Course title E-N-A

Module 4th

Instructor Dr. shahrayar

Student Detail

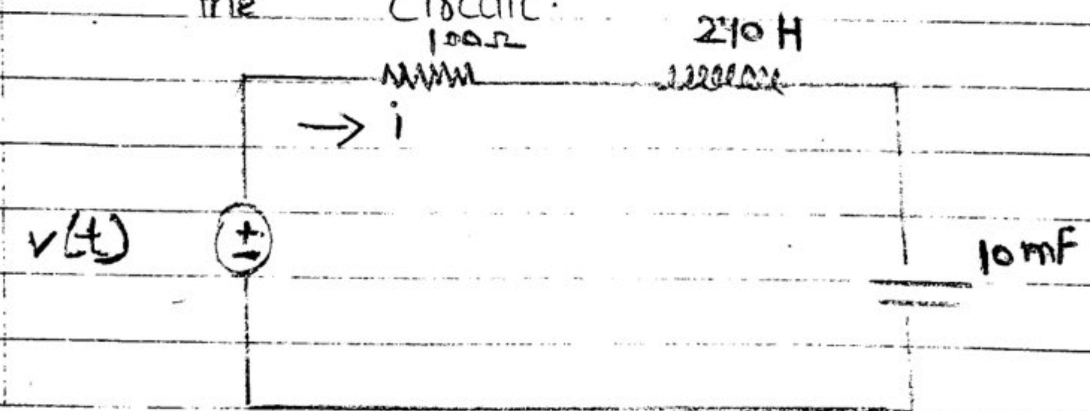
Name Farhan Ali

Student ID 14873

Student Signature

Q4 A Series RLC Circuit has R = 100Ω L = 240 H and C = 10mF if the input voltage is v(t) = 10 Cos 2t, find the current flowing through the Circuit?

Ans:4: A Series RLC Circuit has R = 100Ω, C = 240 H and C = 10mF If the input voltage is v(t) = 10 Cos 2t Find the current flowing through the Circuit.



input voltage is v(t) = 10 Cos 2tv

Here

Amplitude = V_m = 10 v

Angular frequency ω = 2 rad/s

phase angle, ϕ = 0°

So phasor for the voltage v(t)

$v(t) = 10 \angle 0° V$

Now for inductive reactance $X_L = \omega L$

So ω = 2 radious/s , L 240 H

$X = (2)(240)$

$480\ \Omega$

Now for Capacitive reactance

$X_C = \frac{1}{\omega_C}$

$\omega = 2$ radious/s , $C = 10$ mf

$$\frac{1}{2(10 \times 10^{-3})}$$

$$2 \quad \frac{1}{2 \times 10^{-2}}$$

$$\frac{1}{2} \times 10^2$$

$$\frac{100}{2}$$

$X_C = 50\ \Omega$

Now for impedence;

$\mathbf{Z} = R + j{+}L - jX_C$

$R = 100\ \Omega$, $X_L = 480\ \Omega$, $X_L = 50\ \Omega$

Putting in equation

$Z = (100 + 480 - 5)\,\Omega$

$Z = (100 + j430)\,\Omega$

Represent "Z" in prasor form

$Z = \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$

$= \sqrt{10,000 + 184,900} \angle \tan^{1}(4.3)$

$= \sqrt{194,900} \angle \tan^{-1}(4.3)$

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$= \sqrt{194,900} \angle \tan^{-1}(4.3)$

