Department of Electrical Engineering

Assignment

Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	30

Student Details

Name:	Hamza	**Student ID:**	16469

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$	Marks 5 CLO 2

(b) Compute the convolution y(n) of the following signal

Marks 5
CLO 2

$$x(n) = \begin{cases} a^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$

$$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$

Q3. Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC).

Marks 10
CLO 2

i. $$x(n) = \begin{cases} \left(\frac{1}{4}\right)^{n}, & n \ge 0 \\ \left(\frac{1}{3}\right)^{n}, & n < 0 \end{cases}$$

ii. $$x(n) = \begin{cases} (\cdot)^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$$

Q1 (a) consider the following analog signal.

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine the minimum sampling rate required to avoid aliasing.

(ii) suppose that the signal is sampled at the rate $F_s = 100\,Hz$. what is the discrete time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.

(iii) what is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation.

Sol:-

(i) :-

given signal is $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

The frequencies present in the signal above are;

$$f_1 = 50\,Hz \quad \& \quad f_2 = 100\,Hz$$

So $F_{max} = 100\,Hz$ and the minimum sampling rate will be $F_s \geq 2$

$F_{max} = 200\,Hz$

ii) :- Given that $F_s = 100\,Hz$

Thus discrete time signal will be;

$$x(n) = 3\cos \frac{100\pi n}{100} + 4\sin \frac{200\pi n}{100}$$

$$x(n) = 3\cos \pi n + 4\sin 2\pi n$$

(After sampling)

→ At $F_s = 100Hz$, there will be aliasing i.e., there will be ambiguities in signal after reconstructing of original signal from sampled discrete signal.

(iii) The analoge signal we can recover as

$$y_a(t) = 3\cos \pi n + 4\sin 2\pi n$$

change the above equation to time domain

$$y_a(t) = 3\cos \pi n \times \frac{100}{100} + 4\sin 2\pi n \times \frac{100}{100}$$

$$\because t = \frac{n}{F_s} = \frac{n}{100}$$

$$\therefore y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

which is same to same the farther signal (original signal).

③

Q1 (b) consider a discrete time signal which is given by

$$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

This is signal is sampled at the rate $F_s = 2Hz$.

(i) Draw the sampled signal.

(ii) The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i.

(iii) perform the process of Truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.

