# Department of Electrical Engineering
## Assignment
### Date: 25/06/2020

## Course Details

**Course Title:** Signals & Systems **Module:** 04
**Instructor:** ____________ **Total Marks:** 50

## Student Details

**Name:** ____________ **Student ID:** ____________

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|---|---|---|---|
| Q1. | (a) | **Show** with a help of an equation that the differentiation of a function in time domain results in the multiplication by jw in frequency domain. | Marks 06+08 |
| | (b) | If $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$<br>$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$<br>**Produce** Y(z) and y[n] | CLO 3 |
| Q2. | | $f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$<br>**Retrieve** the Fourier series for the given function. | Marks 10<br>CLO 3 |
| Q3. | | If $X(z) = 2z^2 + 2z/(z^2 + 2z - 3)$<br>**Retrieve** x[n] using inverse Z-transform method. | Marks 10<br>CLO 3 |
| Q4. | | **Express** the transfer function using the given data.<br>$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $C = [1 \quad 2]$ $D = [0]$ | Marks 09<br>CLO 3 |
| Q5. | | **Apply** Fourier transform on the signal, $x(t) = e^{-a\lvert t\rvert} u(t)$ where u (t) is a unit step function. | Marks 07<br>CLO 3 |

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x ~ * ~ x.

Q1:-
a)-
Ans:- Let $x(t)$ be a continuous time signal with a fourier transform of $x(j\omega)$
i.e.

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)\, e^{j\omega t}\, d\omega.$$

Differentiating b.s w.r.t "t".

$$= \frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)\frac{d}{dt}\{e^{j\omega t}\}\, d\omega$$

$$= \frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)\{e^{j\omega t}\cdot j\omega\}\cdot d\omega$$

$$= \frac{dx}{dt}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\{j\omega\, x(j\omega)\}\, e^{j\omega t}\, d\omega$$

$\Rightarrow$

$$j\omega\,\frac{1}{2\pi}\int_{-\infty}^{\infty} x(j\omega)\, e^{j\omega t}\, d\omega.$$

$$\frac{d}{dt}x(t) = j\omega\, x(t).$$

$$f\{\tfrac{d}{dt}x(t)\} = j\omega\cdot x(j\omega).$$

The above condition is proved.

x ——— * ——— x

(2)

Q1:-

b)- $x[n] = 2\delta[n] - 4\delta[n-2] + 2\delta[n-3]$
$h[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2]$.

Soln

$Y(z) = H(z)\,X(z)$.

Find $Y[n]$.

$X(z) = 2 - 4z^{-2} + 2z^{-3}$.

$H(z) = 3 + 1z^{-1} + 2z^{-2}$

Now:-

$Y(z) = H(z) * X(z)$.

$= (2 - 4z^{-2} + 2z^{-3})(3 + 1z^{-1} + 2z^{-2})$.

$= 6 + 2z^{-1} + 4z^{-2} - 12z^{-2} - 4z^{-3} - 8z^{-4}$
$+ 6z^{-3} + 2z^{-4} + 4z^{-5}$.

$= 6 + 2z^{-1} - 8z^{-2} - 2z^{-3} + 6z^{-4} + 4z^{-5}$.

To find $Y[n]$ use Time delay property

$$\left\{ \begin{array}{l} Y[n] = 6\delta[n] + 2\delta[n-1] - 8\delta[n-2] \\ - 2\delta[n-3] + 6\delta[n-4] + 4\delta[n-5]. \end{array} \right\}$$

Answer

Q2:-

Ans:-

$$f(x) = \begin{cases} -\pi/2 & -\pi \le x \le 0 \\ \pi/2 & 0 \le x \le \pi \end{cases}$$