$Z = 441.47 \angle 76.90^\circ\,\Omega$

Now for current flowing in the circuit

$i = \frac{v(t)}{Z}$

$vt = 10 \angle 0^\circ \quad Z = 441.47 \angle 76.90^\circ\,\Omega$

Putting in equation

$$i = \frac{10\angle 0^\circ\,V}{441.47\angle 76.90^\circ\,\Omega}$$

$$i = \frac{10}{441.47}\angle[0 - 76.90^\circ]\,A$$

$$= 22.6\times 10^{-3}\angle -76.90^\circ\,A$$

$$= 22.6\angle -76.90^\circ\,mA$$

$$i = \frac{10\angle 0^\circ\,V}{441.47\angle 76.90^\circ\,\Omega}$$

$$i = \frac{10}{441.47}\angle[0 - 76.90^\circ]\,A$$

$$= 22.6\times 10^{-3}\angle -76.90^\circ\,A$$

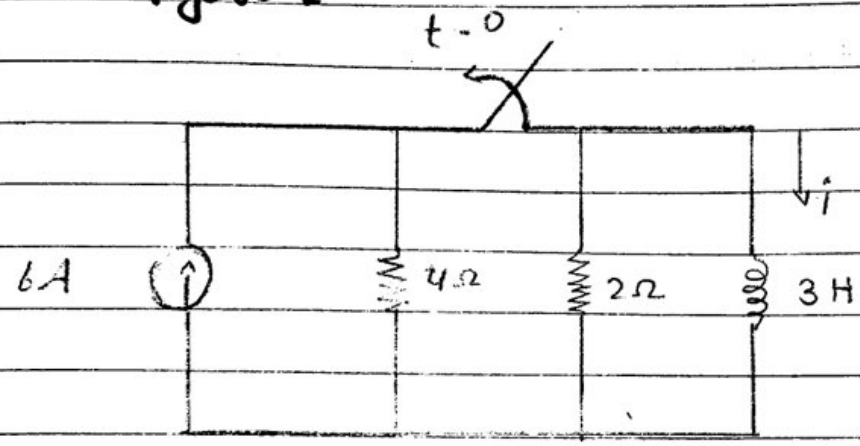
$$= 22.6\angle -76.90^\circ\,mA$$

So

The general expression for "i"

$$i = 22.6\cos(2t - 76.90^\circ)\,mA$$

Q2: Determine the inductor current for both $t > 0$ and $t > 0$ for the circuit in figure 2.



Solution :

For $t < 0$:

The switch is closed and inductor acts as short circuit. Therefore inductor current

$$i = 6A$$

For $t > 0$:

The switch is opened and time constant $\tau = \frac{L}{R}$

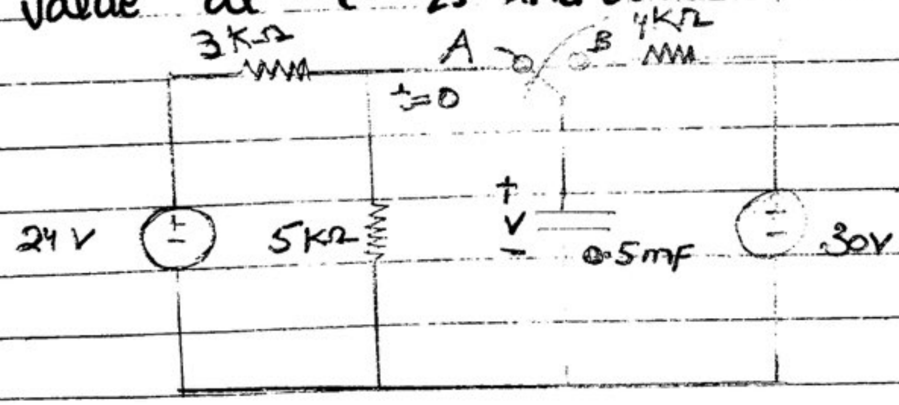
$$\tau = \frac{3}{2}$$

Now the inductor current $i(t) = \frac{-t}{\tau}$

$$i = (t) = 6e^{\frac{-t}{3/2}}$$

$i(t) = 6e^{-2t/3} u(t)\ A$

Q1: The switch in fig 1 has been in position A for a long At $t=0$ the switch moves to B. Determine $v(t)$ for $t>0$ and calculate it value at $t = 2s$ and $8s$.



