Final Term Exam Summer 2020

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Subject: Structural Analysis-II

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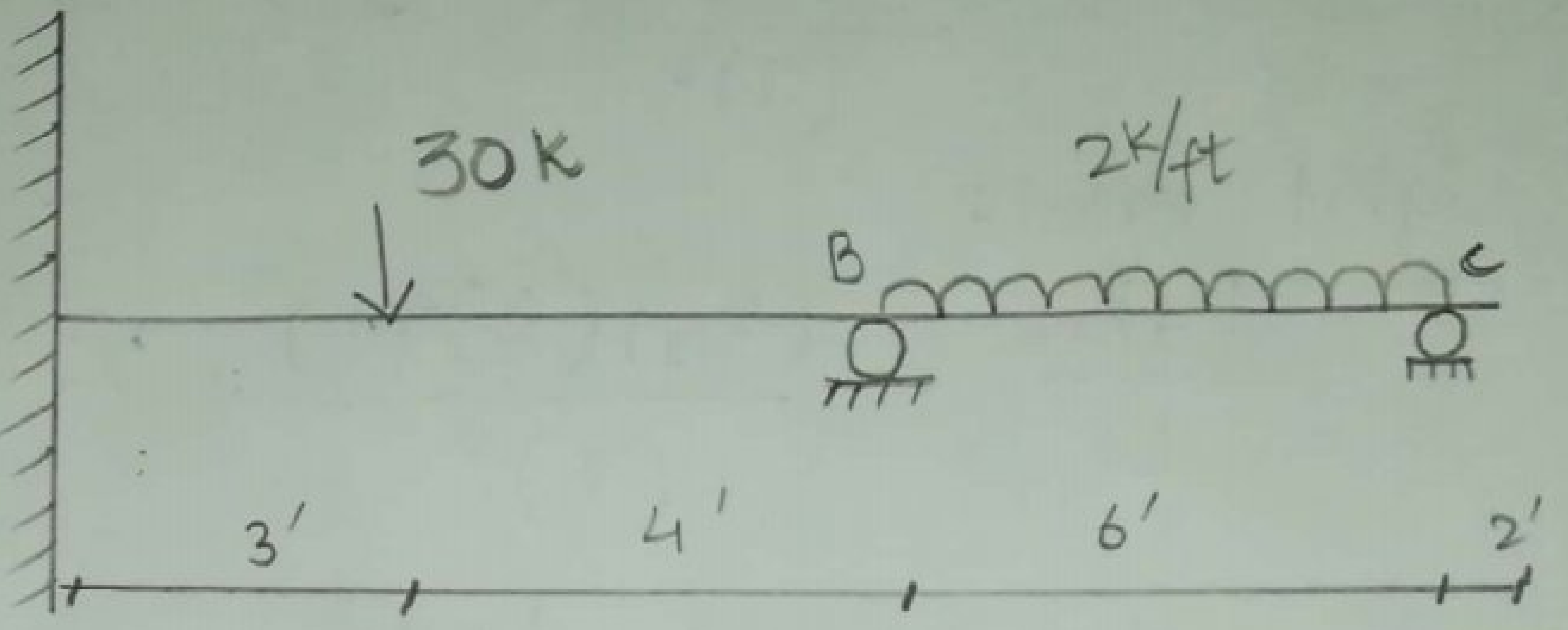
Q1: Analyze the beam shown in figure 1 by stiffness method. Assume EI is constant.