①:- $x(n) = 0.5^n$ for $n \geq 0$

$\therefore F_s = 2Hz$

$\therefore T = \frac{1}{F_s} = 0.5$

$x(0) = 1$

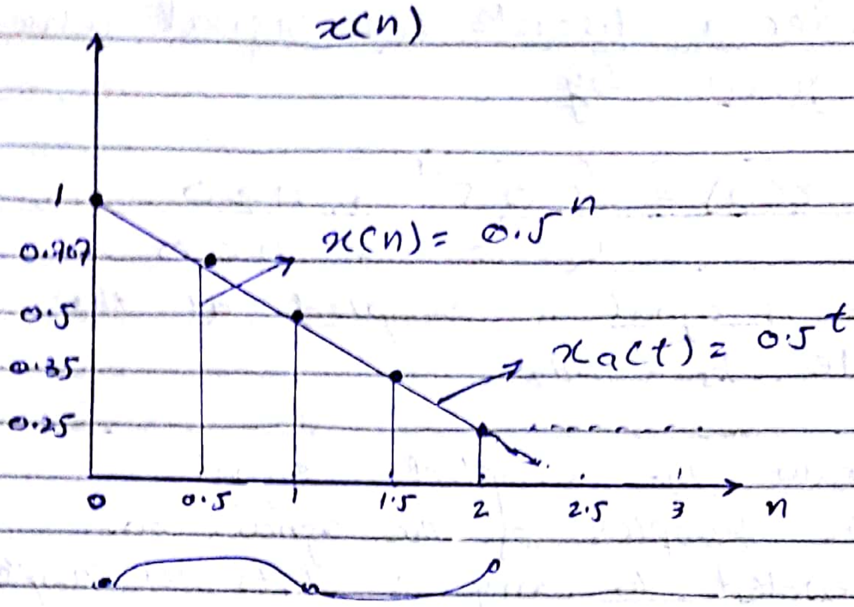
$x(0.5) = (0.5)^{0.5} = 0.707$

$x(1) = 0.5$

$x(1.5) = (0.5)^{1.5} = 0.35$

$x(2) = (0.5)^2 = 0.25$

④



(ii) Given that number of bits per sample is 3. let $n = 3$ bits then we need

$L \geq 2^n$; L is quantization level

$L \geq 2^3$

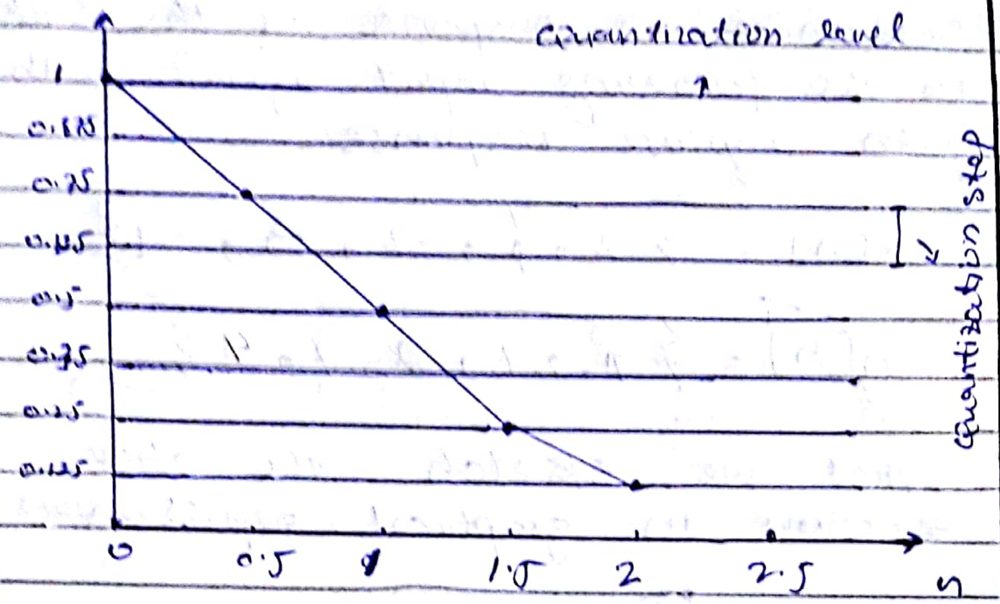
$L \geq 8$ levels

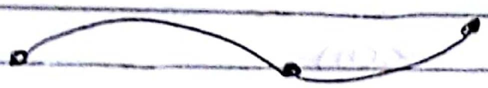
Thus the level of quantization must be equal or greater than 8.

Quantization resolution :-

$$\Delta = \frac{x_{max} - x_{min}}{L - 1}$$

$$\Delta = \frac{1 - 0}{8 - 1} = \frac{1}{7} = 0.142$$





(iii)

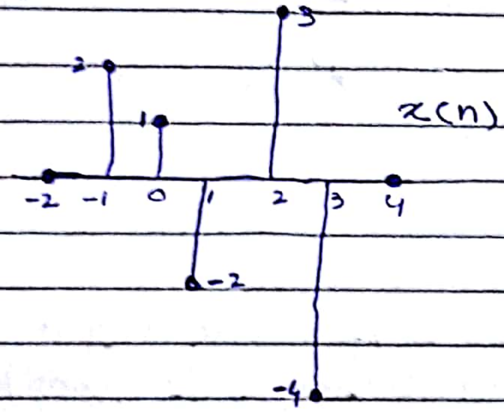
n	x(n) Discrete time signal	$x_q(n)$ Truncation	$x_q(n)$ (Rounding)	$e_q(n) = x_q(n) - x(n)$ (Rounding)
0	1	1	1	0
0.5	0.707	0.7	0.7	0
1	0.5	0.5	0.5	0
1.5	0.35	0.3	0.4	-0.1
2	0.25	0.2	0.3	-0.1
2.5	0.176	0.1	0.2	-0.1
3	0.125	0.1	0.1	0
3.5	0.088	0.0	0.1	-0.1
4	0.062	0.0	0.1	-0.1
4.5	0.044	0.0	0.0	0

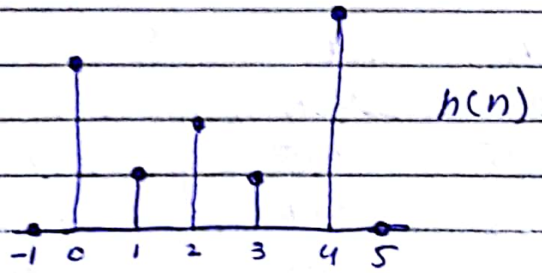
Q2 (a) Determine the response of the system to the following input signal with given impluse response.