Solution:

For $t<0$

The switch is at position A

The capacitor acts like an open circuit to dc, but v is the same as the voltage across 5kΩ resister. Hence the across the capacitor just before $t=0$ is obtained by by voltage division as;

$$v(0^-) = \frac{5}{5+3}(24) = 15v$$

As the capacitor can't change instantaneously.

$$v(0) = v(0^+) = 15\,v$$

For $t > 0$:

The switch is in position B. The $R_{Th} = 4k\Omega$

Time constant is ;

$$\tau = R_{Th}C = 4\times10^3 \times 0.5\times10^{-3}$$

$$\tau \quad 2s$$

since the Capacitor acts like an open Circuit to dc at steady state

$$v = (\infty) = 30v$$

Thus

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{\frac{-t}{\tau}}$$

$$= 30 + (15 - 30)e^{-t/2}.$$

$$= (30 - 15e^{-0.5t})\,v$$

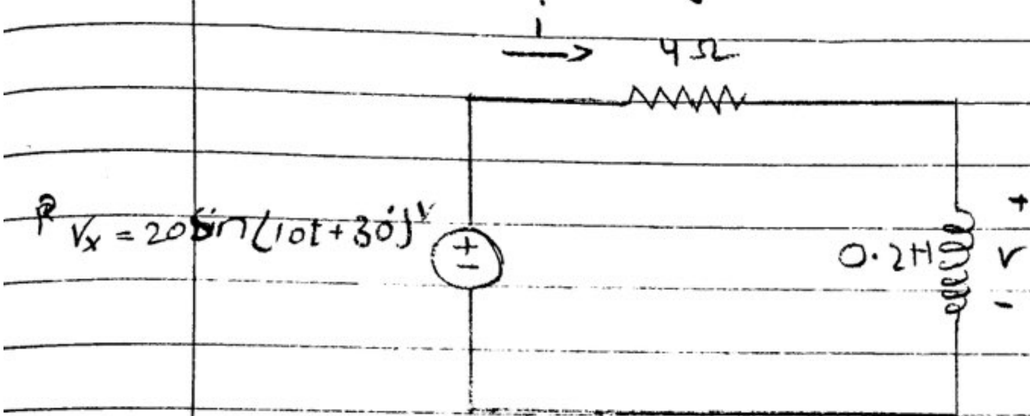
At $t = 2$

$$v(2) = 30 - 15e^{-\frac{2}{2}}$$
$$= 30 - 15e^{-1}$$
$$v(2) = 24.48\,V$$

At $t = 8$:

$$v(8) = 30 - 15e^{-\frac{8}{2}}$$
$$= 30 - 15e^{-4}$$
$$v(8) = 29.72$$

Q5: Find v(t) and i(t) in the current show in figure 3.



Solution:

For i(t)

from the voltage source

$V_s = 20 \sin(10t + 30^\circ)$ V

$V_s = 20 \cos(10t + 30^\circ - 90^\circ)$ V

$V_s = 20 \cos(10t - 60^\circ)$ V

$V_s = 20 \angle -60^\circ$ V

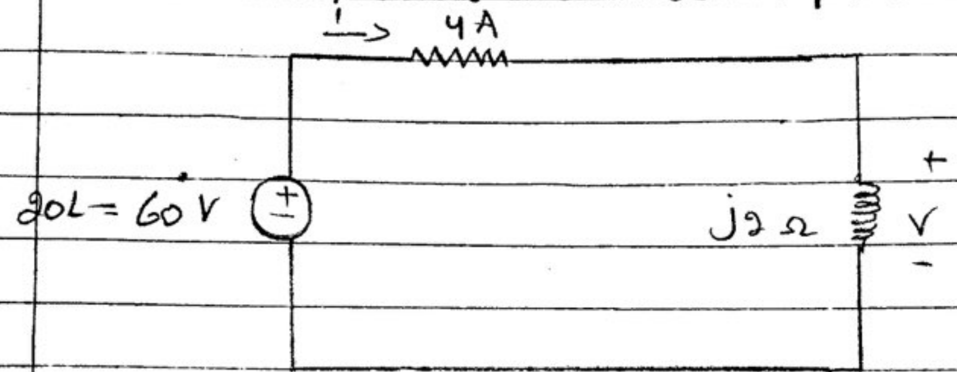
$\omega = 10$ rad/sec

$X_L = j\omega L$

$0.2H = j \times 10 \times 0.2$

$0.2H = j2\ \Omega$

Given circuit can be represented As ;