As:-

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)\, dx.$$

$$= \frac{1}{2\pi} \left[ \int_{-\pi}^{0} f(x)\, dx + \int_{0}^{\pi} f(x)\, dx \right]$$

$$= \frac{1}{2\pi} \int_{-\pi}^{0} -\frac{\pi}{2}\, dx + \int_{0}^{\pi} \frac{\pi}{2}\, dx.$$

$$= \frac{1}{2\pi} \left[ -\frac{\pi}{2} \int_{-\pi}^{0} 1\, dx + \int_{0}^{\pi} 1\, dx \right]$$

$$= \frac{1}{2\pi} \left[ -\frac{\pi}{2} x \Big|_{-\pi}^{0} + \frac{\pi}{2} x \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[ \frac{\pi}{2} [0-(-\pi)] + \frac{\pi}{2} [\pi - 0] \right].$$

$$= \frac{1}{2\pi} \left[ -\frac{\pi}{2} (\pi) + \frac{\pi}{2} (\pi) \right]$$

$$= \frac{1}{2\pi} \left[ -\frac{\pi^2}{2} + \frac{\pi^2}{2} \right].$$

$$= \frac{1}{2\pi} \left[ \frac{0}{2} \right] \Rightarrow (a_0 = 0).$$

Now:-

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} f(x) \cos nx\, dx + \int_{0}^{\pi} f(x) \cos nx\, dx.$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} \frac{\pi}{2} \cos nx\, dx + \int_{0}^{\pi} \frac{\pi}{2} \cos nx\, dx.$$

$$= \frac{1}{\pi} \left[ \frac{\pi}{2} \frac{\sin nx}{n} \Big|_{\pi}^{0} + \frac{\pi}{2} \frac{\sin nx}{n} \Big|_{0}^{\pi} \right].$$

$$= \frac{1}{n\pi} \left[ -\frac{\pi}{2} \sin n(0) - \sin n(-\pi) + \frac{\pi}{2} [\sin n(\pi) - \sin n(0)] \right].$$

P.T.O.

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}(0) + \frac{\pi}{2}(0)\right].$$

$$= \frac{1}{n\pi}(0).$$

$(a_n = 0)$

Now:-

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nx\,dx.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx\,dx + \int_{0}^{\pi} f(x)\sin nx\,dx.\right.$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} -\frac{\pi}{2}\sin nx\,dx + \int_{0}^{\pi}\frac{\pi}{2}\sin nx\,dx\right].$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2}\int_{-\pi}^{0}\sin nx\,dx + \frac{\pi}{2}\int_{0}^{\pi}\sin nx\,dx\right.$$

$$= \frac{1}{\pi}\left[-\frac{\pi}{2} - \frac{\cos nx}{n}\Big|_{-\pi}^{0} + \frac{\pi}{2} - \frac{\cos nx}{n}\Big|_{0}^{\pi}\right].$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-\cos n(0) + \cos n(-\pi)\right]\right] + \frac{\pi}{2}\left[-\cos n(\pi) + \cos n(0)\right]$$

$$= \frac{1}{n\pi}\left[-\frac{\pi}{2}\left[-1 + \cos n(-\pi)\right] + \frac{\pi}{2}\left[-\cos\pi + \cos n(0)\right]\right]$$

$$\frac{\pi}{2} = \frac{1}{n\cancel{\pi}}\left[-1\left[-1\cos n(-\pi)\right] + 1\left[-\cos nx + 1\right]\right]$$

$$= \frac{1}{2n}\left[1 - \cos n\pi - \cos n\pi + 1\right]$$

$$= \frac{1}{2n}\left[2 - 2\cos n\pi\right].$$

Now:-

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P.T.O

Now:-

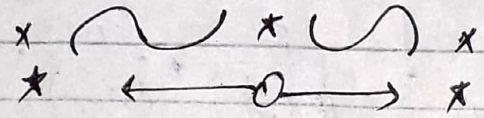
$$b_n = \begin{cases} 0 & \text{if } n \text{ is even} \\ \frac{4}{2n} & \text{if } n \text{ is odd.} \end{cases}$$

$$\left\{ b_n = \frac{4}{2n} \right\}.$$

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \cdots$$
$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \cdots$$

$$f(x) = (0) + (0)\cos x + 0 \cos(2x) + 0 \cos(3x) + \cdots$$
$$+ \frac{4}{2}\sin x + (0)\sin 2x + \frac{4}{3(2)}\sin 3x + \cdots$$

$$\left( f(x) = \frac{4}{2}\sin x + \frac{4}{6}\sin 3x + \cdots \right) \text{ Ans}$$