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

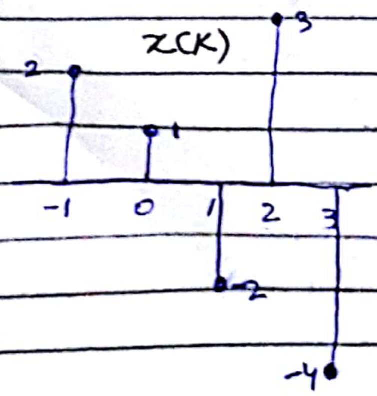
$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

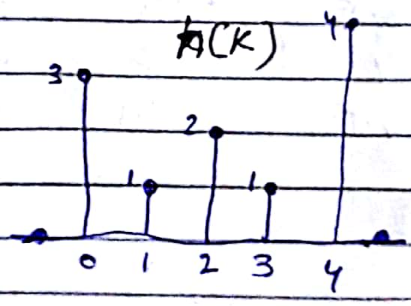
Sol:- first we sketch the above equation in graphical representation,



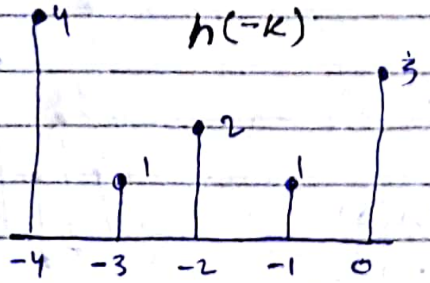


Replace n by k



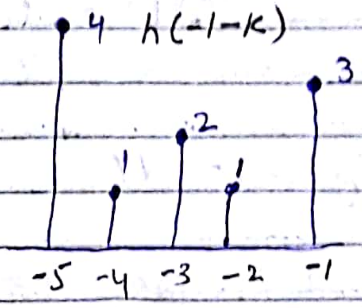


Now fold the impulse signal $(h(k))$.



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

for $n = -1$



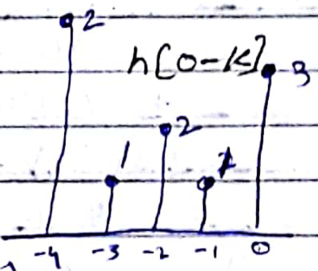
$$y(-1) = \sum_{k=-1}^{-1} x[k]\, h[-1-k]$$

$$= x[-1]\, h[-1+1]$$

$$= (2)(3)$$

$$= 6$$

Now for $n = 0$



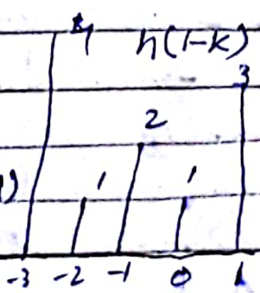
$$y(0) = \sum_{k=-1}^{0} x(k)\, h(-k)$$

$$= x(-1)h(-1) + x(0)h(0)$$

$$= (2)(1) + (1)(3) = 2+3$$

$$\boxed{y(0) = 5}$$

Then for $n = 1$



$$y(1) = \sum_{k=-1}^{1} x(k)\, h(1-k)$$

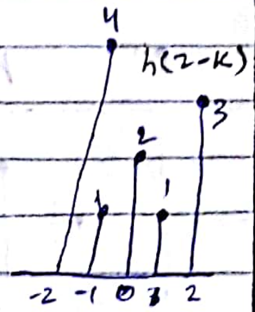
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1)$$

$$= (2)(2) + (1)(1) + (3)(-2)$$

$$= 4+1-6 = -1$$

$y(1) = -1$

for $n = 2$



$$y(2) = \sum_{k=-1}^{2} x(k)\,h(2-k)$$

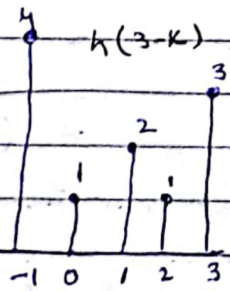
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$$

$$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$$

$$= 2 + 2 - 2 + 9 = 11$$

$y(2) = 11$

Now for $n = 3$



$$y(3) = \sum_{k=-1}^{3} x(k)\,h(3-k)$$

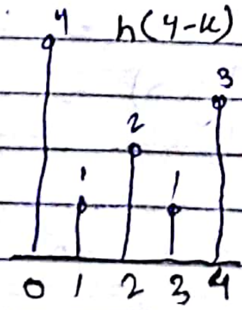
$$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$$

$$= 8 + 1 - 4 + 3 - 12$$

$y(3) = -4$

for $n = 4$



$$y(4) = \sum_{k=0}^{3} x(k)\,h(4-k)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