From the above circuit

$$Z = 4 + j2\,\Omega$$

Hence the current is ;

$$i = \frac{20\angle -60^\circ}{4 + j2}$$

$$i = \frac{20\angle -60^\circ}{\sqrt{4^2+2^2}\angle \tan^{-1}\frac{2}{4}}$$

$$i = \frac{20\angle -60^\circ}{4.472\angle 26.57^\circ}$$

$$i = 4.472\angle -86.57^\circ$$

Conversting this into time domain

$i(t) = 4.472 \cos(20t - 86.57°)$

$i(t) = 4.472 \sin(10t - 86.57° + 90°)$

$i_t = 4.472 \sin(10t + 3.43°)$ A

For v(t) from the circuit voltage across the inducter is

$V = j2 \times i$

$V = j2 \times (4.472 \angle 86.57°)$

Converting polar form to ~~reg~~ rectangular form we get;

$V = j2 \times (0.26756 - j4.464)$

$V = 8.928 + j0.53512$

Converting rectangular form to polar form

$$V = \sqrt{(8.926)^2 + (0.53512)^2} \angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$$

$$V = 8.944 \angle 3.4 V$$

Converting this into time domain

$$v(t) = 8.944 \cos(10t + 3.4^\circ)$$

$$V(t) = 8.944 \sin(10t + 3.4^\circ + 90^\circ)$$

$$V(t) = 8.944 \sin(10t + 93.4^\circ) V$$

Q3 A Series RLC Circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = 10$$

Find the response when $L = 0.5H$, $R = 4\Omega$

and $C = 0.2F$. let $i(0) = 1$, $di(0)'\, dt = 0$

Solution:

The Step response of the branch voltage of the given RLC Circuit is described by:

$$\frac{Ld^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = 10$$

Divide by L

$$\frac{d^2i}{dt^2} + \frac{Rdi}{Ldt} + \frac{i}{LC} = \frac{10}{L}$$

Xing by $\frac{c}{c}$

$$\frac{d^2t}{dt^2} + \frac{Rdi}{Ldt} + \frac{i}{LC} = \frac{100}{LC}$$

As = C = 0.2F thus ;

$$\frac{d^2 i}{dt^2} + \frac{R di}{L dt} + \frac{i}{LC} = \frac{2}{LC}$$

Substitute

$$\frac{d^2 i}{dt^2} + \frac{8 di}{dt} + 10 i = 20 \ldots (i)$$

The general form for source-free RLC given by

$$\frac{d^2 i}{dt^2} + \frac{R di}{L dt} + \frac{1 i}{LC} = \frac{I_s}{LC}$$

Comparing 1 & 2 we get

$$RL = 8 \rightarrow (3)$$

$$\frac{1}{LC} = 10 \rightarrow (4)$$

$$\frac{I_s}{LC} = 20 \rightarrow (5)$$

From (3), α is given by ;

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/sec} \rightarrow (6)$$

The natural frequency ω_0 is given by;

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

From (4)

$$\omega_0 = \sqrt{10} \text{ rad/sec} \rightarrow (7)$$

From (6) & (7)

$\because \alpha > \omega_0$

$\therefore$ The circuit is over damped

The roots of characteristics equation are given by;

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$
$$= -4 + \sqrt{6} \text{ rad/sec}$$

And,

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$
$$= -4 - \sqrt{4^2 - \sqrt{10}^2}$$
$$= -4 - \sqrt{6} \text{ rad/sec}$$

From (5) the steady current is given by

$$I_S = 20 \times LC = 20 \times 0.5 \times 0.2 = 2A \rightarrow (8)$$

The current for over damped is given by;

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0 \rightarrow (9)$$

$$i(0) = I_s + A_1 + A_2$$
$$1 = 2 + A_1 + A_2$$
$$A_1 + A_2 = -1 \rightarrow (10)$$

From 9, find $\frac{di(t)}{dt}$

$$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$$

Subsitute too;

$$\frac{di}{dt}(0) = A_1 s_1 + A_2 s_2$$

Subtitute the value

$$(-4 + \sqrt{6}) A_1 + (-4 - \sqrt{6}) A_2 = 0 \rightarrow (11)$$

solving (10) & (11) Simultaneously

$$A_1 = -1.316$$
$$A_2 = 0.316$$

Subsitute in (9)

$$i(t) = 2 - 1.316 e^{(-4+\sqrt{6})t} + 0.316 e^{(4-\sqrt{6})t} A$$