

NAME : AQIB

ID : 15415

Subject : Probability and stat

Degree : BSC.

Q1

a)

As we know

$$\text{Mean } (np) = 4 \quad \dots (i) \quad \text{Variance } (npq) = 3 \quad \dots (ii)$$

Dividing the LHS and RHS of equation (ii) by equation (i) we have

$$Npq/np = 3/4$$

$$\Rightarrow q = 3/4$$

Therefore, we have $p = 1 - q = 1 - 3/4 = 1/4$

Putting the value of $p = 1/4$ in equation (i),

We have $n = 16$.

c)

A **critical region**, also known as the rejection **region**, is a set of values for the test statistic for which the null hypothesis is rejected. I.e. if the observed test statistic is in the **critical region** then we reject the null hypothesis and accept the alternative hypothesis.

d)

The **t distribution** has the following **properties**:

The mean of the **distribution** is equal to 0.

The variance is equal to $v / (v - 2)$, where v is the degrees of freedom (see last section) and $v > 2$.

The variance is always greater than 1, although it is close to 1 when there are many degrees of freedom.

e)

Analysis of variance, or ANOVA, is a statistical method that separates observed **variance** data into different components to use for additional tests. A one-way ANOVA is used for three or more groups of data, to gain information about the relationship between dependent and independent variables

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f)

RBD: A diagram that gives the relationship between component states and the success or failure of a specified system function. The logical layout in an **RBD** can be as series system, parallel system, or a combination.

g)

Statistical quality control, the use of **statistical** methods in the monitoring and maintaining the **quality** of products and services. One method, referred to as acceptance sampling, can be used when a decision must be made to accept or reject a group of parts or items based on the **quality** found in a sample

h)

Chance cause: a process that is operating with only chance causes of variation present is said to be in statistical control.

Assignable cause is a type of variation in which a specific activity or event can be linked to inconsistency in a system..

i)

traffic intensity: A measure of the average occupancy of a facility during a specified period of time, normally a busy hour, measured in **traffic units** (erlangs) and defined as the ratio of the time during which a facility is occupied (continuously or cumulatively) to the time this facility is available for occupancy

Q2

Part A)

$$\begin{aligned} E(X) &= \sum_{x=0}^n x \binom{n}{x} p^x (1-p)^{n-x} \\ &= \sum_{x=0}^n x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\ &= \sum_{x=1}^n \frac{n!}{(x-1)!(n-x)!} p^x (1-p)^{n-x} \end{aligned}$$

since the $x = 0$ term vanishes. Let $y = x - 1$ and $m = n - 1$. Subbing $x = y + 1$ and $n = m + 1$ into the last sum (and using the fact that the limits $x = 1$ and $x = n$ correspond to $y = 0$ and $y = n - 1 = m$, respectively)

$$\begin{aligned} E(X) &= \sum_{y=0}^m \frac{(m+1)!}{y!(m-y)!} p^{y+1} (1-p)^{m-y} \\ &= (m+1)p \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\ &= np \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \end{aligned}$$

The binomial theorem says that

$$(a+b)^m = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y}$$

Setting $a = p$ and $b = 1 - p$

$$\sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} = \sum_{y=0}^m \frac{m!}{y!(m-y)!} a^y b^{m-y} = (a+b)^m = (p+1-p)^m = 1$$

so that

$$E(X) = np$$

Similarly, but this time using $y = x - 2$ and $m = n - 2$

$$\begin{aligned} E(X(X-1)) &= \sum_{x=0}^n x(x-1) \binom{n}{x} p^x (1-p)^{n-x} \\ &= \sum_{x=0}^n x(x-1) \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} \\ &= \sum_{x=2}^n \frac{n!}{(x-2)!(n-x)!} p^x (1-p)^{n-x} \\ &= n(n-1)p^2 \sum_{x=2}^n \frac{(n-2)!}{(x-2)!(n-x)!} p^{x-2} (1-p)^{n-x} \\ &= n(n-1)p^2 \sum_{y=0}^m \frac{m!}{y!(m-y)!} p^y (1-p)^{m-y} \\ &= n(n-1)p^2 (p + (1-p))^m \\ &= n(n-1)p^2 \end{aligned}$$

So the variance of X is

$$\begin{aligned} E(X^2) - E(X)^2 &= E(X(X-1)) + E(X) - E(X)^2 = n(n-1)p^2 + np - (np)^2 \\ &= np(1-p) \end{aligned}$$

Part b)

Let X denote number of cars hired out per day

Poisson distribution mean = $m = 1.5$

$$P(X=x) = \frac{(e^{-m} (m^x))}{(x!)} = \frac{(e^{-1.5} (1.5^x))}{(x!)}$$

1) P (neither car is used):

$$P(X=0) = \frac{(e^{-1.5} (1.5^0))}{0.2231}$$

2) P (Some demand is refused) = P (Demand is more than 2 cars per day)

$$P(x > 2)$$

$$= 1 - P(x \leq 2)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2)]$$

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$$= 1 - \left[\frac{(e^{1.5})(1.5^0)}{0!} + \frac{(e^{1.5})(1.5^1)}{1!} + \frac{(e^{1.5})(1.5^2)}{2!} \right]$$

$$= 1 - e^{1.5} \left[1 + 1.5 + \frac{2.25}{2} \right] = 0.1912$$

Proportion of days on which no demand is used = 0.2231 = 22.31 %

Proportion of days on which some demand is refused = 0.1912 = 19.12 %

No of Defects.

100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

Group No