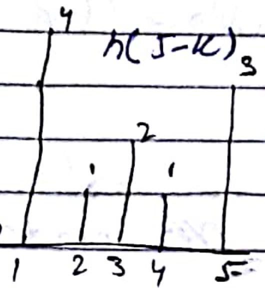
$$= (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$$

$$= 4 - 2 + 6 - 4$$

$y(4) = 4$

for $n = 5$

$$y(5) = \sum_{k=1}^{3} x(k)\,h(5-k)$$



$$= x(1)h(1) + x(2)h(2) + x(3)h(3)$$

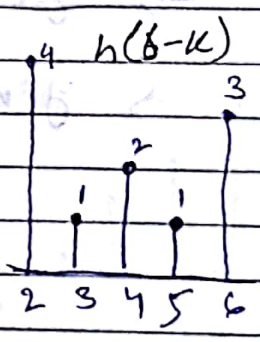
$$= (-2)(4) + (3)(1) + (-4)(2)$$

$$= -8 + 3 - 8$$

$$y(5) = -13$$

for $n = 6$

$$y(6) = \sum_{k=2}^{3} x(k)\,h(6-k)$$



$$= x(2)h(2) + x(3)h(3)$$

$$= (3)(4) + (-4)(1)$$

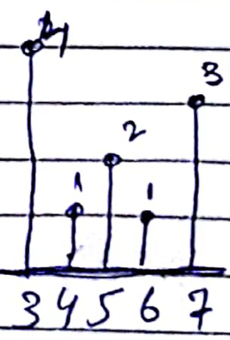
$$= 12 - 4$$

$$= 8$$

$$y(6) = 8$$

Now finally for $n = 7$

$$y(7) = x(3)h(3)$$

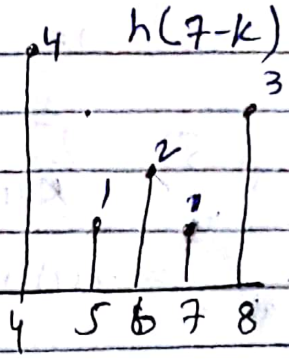


$$= (-4)(4)$$

$$= -16$$

$$y(7) = -16$$

Now for $n = 8$



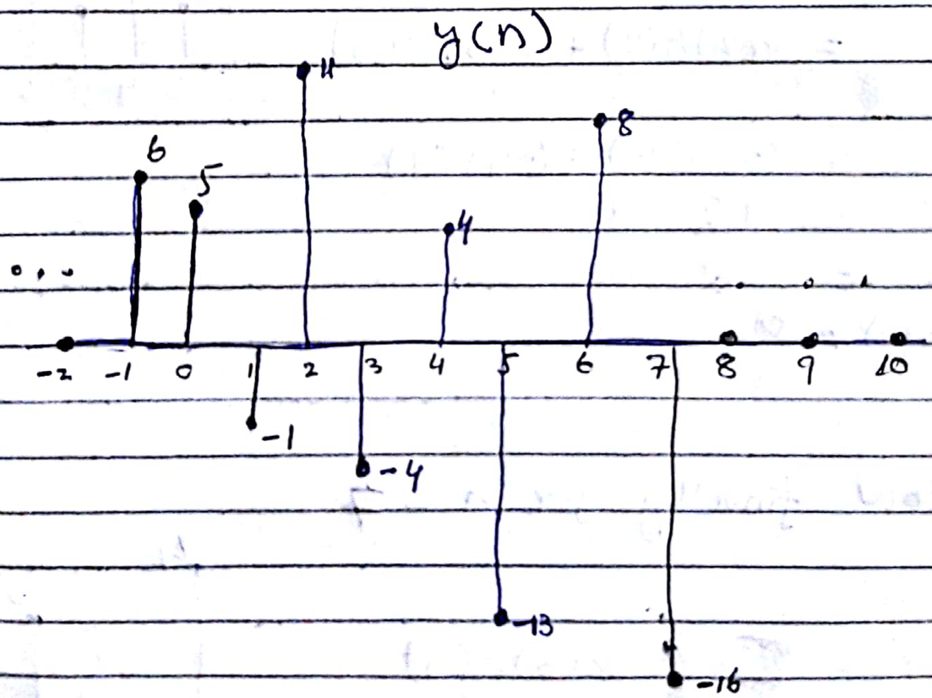
No matching samples

so;

$$\boxed{y(8) = 0}$$

$\rightarrow$ $y(n) = 0$ for $n \le -2$

$\rightarrow$ $y(n) = 0$ for $n \ge 8$



$$y(n) = \{\ldots, 0, 0, \underset{\uparrow}{6}, 5, -1, 11, -4, 4, -13, 8, -16\}$$

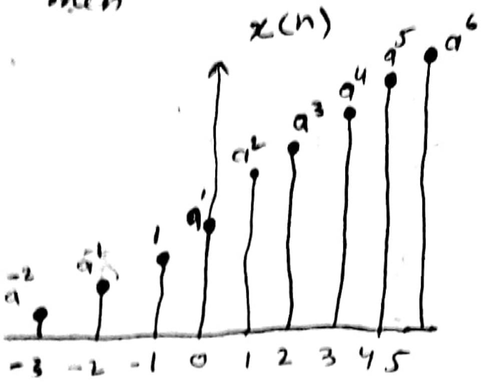
Q2 (B) Compute the convolution $y(n)$ of the following signal.

