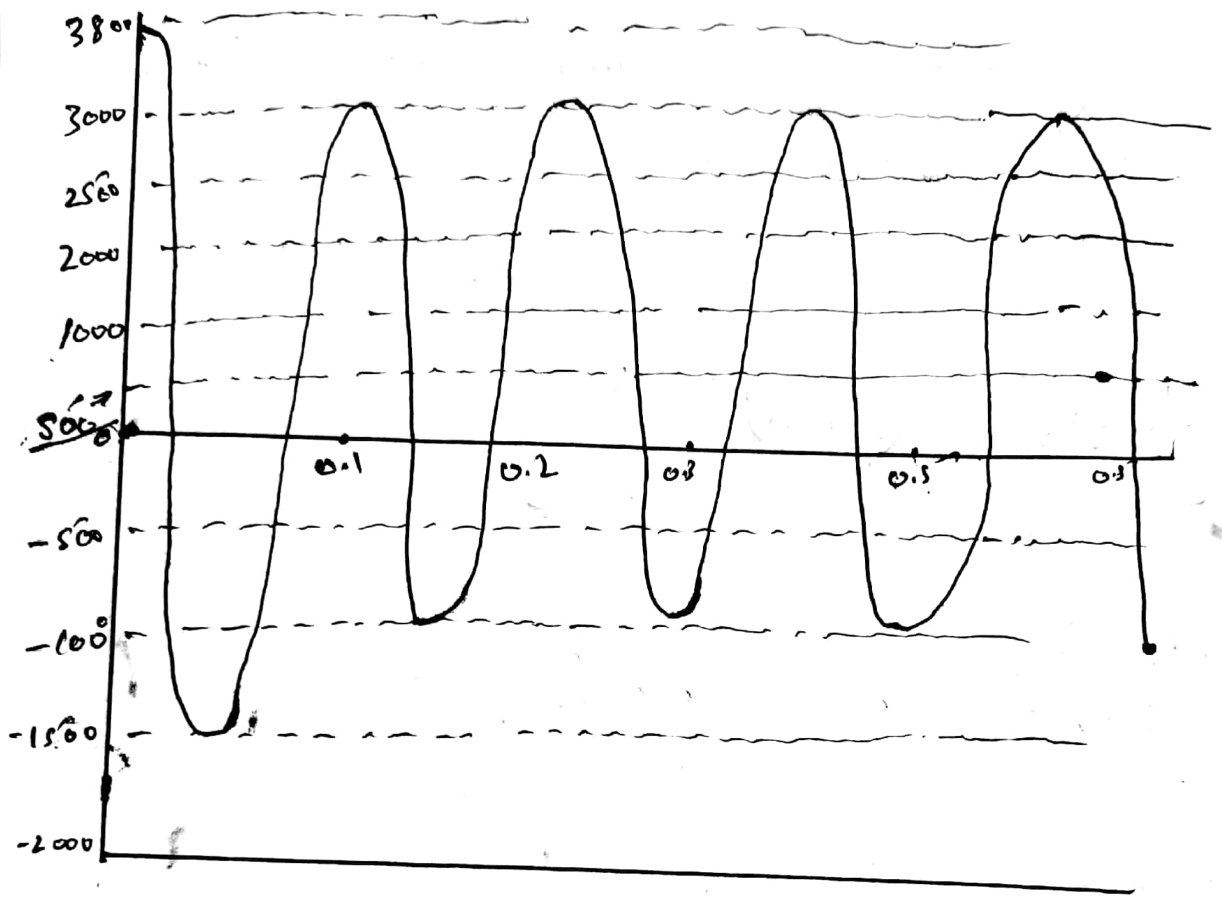
$K u_0 = 90625 \times \frac{1}{24} = $ **3776 lb**

★ Undamped Free vibration



(4)

✱ Undamped Free Vibration



Question No 2:-

Given Data:-

$E = 29000\ ksi$

$I = 150 in^4$

$\delta_{st} = 7695 lb$

Take $\xi_0 = 2.5\%$

Sol:-

E.O.M for damped free vibration

$$ku + c\dot{u} + m\ddot{u} = 0 \quad — (1)$$

We know that

$k = 90625\ lb/ft$

$m = 234.44\ sec^2/ft$

$$C = \xi_0 \times 2m\omega_n$$

$$C = 0.025 \times 2(234.44)(20.238)$$

$$\boxed{C = 237.23\ lb.sec/ft}$$

By Putting Value of K, C and m in eq (1)

$$Ku + C\dot{u} + m\ddot{u} = 0$$

$$90625u + 237.234\dot{u} + 234.44\ddot{u} = 0$$

Solution of E.O.M for damped free vibration is

$$U(t) = e^{-\zeta w_n t}\left[u(0)\cos(w_n t) + \frac{1}{w_D}\left[u(0)\zeta w_n\right] \cdot \sin(w_D t)\right)$$

$$w_D = \sqrt{k/m} = \sqrt{\frac{90625}{234.44}} = \boxed{19.66 \text{ rad/sec}}$$