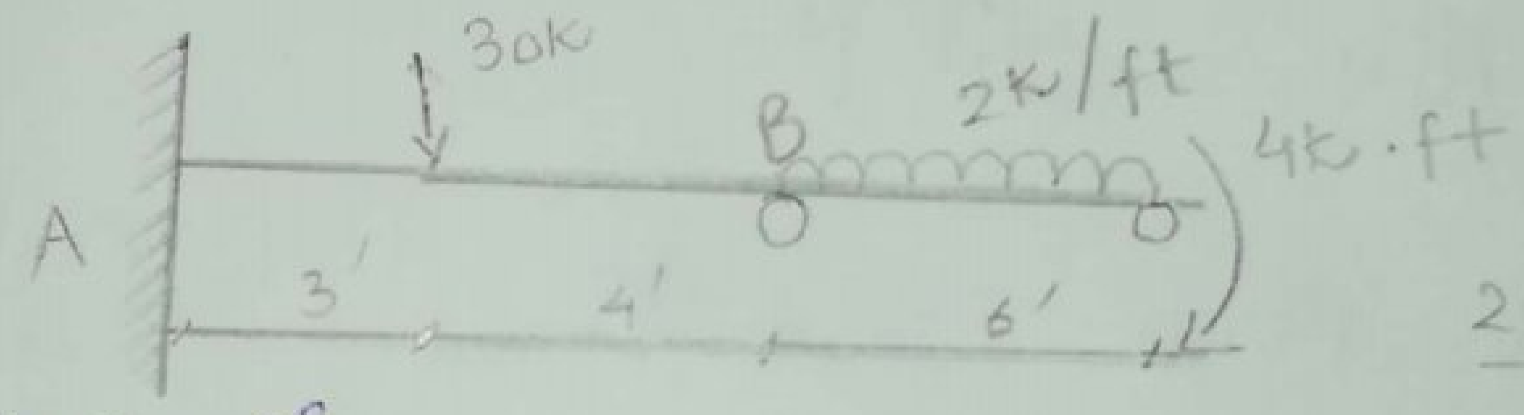



Solution:-

**Step #1:** Determining Kinematic Indeterminacy

K.I = 5°

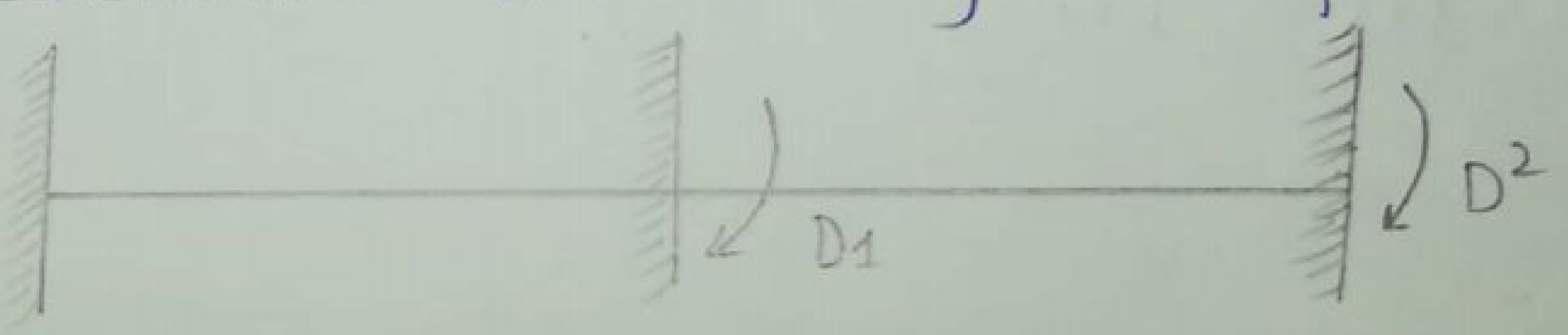
So we have to reduce the extended portion.




$\frac{2(2)}{1} = 4k.ft$

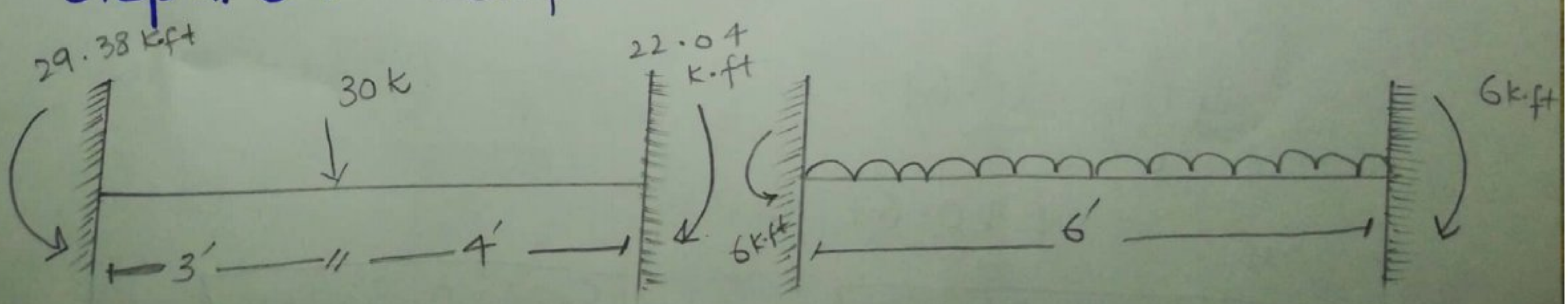
Now K.I = 2°

**Step #2:** Determine unknown joint displacement




$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$ $\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

**Step #3:** Compute ADL Matrix.



⇒ For point load (not at mid).

⇒ For left end:

$$\frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ K.ft}$$

⇒ For right end:

