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Subject :- Differential eq

Q.NO.1

⇒ Solve the initial value problem

$\Rightarrow \frac{dy}{dt} = e^{y-t} \sec(y)(1-t)^2 \quad y(0)=0$

→ Solutions:

$\frac{dy}{dt} = e^{y+t} \sec(y)(1+t^2) \qquad y(0)=0$

By Separating Variables

$\int e^{-y} \cos y \, dx = \int (1+t^2) e^{-t} \, dt \qquad \therefore \cos = \frac{1}{\sec}$

⇒ Integrating by parts (II)

$u = e^{-y} \qquad dv = \sin y \, dy$

$du = -e^{-y} dy \qquad v = -\cos y$

⇒ by integrating 1st parts

$L.H.S = e^{-y} \sin y + \int e^{-y} \sin y \, dy$

By integrating part-2:

$= e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y \, dy.$

Since the last integral is Same as L.H.S.

$L \cdot H \cdot S = e^{-y} (\sin y - \cos y)$

by adding L.H.S

$2 (L.H.S) = e^{-y} (\sin y - \cos y)$

$\div$ ing by 2.

$L.H.S = \frac{e^{-y}}{2} (\sin y - \cos y)$

integrating part (3)

$U = 1 + t^2$ $\quad dv = e^{-t} dt$

$du = 2t dt$ $\quad v = e^{-t}$

integrating by part (4)

$u = 2t$ $\quad dv = e^{-t} dt$

$du = 2dt$ $\quad v = -e^{-t}$

let us evaluate R.H.S

by integrating part 4

$= -(1+t^2) - 2te^{-t} + \int 2e^{t} \, dt$

$= -(t^2 + 2t + 1) e^{-t} + c$

$\Rightarrow$ comparing L.H.S & R.H.S.

③

$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t} + c$

Since $y(0)=0$ We have

$\frac{1}{2}(0-1) = -3+c$

$c = \frac{5}{2}$

Hence the Solution is Implicitly expressed as

$\frac{e^{+y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t} + \frac{5}{2}$

Q No. 2

Solve the following

$$(\sqrt{x+y} + \sqrt{x-y})dx - (\sqrt{x+y} - \sqrt{x-y})dy = 0$$

Solution:

$$(\sqrt{x+y}) + \sqrt{x-y})\, dx - (\sqrt{x+y} - \sqrt{x-y})\, dy = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \quad -(1)$$

⇒ This is homogeneous differential equation in x and y to solve this put.

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

This eq (1) becomes

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} - \sqrt{x+vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{(\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

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$$v + x \cdot \frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x \frac{dv}{dx} = \frac{1+\cancel{v} + 1 - \cancel{v} + 2\sqrt{1-v^2}}{2v}$$

$$v + x\frac{dv}{dx} = \cancel{2}\frac{1+\sqrt{1+v^2}}{\cancel{2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1+v^2}}{v} - v$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2} - v^2}{v}$$

$$x\frac{dv}{dx} = \sqrt{1-v^2}\left(\frac{1+\sqrt{1-v^2}}{x}\right)$$

$$\frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \frac{dx}{x}$$

taking integral Bds.

$$\int \frac{v\,dv}{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})} = \int \frac{dx}{x}$$

put $1+\sqrt{1-v^2} = t$

$\frac{1}{2}(1-v^2)^{1/2}(-2v)\,dv = dt$.

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}$$

$-\ln t = \ln x + \ln c$

$-\ln(1+\sqrt{1-v^2}) = \ln cx$

$\ln(1+\sqrt{1-v^2} = -\ln cx$

$\ln(1+\sqrt{1-v^2} = \ln(cx)^{-1}$

$1+\sqrt{1-v^2} = 1/cx$.

$$1+\sqrt{1-\frac{y^2}{x^2}} = 1/cx$$

$$1+\frac{\sqrt{x^2-y^2}}{x^2} = 1/c$$

$x+\sqrt{x^2-y^2} = 1/c$

$x+\sqrt{x^2-y^2} = C_1 \quad \therefore 1/c = C_1$.

Which is Require

Q No.3.

⇒ Solve:

$$(D^4 + D^2)\,y = 3x^2 + 4\sin x - 2\cos x.$$

Solution:

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow F(D)y = f(x)$$

As it is non Homogineous linear equation.

So Solution will be

$$y = y_c + y_p \quad -①$$

Complementry Solution y_c.

$$D^4 + D^2 = 0 \Rightarrow D^2(D^2+1) = 0$$

Either $D^2 = 0 \Rightarrow D = 0$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \rightarrow D = i, \text{ or } D = 0 + i$$

Roots are real and complex.

$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x$

$y_p = \frac{1}{f(D)} f(x)$

$y_p = \frac{1}{D^4+D^2}(3x^2 + 4\sin x - 2\cos x$

$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$

$f(D) = D^4+D^2$

at $D=0 \Rightarrow f(D)=0$

So, $f'(D) = 4D^3+2D$

Now also for $D=0 \Rightarrow f'(0)=0$

again differentiating

$f''(D) = 12D+2$

So for $D=0$.

$f''(0) = 12(0)+2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_p = \frac{x^2 \, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$$

put $D=0$ in all.

So,

$$y_p = \frac{x^2 \, 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x.$$

So,

putting in equation (i).

$$y = c_1 + c_2\cos x + c_3\cos x + \tfrac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

$$y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \tfrac{3}{2}x^4$$

"THE END"