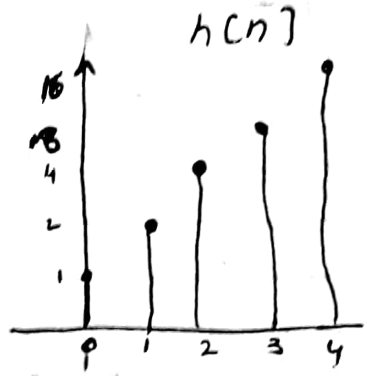
$$x(n) = \begin{cases} a^{n+1}, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^n, & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Sol:-

if $|a| > 1$
then



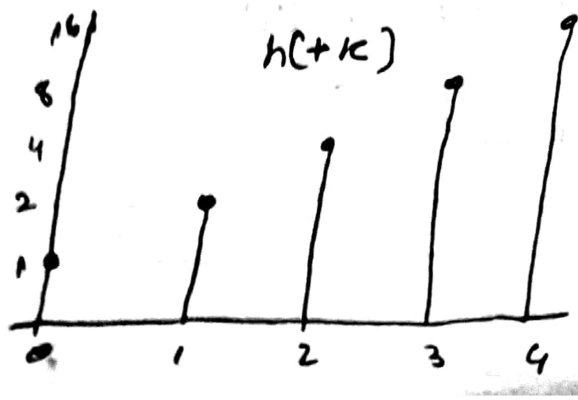


$$x(n) = \{a^{-2}, a^{-1}, 1, \underset{\uparrow}{a}, a^2, a^3, a^4, a^5, a^6\}$$

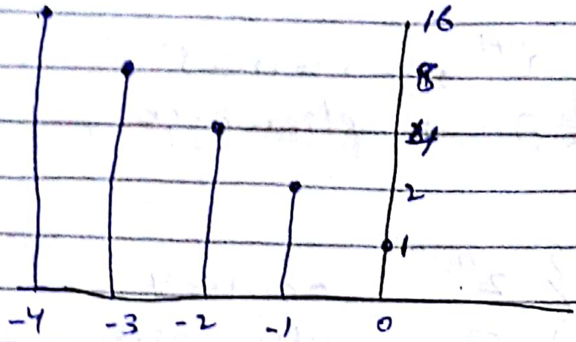
$$h[n] = \{1, 2, 4, 8, 16\}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

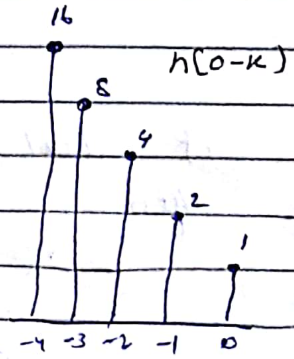
Replace n by k



Now Replace "k" by "-k"



for $n = 0$



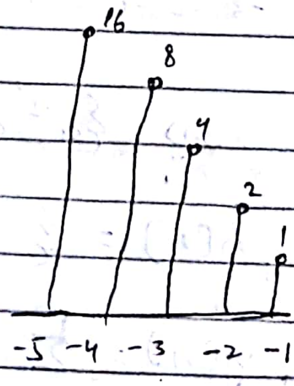
$$y(0) = \sum_{k=0}^{3} x(k)\,h(0-k)$$

$$= x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3)$$

$$= (a^{1})(1) + (1)(2) + (a^{-1})(4) + (a^{-2})(8)$$

$$\boxed{y(0) = a + 2 + 4a^{-1} + 8a^{-2}}$$

for $n = -1$



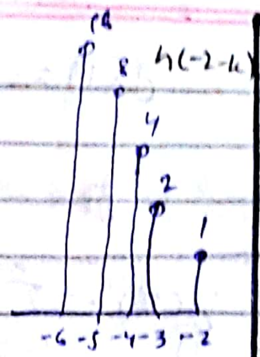
$$y(-1) = \sum_{k=0}^{2} x(k)\,h[-1-k]$$