$$\frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

For uniformly distributed Load

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = 6 \text{ K.ft}$$

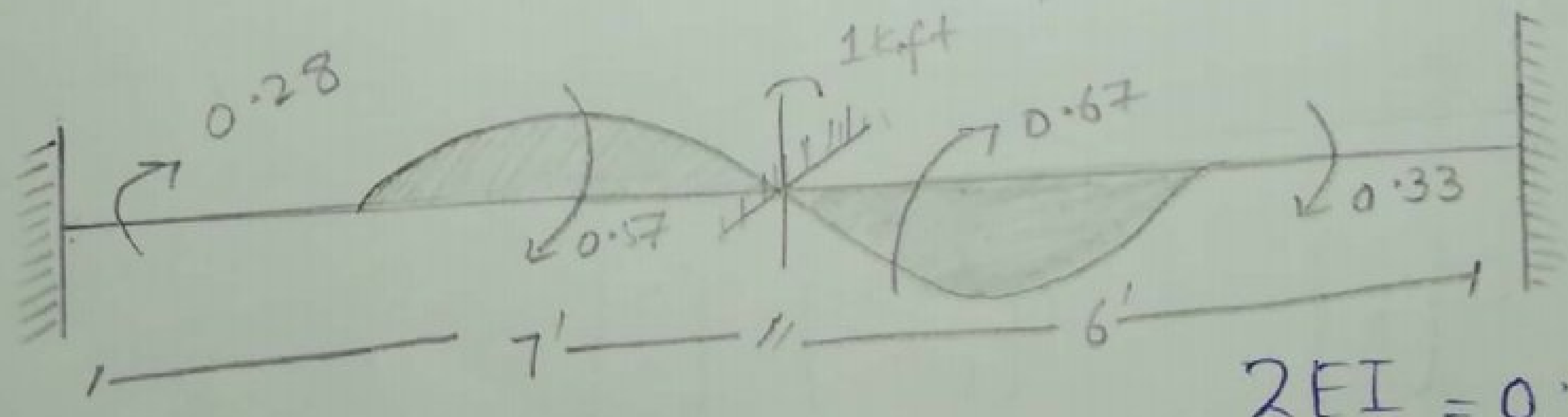
$ADL_1 = +22.04 - 6 = 16.04$ K.ft

$ADL_2 = 6$ K.ft

**Step #4:** Now Compute [S] matrix.

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a) $D1 = 1k$, $D_2 = 0$



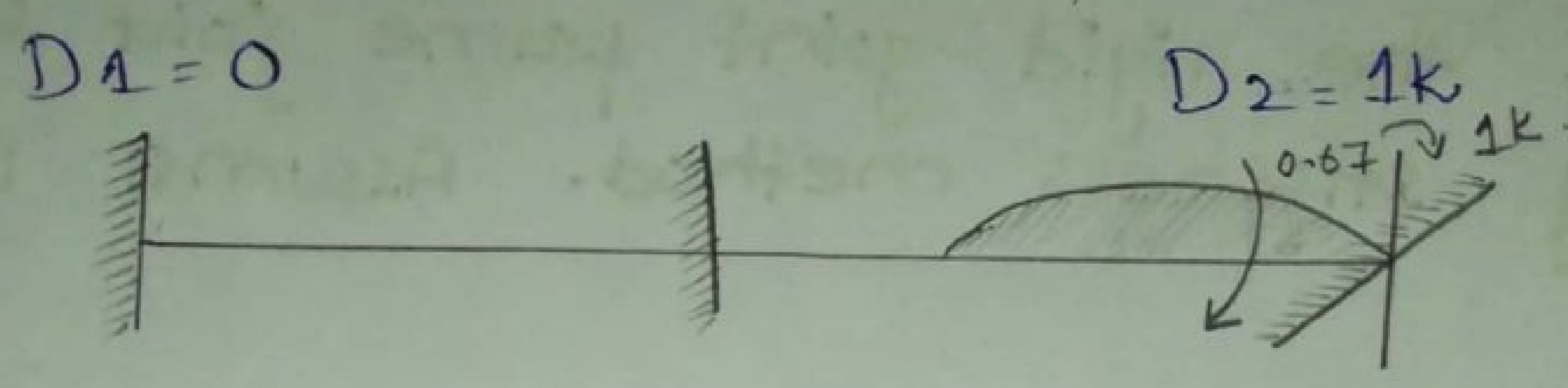

$$\frac{4EI}{7} = 0.57 \qquad \frac{2EI}{6} = 0.33$$

$$\frac{4EI}{6} = 0.67 \qquad \frac{2EI}{7} = 0.28$$

$S_{11} = 0.57 + 0.67$

$\boxed{S_{11} = 1.24 \text{ EA}}$ $\boxed{S_{21} = 0.33 \text{ EA}}$

b)




$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

$S_{12} = 0.33$ $\qquad$ $S_{22} = 0.67$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

**Step #5:** Now compute [D] matrix.

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}.$$

$$= \frac{1}{\begin{vmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{vmatrix}} \times \text{Adj}\, A \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}.$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219.$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now, $\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$

$$\rightarrow \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{1}{|S|} \times \text{Adj } A \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.919 & -0.452 \\ -0.452 & 1.70 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -13.83 \\ 3.85 \end{bmatrix}$$

QNo:2 Analyse the pin-jointed frame shown by stiffness method ---- shown in fig-3
Take E = 2000 t/cm².