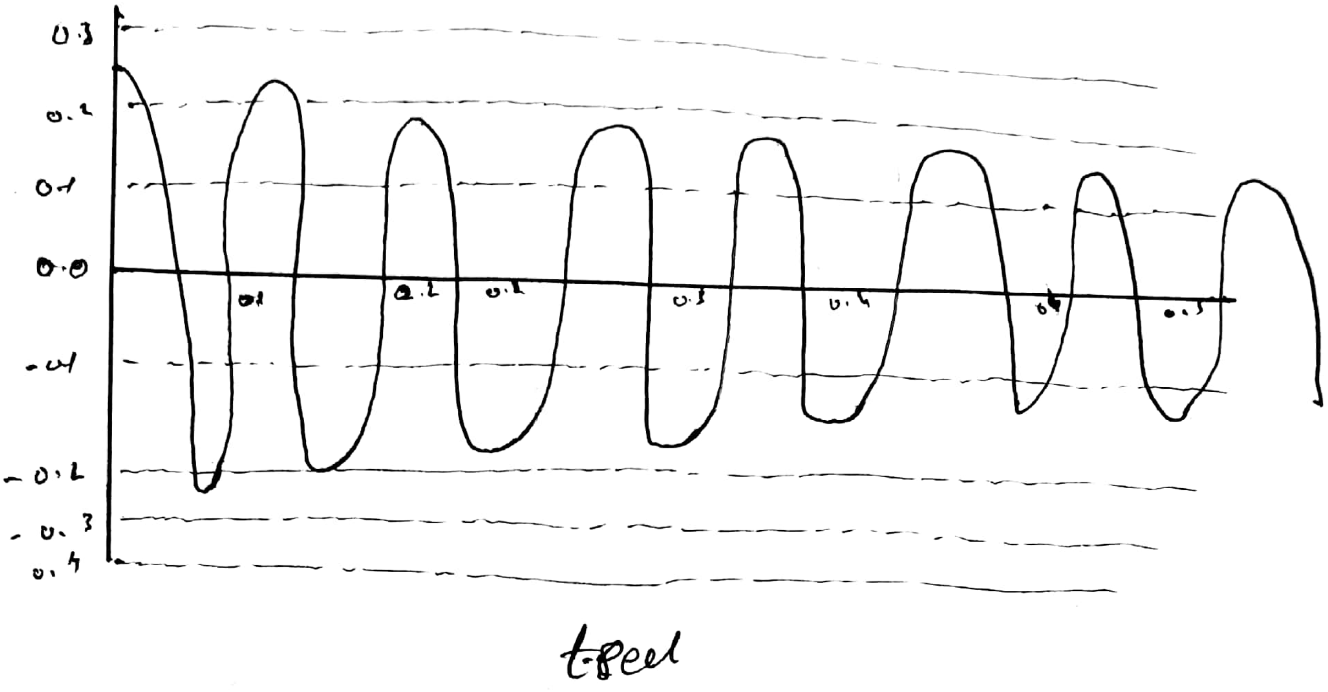
$$u(t) = e^{-0.025 \times 19.66}\left(\frac{1}{24} \times \cos(19.66) + \frac{1}{19.66} \times \left[0 + \frac{1}{24} \times 0.025 \times 19.66 \times \sin(19.66)\right]\right)$$