$$= x(0)h(-1-0) + x(1)h(-1-1) + x(2)h(-1-2)$$

$$= (1)(1) + (a^{-1})(2) + (a^{-2})(4)$$

$$\boxed{y(-1) = 1 + 2a^{-1} + 4a^{-2}}$$

for $n = -2$



$$y(-2) = \sum_{k=-2}^{-3} x(k)\, h(-2-k)$$

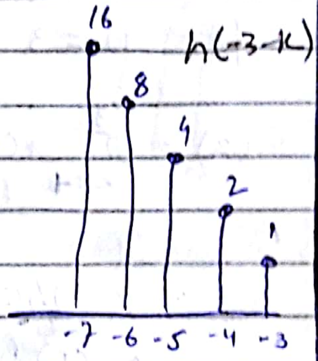
$$= x(-2)h(-2-2) + x(-3)h(-2-3)$$

$$= (a^{-1})(1) + (a^{-2})(2)$$

$$= a^{-1} + 2a^{-2}$$

$$\boxed{y(-2) = a^{-1} + 2a^{-2}}$$

for $n = -3$



$$y(-3) = \sum x(k)\, h(-3-k)$$

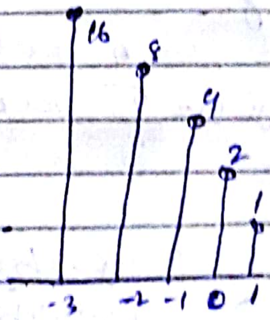
$$= x(-3)h(-3-3)$$

$$= (a^{-2})(1)$$

$$\boxed{y(-3) = a^{-2}}$$

Now for $\boxed{y(n) = 0 \quad n < -3}$

Now for $n = 1$



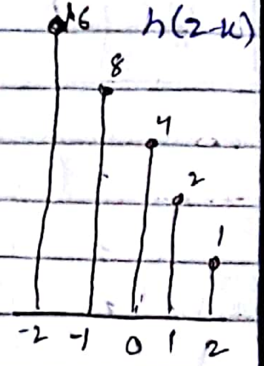
$$y(1) = \sum_{k=-3}^{1} x(k)\, h(1-k)$$

$$= x(-3)h(1+3) + x(-2)h(1+2) + x(-1)h(1+1) + x(0)h(1-0) + x(1)h(1-1)$$

$$= (a^{-2})(16) + (a^{-1})(8) + (1)(4) + (a)(2) + (a^{2})(1)$$

$$y(1) = 16a^{-2} + 8a^{-1} + 4 + 2a + a^2$$

for $n = 2$

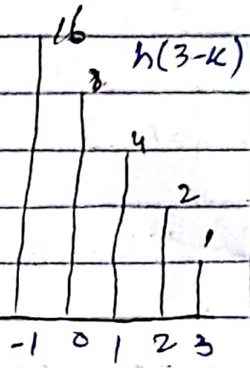


$$y(2) = \sum_{k=-2}^{2} x(k)\,h(2-k)$$

$$= x(-2)h(2+2) + x(-1)h(2+1) + x(0)h(2-0) + x(1)h(2-1) + x(2)h(2-2)$$

$$y(2) = 16a^{-1} + 8 + 4a + 2a^2 + a^3$$

for $n = 3$

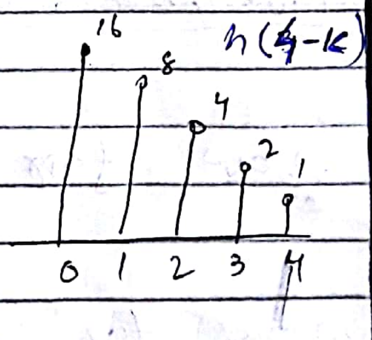


$$y(3) = \sum_{k=-1}^{3} x(k)\,h(3-k)$$

$$= x(-1)h(3+1) + x(0)h(3-0) + x(1)h(3-1) + x(2)h(3-2) + x(3)h(3-3)$$

$$y(3) = 16 + 8a + 4a^2 + 2a^3 + a^4$$

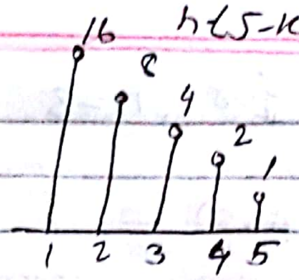
for $n = 4$



$$y(4) = \sum_{k=0}^{4} x(k)\,h(4-k)$$

$$= x(0)h(4-0) + x(1)h(4-1) + x(2)h(4-2) + x(3)h(4-3) + x(4)h(4-4)$$

$$y(4) = 16a + 8a^2 + 4a^3 + 2a^4 + a^5$$

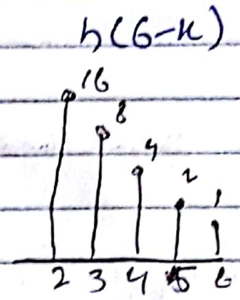


for $n = 5$

$$y(5) = \sum_{k=1}^{5} x(k)\, h(5-k)$$

$$= x(1)h(5-1) + x(2)h(5-2) + x(3)h(5-3)$$
$$x(4)h(5-4) + x(5)h(5-5)$$

$$y(5) = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$$

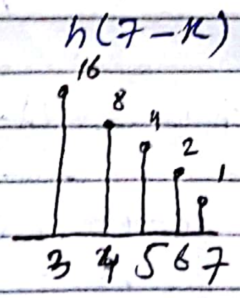


for $n = 6$

$$y(6) = \sum_{k=2}^{6} x(k)\, h(6-k)$$

$$= x(2)h(6-2) + x(3)h(6-3)$$
$$+ x(4)h(6-4) + x(5)h(6-5) \cancel{+ x(6)h(6-6)}$$