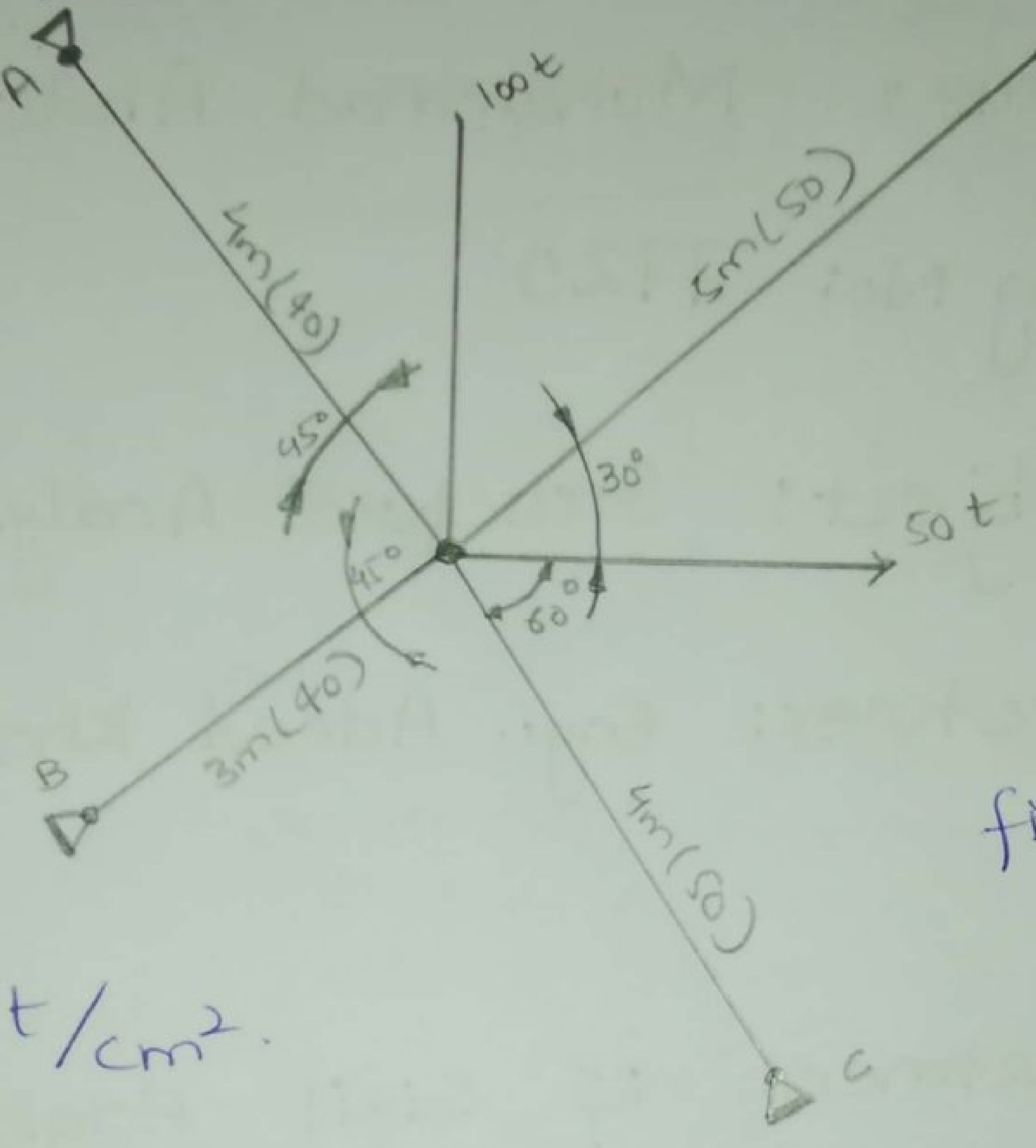



figure 3.

E = 2000 t/cm².

Solution:

⇒ For A :- $\sin 45° = \frac{P}{H} = \frac{P}{4}$

$\Rightarrow P = 2.828m$

$\cos 45° = \frac{b}{H} = \frac{b}{4}$

$\Rightarrow b = 2.828 m$

⇒ For B :- $\sin 45° = \frac{P}{H} = \frac{P}{3}$

$\Rightarrow P = 2.12 m$

$\cos 45° = \frac{b}{H} = \frac{b}{3}$

$\Rightarrow b = 2.12 m$

⇒ For C :- $\sin 60 = \frac{P}{H} = \frac{P}{4}$

$\sin 60 (4) = P$

$\rightarrow P = 3.46.$

$\cos 60^\circ = \frac{b}{H} = \frac{b}{4}$

$\cos 60 \times 4 = b$

$\rightarrow b = 2.$

$\Rightarrow$ for D :- $\sin 30 = \frac{P}{5}$

$P = 2.5m.$

$\cos 30^\circ = \frac{b}{5}$

$b = 4.33m$

Now
$EA(A) = 2000 \times 40 = 80000t.$

$EA(B) = 2000 \times 40 = 80,000t$

$EA(C) = 2000 \times 50 = 100,000t$

$EA(D) = 2000 \times 50 = 100,000t.$

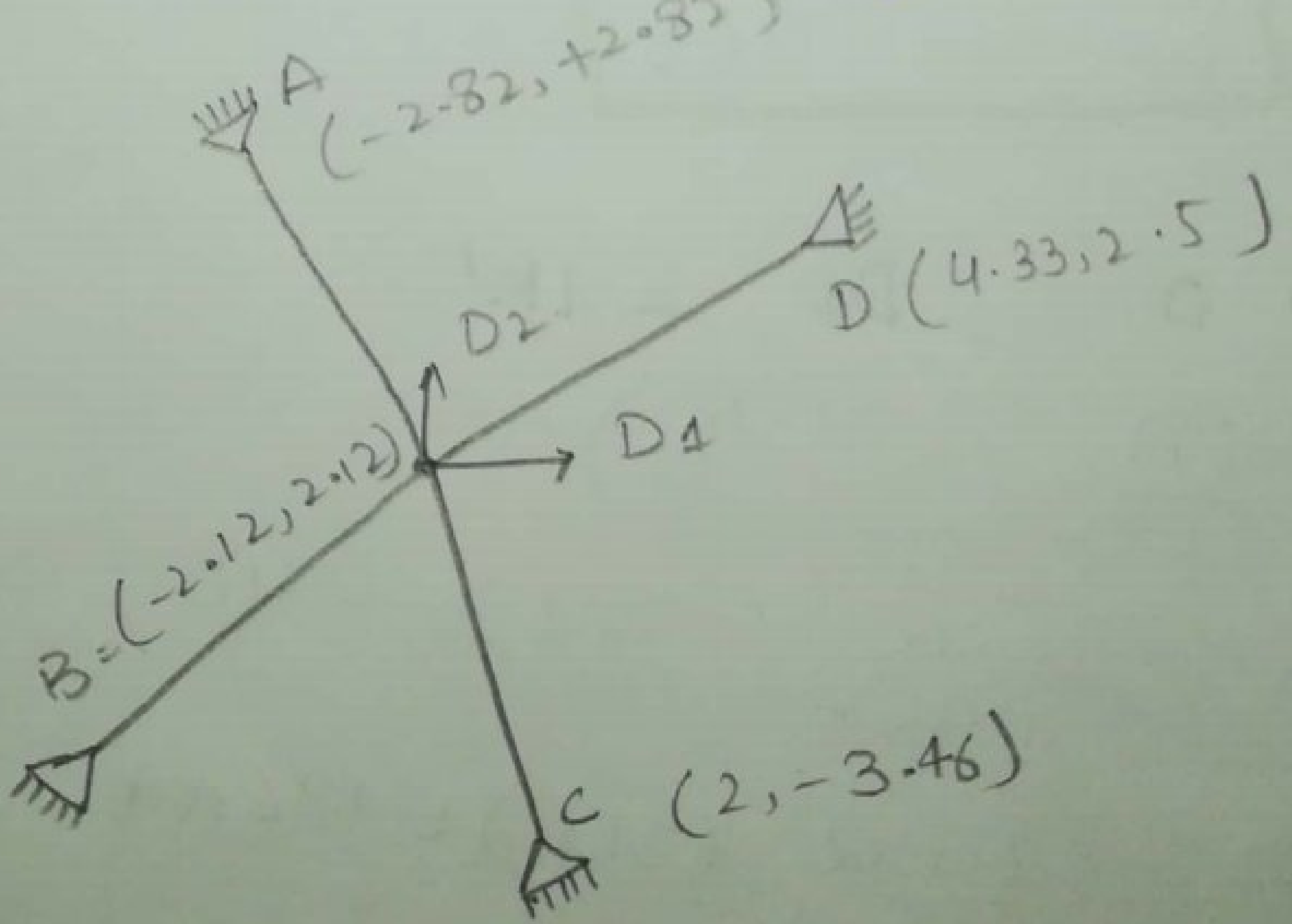
**Step 1:-** K I.