$$u(t) = e^{-0.4915}\left[0.04165 \cos(19.66t) + 0.001041 \sin(19.66t)\right]$$

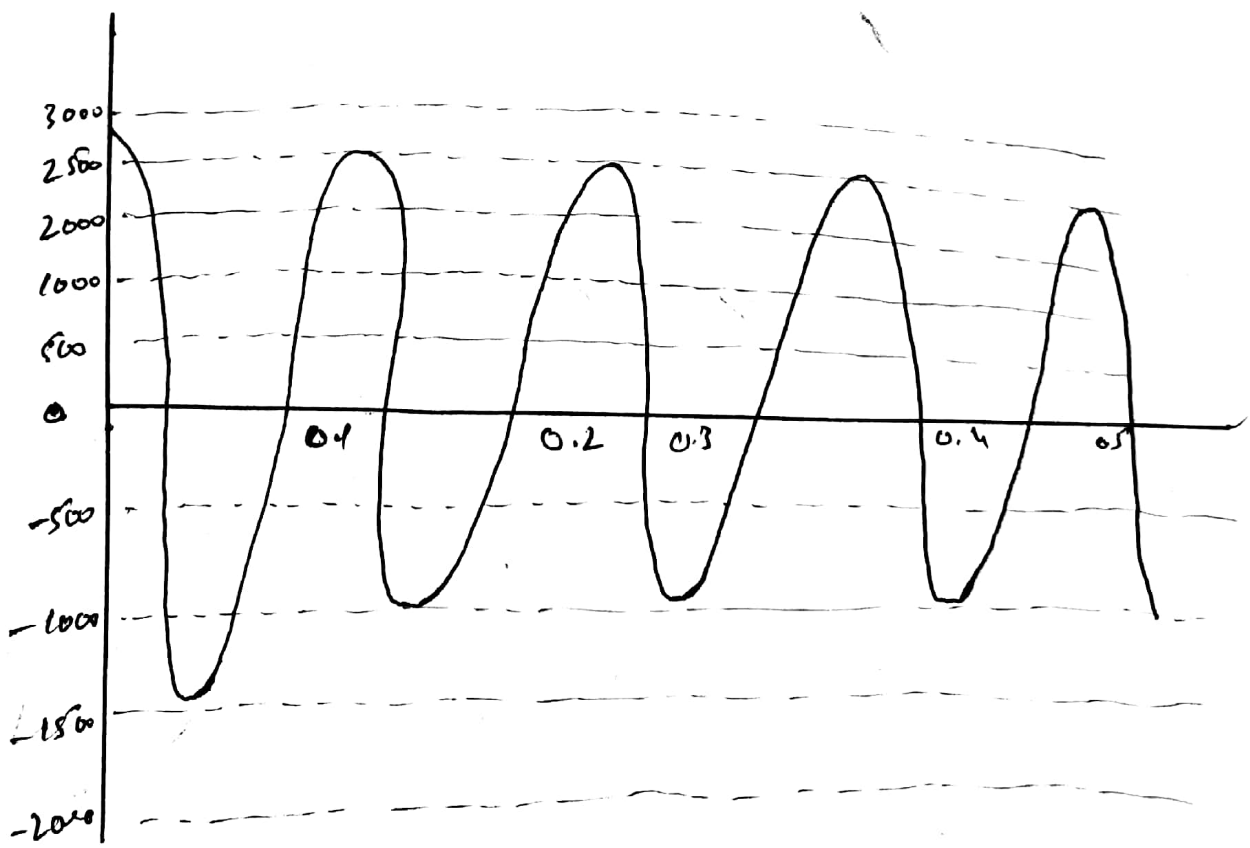
$$f_s(t) = k \cdot U(t) = 90625 \times u(t)$$

$$f_s(t) = e^{-0.4915}\left[3776 \cos(19.66t) + 94.34 \sin(19.66t)\right]$$

★ Damped free vibration



★ Damped free Vibration



Question # 3:-

Given Data:-

Force = 60 kip

Horizontal Distplacement = $\frac{7549}{1000}$" = 7.549"

Cycle = 7

time = 3.57 sec

amplitude displacement = 2.286 cm = 0.9"

Sol:-

(a) Damping Ratio :-

$j = 7, \quad U_{j+1} = U_8 = 2.286$

$U_1 = 7.549"$

$$\Rightarrow \quad 7 = \frac{1}{2\pi\delta} \ln\left(\frac{7.549}{0.9}\right)$$

$$7 = \frac{1}{2\pi\delta}(2.127)$$

$$\delta = \frac{1}{7 \times 2 \times 3.14}(2.127)$$

$$\delta = 0.048 = \boxed{4.8\%}$$

(b) Natural Period of Un-damped Vibration:-

$T_n = ?$

7 cycle of vib complete in 3.57 sec

Time required to complete one circle $\frac{3.57}{7} = T_D$

$T_D = 0.51 \text{ sec}$

Now

$$W_D = W_n \sqrt{(1-\delta^2)}$$

$$2\pi / W_D = 2\pi / W_n \sqrt{(1-\delta^2)}$$

$$T_D = \frac{T_n}{\sqrt{1-\delta^2}}$$

$$T_n = T_D \times \sqrt{1-\delta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.048)^2} = \boxed{0.509 \text{ Sec}}$$

(c) Stiffness of Structure:-

$$K = \frac{60 \times \cos(60)}{2} = 15 \text{ k/in} = 1800 \text{ lb/ft}$$

(d) weight of tank:-

$$W_n = \sqrt{k/m} = \sqrt{k/w/g} = \sqrt{\frac{k \cdot g}{W}}$$

$$W_n^2 = \frac{K \times g}{W_n} \qquad \therefore W_n = \frac{2\pi}{T_n}$$

$$W = Kg\left(\frac{4\pi^2}{T_n^2}\right) = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{\frac{100000 \text{ lb}}{72} \times \frac{2.22 \text{ ft}}{\text{sec}^2} (0.51)^2}{4\pi^2}$$

$$W = 74875 \text{ lb} = \boxed{74.9K}$$

(e) Damping Coefficient:-

$$c = ?$$

We known that

$$J = \frac{c}{2 m \omega n}$$

$C = \zeta \times 2 m \omega_n = \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$

$C = \frac{0.048 \times 4 \times 3.14 \left(\frac{74875}{32.2}\right)}{0.51}$

$\boxed{C = 1455.3\ lb/ft}$

(f) No of cycle to reduce displacement amplitude 7.75in to 0.5"

$j = \frac{1}{2\pi\zeta} \ln\left[\frac{V_1}{V_{j+1}}\right]$

$j = \frac{1}{2 \times 3.14 \times 0.048} \ln\left(\frac{7.549}{0.5}\right)$

$j = 3.32 \ln\left(\frac{7.549}{0.5}\right)$

$j = 9.012 \approx \boxed{9\ cycle}$