$$y(6) = 16a^3 + 8a^4 + 4a^5 + 2a^6 \cancel{+ a^7}$$



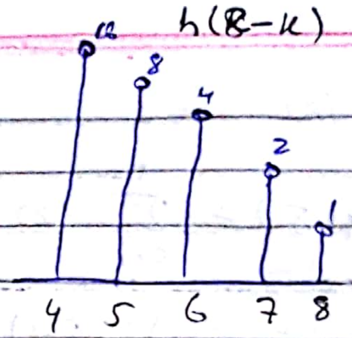
for $n = 7$

$$y(7) = \sum_{k=3}^{7} x(k)\, h(7-k)$$

$$= x(3)h(7-3) + x(4)h(7-4)$$
$$+ x(5)h(7-5) \cancel{+ x(6)h(7-6) + x(7)h(7-7)}$$

$$\boxed{y(7) = 16a^4 + 8a^5 + 4a^6} \cancel{+ 2a^7 + a^8}$$

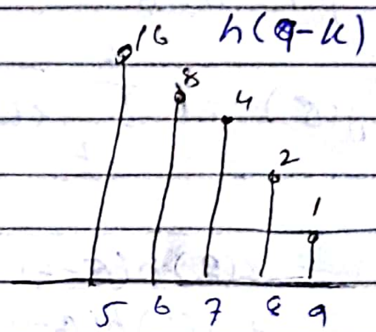
for $n=8$



$$y(8) = \sum_{k=4}^{5} x(k)h(8-k)$$

$$= x(4)h(8-4) + x(5)h(8-5)$$

$$\boxed{y(8) = 16a^5 + 8a^6}$$



for $n=9$

$$y(9) = x(k)h(9-k)$$

$$= x(9)h(9-9)$$

$$= 16a^6$$

for $y(n)=0$ where $n>9$

$$y(-4) = 0$$
$$y(-3) = a^{-2}$$
$$y(-2) = 2a^{-2} + a^{-1}$$
$$y(-1) = 0 \quad 4a^{-2} + 2a^{-1} + 1$$
$$y(0) = 8a^{-2} + 4a^{-1} + 2 + a$$
$$y(1) = 16a^{-2} + 8a^{-1} + 4 + 2a + a^2$$
$$y(2) = 16a^{-1} + 8 + 4a + 2a^2 + a^3$$
$$y(3) = 16 + 8a + 4a^2 + 2a^3 + a^4$$
$$y(4) = 16a + 8a^2 + 4a^3 + 2a^4 + a^5$$
$$y(5) = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$$

$$y(6) = 16a^3 + 8a^4 + 4a^5 + 2a^6$$
$$y(7) = 16a^4 + 8a^5 + 4a^6$$
$$y(8) = 16a^5 + 8a^6$$
$$y(9) = 16a^6$$
$$y(10) = 0$$

Q3 (a) Determine the z-transform of the following signals and also sketch its Region of convergence (ROC)

(i) $$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

Sol:-

As we know that

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$x(n) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n + \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{-1} \left(\frac{1}{3}\right)^{-n} z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4} z^{-1}\right)^n + \sum_{n=-\infty}^{-1} \left(\frac{1}{3} z^{-1}\right)^{-n}$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=-\infty}^{1} \left(\frac{1}{3} z^{-1}\right)^n$$

Let $n = -k$

$k = -\infty \Rightarrow n = \infty$

$k = -1 \Rightarrow n = 1$

$k = -n$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=\infty}^{1} \left(\frac{1}{3} z^{-1}\right)^n$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=1}^{\infty} \left(\frac{1}{3}z^{-1}\right)^n$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}z^{-1}\right)^n - 1$$