$KI = 2j - c.$

$2(5) - 8$

$K.I = 20$

**Step#2:** select unknown joint displacement.




$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$ $\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}.$

Step # 3 $[AMD]_{4\times2}$ $[S]_{2\times2}$.

i/ $D_1 = 1k$ , $D_2 = 0$.

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80{,}000}{(300)^2} \times (0 + 212) = 188.44.$$

$$AMD_{31} = \frac{1001000}{(500)^2} \times (0 - 433) = -173.2.$$

$$AMD_{41} = \frac{1001000}{(400)^2} \times (0 - 200) = -125$$

Now,

$$S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_k - X_j)^2$$

$$= \frac{801000}{(400)^3}(282)^2 + \frac{801000}{(300)^3} \times (212)^2.$$

$$+ \frac{1001000}{(5000)^3}(282)^2 + \frac{1001000}{(400)^3} \times (-200)^2$$

$$\Rightarrow S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

ii/ $D_1 = 0$ $D_2 = 1k'$

$$AMD = \frac{EA}{L^2}(Y_k - Y_j).$$

$$AMD_{12} = \frac{801000}{(400)^2}(-282) = -141$$

$$AMD_{22} = \frac{801000}{(300)^2}(212) = 188.44.$$

$$AMD_{32} = \frac{1001000}{(500)^2} \times (-250) = -100$$

$$AMD_{42} = \frac{1001000}{(400)^3}(346) = 216.25.$$

Now, $S_{22} = \sum_{s=1}^{m} \frac{EA}{L^3} (\gamma_k - \gamma_j)^2$.

$= \frac{801000}{(400)^3} (-282)^2 + \frac{801000}{(300)^3} (212)^2 + \frac{1001000}{(500)^3} + \frac{1001000}{(400)^3} (346)^2$

$$S_{22} = 469.628$$

**Step 4:** $[D] = [S]^{-1} \times [AD]$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}.$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

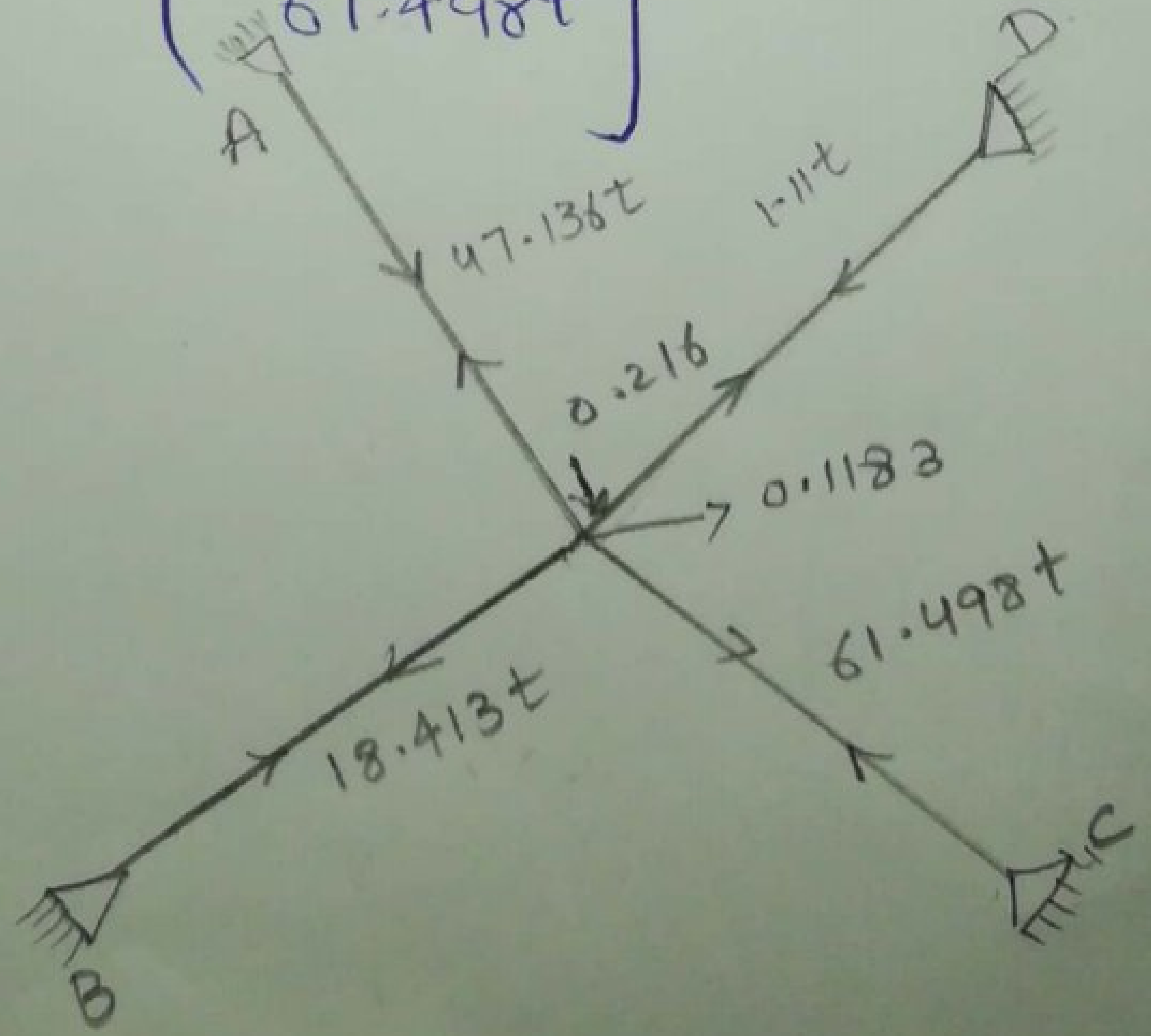
**Step # 5:** [AM].