Q3:-

Ans:

$$X(z) = \frac{2z^2+2z}{z^2+2z-3}.$$

$$X(z) = \frac{2z(z+1)}{z^2+3z-z-3}.$$

$$= 2z(z+1) \Big/ z(z+3)-1(z+3)$$

$$\frac{x(z)}{z} = 2\cancel{z}(z+1) \Big/ (z+3)(z-1)$$

or

$$\frac{2(z+1)}{z^2+2z+3} = \frac{A}{(z+3)} + \frac{B}{(z-1)} \cdot \longrightarrow (1)$$

put $z=1$ & multiplying b.s by $(z+3)(z-1)$.

OR

This (1) becomes:

$$2(z+1) = A(z-1) + B(z+3) \longrightarrow (ii)$$

put $z=1$ in (ii).

$2(1+1) = B(1+3)$.

$4 = 4B$.

$(B=1)$

put $z=-3$ in eq (ii)

$2(-3+1) = A(-3-1)$.

$-4 = -4A$

$(A=1)$

Now put (A) & (B) in (i).

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{(z+3)} + \frac{1}{(z-1)}.$$

$$x(z) = \frac{z}{z+3} + \frac{z}{z-1}.$$

So inverse z- transform.

$$x(z^{-1}) = u[B] + 1(-1)^k.$$

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Q4:-

Ans:- As we are given:-

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = [1, 2], \quad D = [0].$$

$$G(s) = C[SI - A]^{-1} B + D.$$

$$= [1 \ 2] \left[ s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + [0].$$

$$= [1 \ 2] \frac{1}{s(s+2)+1} \begin{bmatrix} s & -1 \\ 1 & s+2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

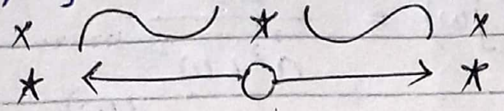
$$= [1 \ 2] \frac{1}{s^2+2s+1} \begin{bmatrix} s \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+2s+1}\begin{bmatrix}1 & 2\end{bmatrix}\begin{bmatrix}s \\ 1\end{bmatrix}.$$

$$= G(s) = \frac{1}{s^2+2s+1}\begin{bmatrix}s & 2\end{bmatrix}.$$

[num, den] = ss2tf (A, B, C, D).

[A, B, C, D] = tf2ss [num, den].



Q5:-
Sol:-

The fourier transform of the given fln.
$x(t)$ is given by:-

$$x(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt.$$

$$x(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{\cdot j\omega t}\, dt.$$

$$\because e^{-a|t|} = \begin{cases} e^{-at} & \text{for } t \geq 0 \\ e^{-a(-t)} = e^{at} & \text{for } t < 0. \end{cases}$$

$$x(j\omega) = \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt.$$

$$= \frac{e^{(a-j\omega)t}}{a-j\omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\Bigg|_{0}^{\infty}.$$

$$= \frac{1}{(a-j\omega)}\left[e^{0} - e^{-\infty}\right] - \frac{1}{a+j\omega}\left[e^{-\infty} - e^{0}\right].$$

$$= \frac{1}{(a-j\omega)}(1-0) - \frac{1}{a+j\omega}(0-1)$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{a+\cancel{j\omega}+a-\cancel{j\omega}}{a^2-(j\omega)^2}.$$

$$= X(j\omega) = \frac{2a}{a^2+\omega^2}.$$