"-1" occurs due to introduced the extra sum term in the summation

$$x(z) = \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z^{-1}} - 1$$

Now

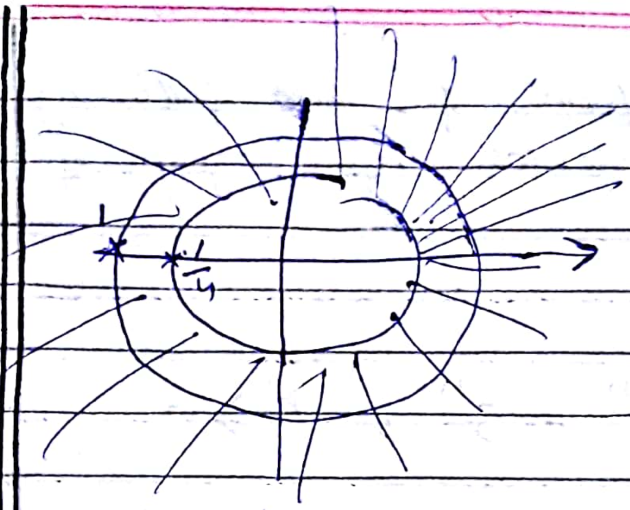
$$\left|\frac{1}{4}z^{-1}\right| < 1 \Rightarrow \left|\frac{1}{4z}\right| < 1$$

$$\frac{1}{4} < |z|$$

and also $3 > |z|$ $\rightarrow$ $|\frac{1}{3}z| < 1$

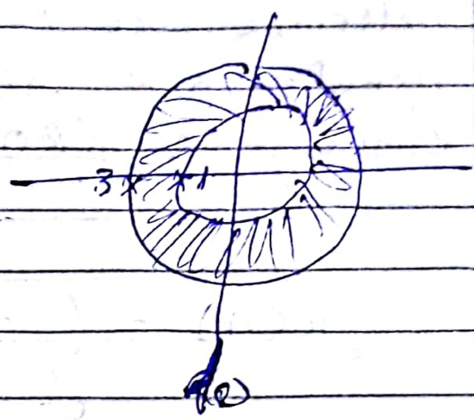
$|z| < 3$

So ROC is $\frac{1}{4} < |z| < 3$



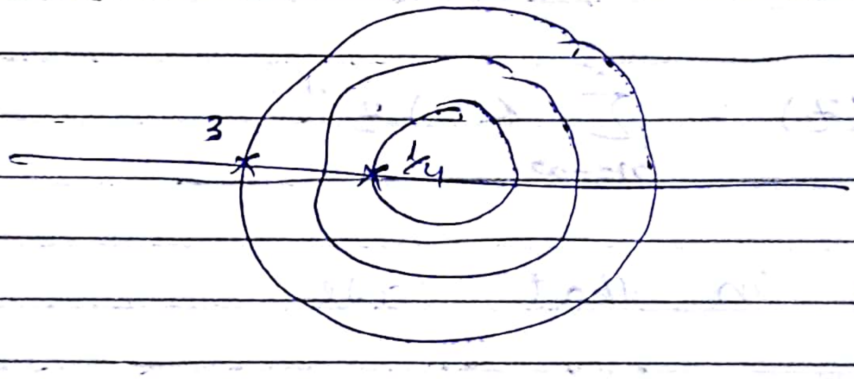
ROC

$|z| > \frac{1}{4}$



ROC

$|z| < 3$



ROC.

$\frac{1}{4} < |z| < 3$

Q3 Determine the z-transform of the following signals and also sketch its Region of convergance (ROC)

(ii)

$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

Sol:-

We know that

$$x(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

while in that case

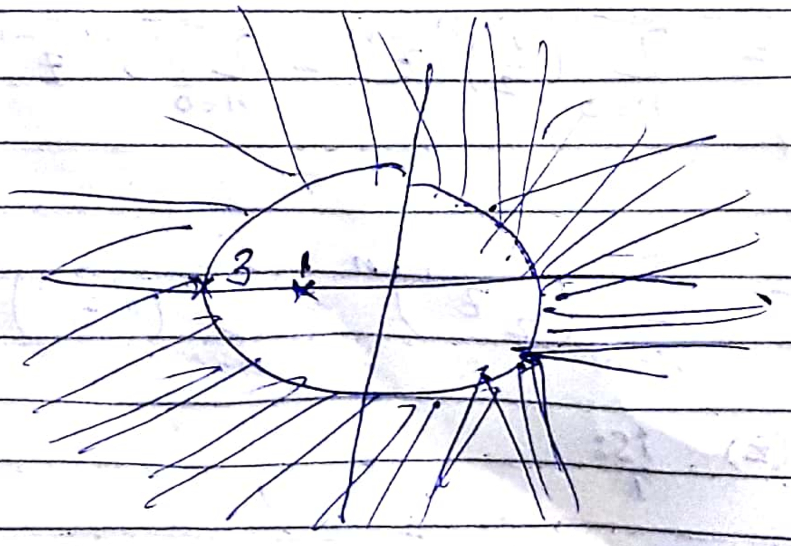
$$x(z) = \sum_{n=0}^{\infty} \left(\left(\frac{1}{2}\right)^n - 3^n\right) z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n - \sum_{n=0}^{\infty} \left(3z^{-1}\right)^n$$

our $x(z)$ is:

$$x(z) = \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$



ROC

$|z| > 3$

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