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (0.216) \\ 188.44 \times 0.1183 + (1.88.44) \times (-0.216) \\ -173.2 \times 0.1183) + (-100)(-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 + 46.71 \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ 61.498t \end{bmatrix}$$



**Q No: 3** Analyze the rigid - joint frame joint shown in fig-2 by stiffness method. Assume EI is constant.

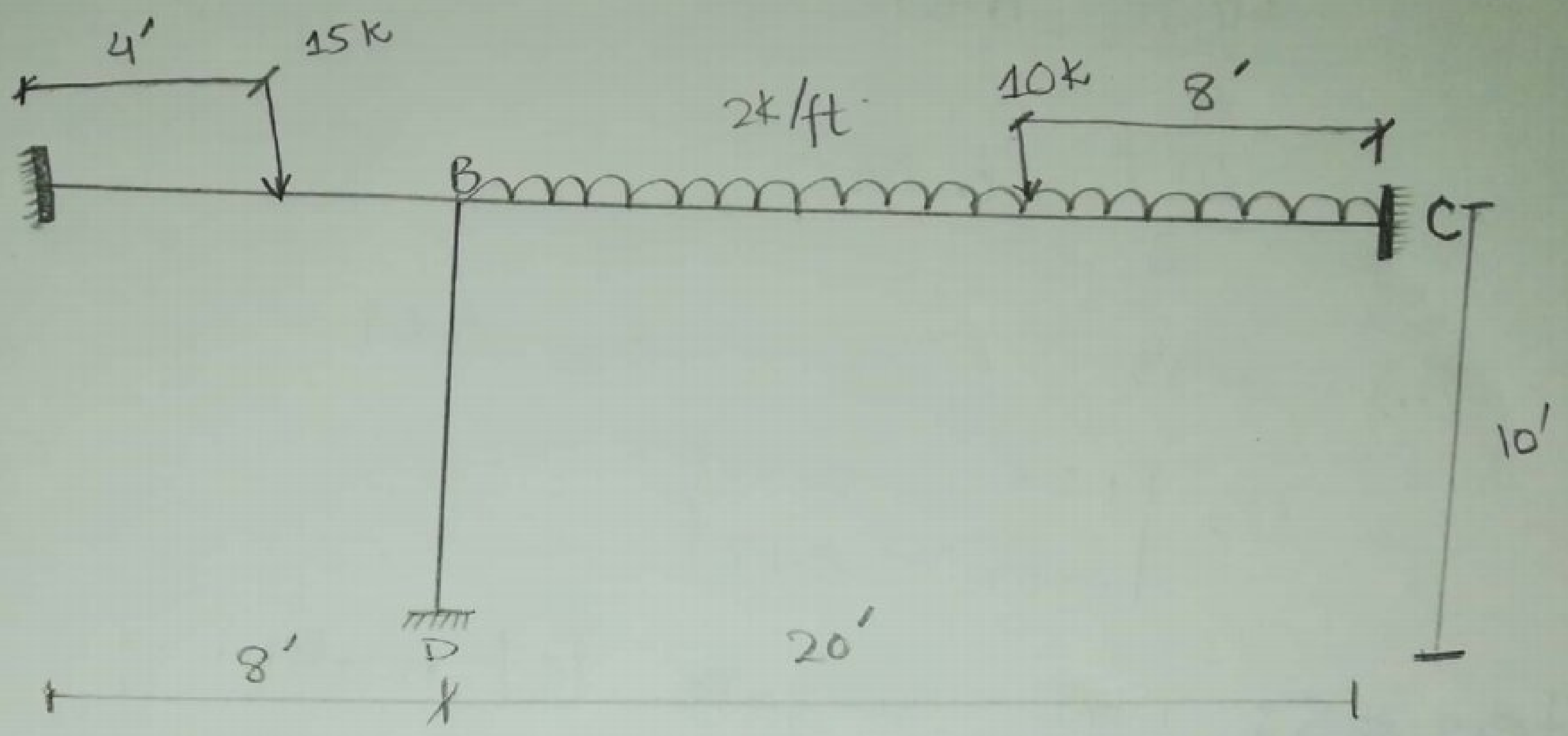


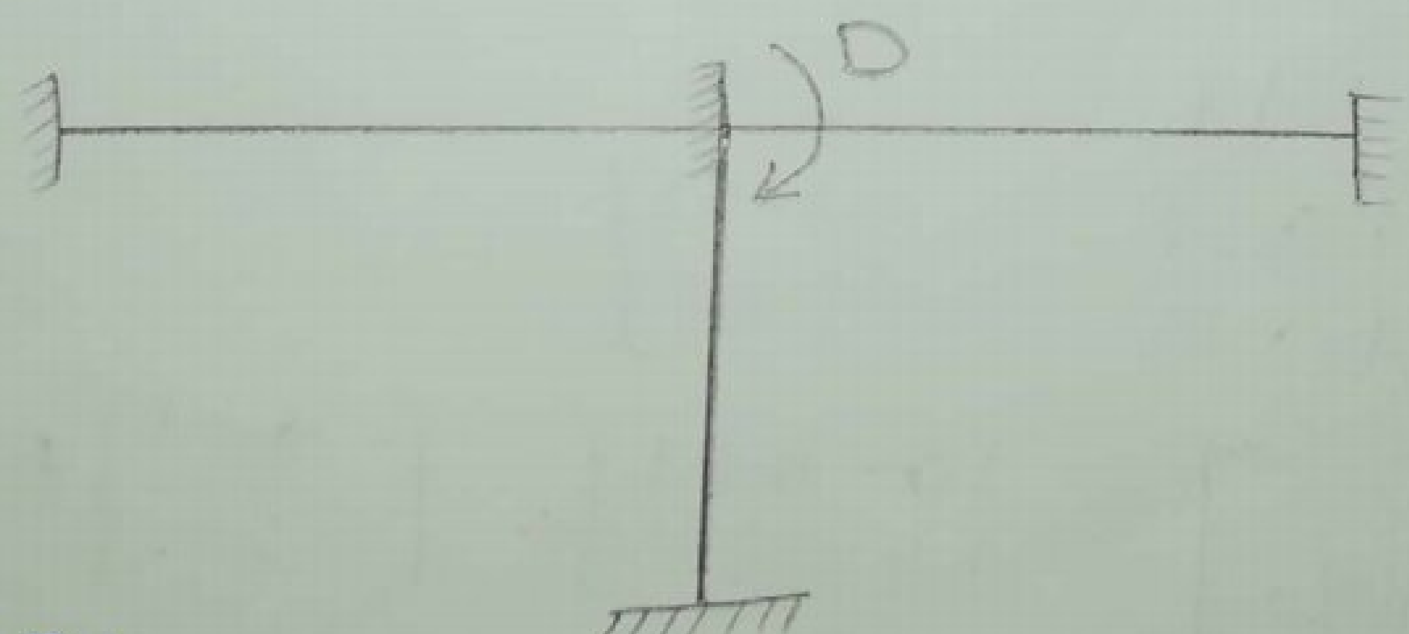

figure - 2

Solution :-

**Step #1 :** Determine kinematic Indeterminancy

K.I = 1°.

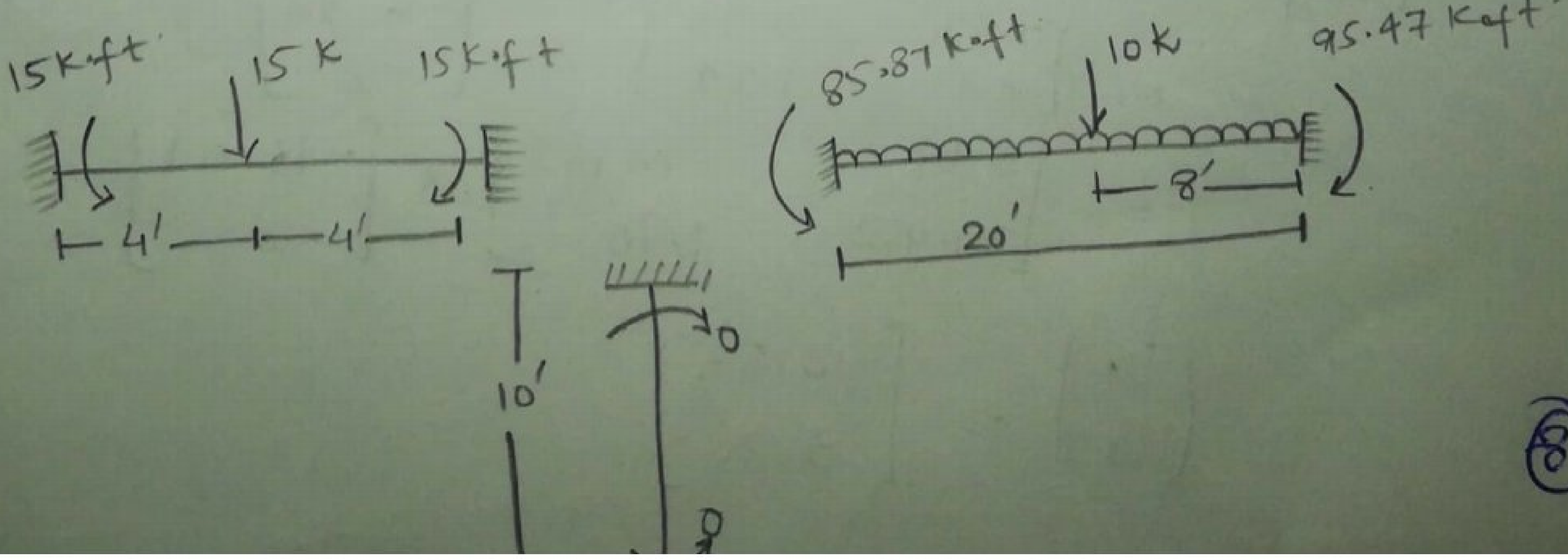
**Step #2:** Determine unknown joint displacement




[D] = ?

[AD] = [0].

**Step #3:** Compute ADL Matrix



⇒ Point load at center:

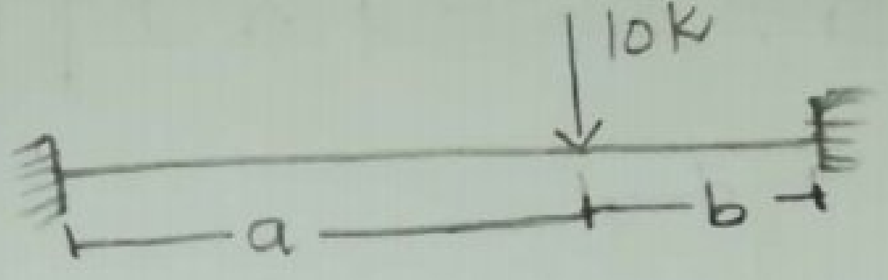
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip-ft}$$

⇒ Uniformly distributed load:

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k-ft}$$

⇒ Point load (Not at mid):—

Suppose:




For left End:-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ K-ft}$$

For Right End:-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ K-ft}$$

So, Total Moment at left end:

$$19.2 + 66.67 = 85.87 \text{ kft}$$

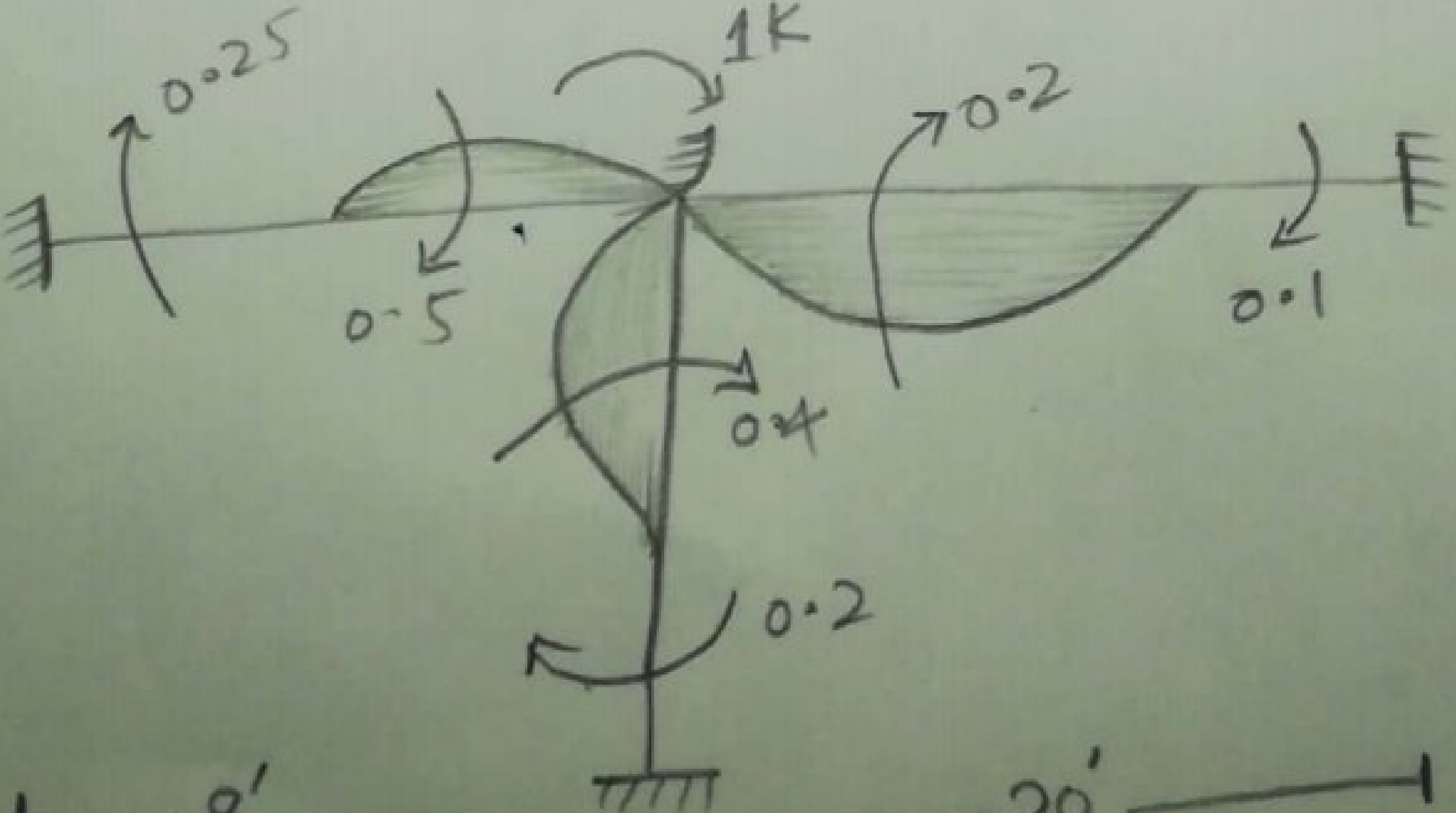
Similarly at right end:

$$28.8 + 66.67 = 95.47 \text{ k-ft}$$

So, $[ADL] = -85.87 + 15 = -70.87$ k-ft

**Step #4:** Determine [S] matrix.

$[S] = [S_{11}]$ Now, D = 1k.



$\Rightarrow \frac{4EI}{8} = 0.5$ $\qquad \frac{2EI}{8} = 0.25$

$\Rightarrow \frac{4EI}{20} = 0.2$ $\qquad \frac{2EI}{20} = 0.1$

$\Rightarrow \frac{4EI}{10} = 0.4$ $\qquad \frac{2EI}{10} = 0.2$

$$[S] = [0.5 + 0.4 + 0.2]\, EI$$

$$= 1.1\ EI$$

$$[S] = 1.1\,EI$$

**Step # 5:** Compute D matrix.

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87].$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42]\ {}^{1}/_{EI}.$$

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