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Subject is Differential eq.

QNO.1

⇒ Solve the initial value problem

$$\Rightarrow \frac{dy}{dt} = e^{t^2} \sec(y)(1+t^2) \quad y(0) = 0$$

→ Solutions

$$\frac{dy}{dt} = e^{t^2} \sec(y)(1+t^2) \quad y(0) = 0$$

By Separating Variables

$$\int e^{-y} \cos y dy = \int (1+t^2) e^{-t} dt \quad \because \cos = y \sec$$

⇒ Integrating by parts ①

$$u = e^{-y} \quad du = -\sin y dy$$

$$dv = -e^{-y} dy \quad v = -\cos y$$

⇒ by integrating 1st part

$$\text{L.H.S} = e^{-y} \sin y + \int e^{-y} \sin y dy$$

By integrating part-2

$$= e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y dy$$

Since the last integral is same as L.H.S.

Q

L.H.S =  $e^{-y} (\sin y - \cos y)$   
by adding L.H.S

$$2(L.H.S) = e^{-y} (\sin y - \cos y)$$

dividing by 2.

$$L.H.S = \frac{e^{-y}}{2} (\sin y - \cos y)$$

Integrating part (3)

$$u = 1+t^2 \quad du = e^{-t} dt$$

$$du = 2t dt \quad v = e^{-t}$$

Integrating by part (4)

$$u = 2t \quad dv = e^{-t} dt$$

$$du = 2 dt \quad v = -e^{-t}$$

Let us evaluate R.H.S

by integrating part 4

$$= -(1+t^2) - 2te^{-t} + \int 2et^2 dt$$

$$= -(t^2 + 2t + 1)e^{-t} + C$$

⇒ Comparing L.H.S & R.H.S.

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$$\frac{e^{-y}}{2} (\sin y - \cos y) = -(t^2 + 2t + 3) e^{-t}$$

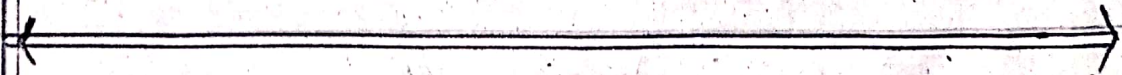
Since  $y(0) = 0$  we have

$$\frac{1}{2} (0 - 1) = -3 + c$$

$$c = 5/2$$

Hence the solution is implicitly expressed as

$$\frac{e^{-y}}{2} (\sin y - \cos y) = -(t^2 + 2t + 3) e^{-t + 5/2}$$



④

QNO.2

Solve the following.

$$(\sqrt{x+y} + \sqrt{x-y}) dx - (\sqrt{x+y} - \sqrt{x-y}) dy = 0$$

Solution

$$(\sqrt{x+y} + \sqrt{x-y}) dx - (\sqrt{x+y} - \sqrt{x-y}) dy = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \quad \text{--- (1)}$$

⇒ This is homogeneous differential equation in  $x$  and  $y$   
to solve this put.

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

This eq (1) becomes.

$$v + x \frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x \frac{dv}{dx} = \frac{(\sqrt{1+v} + \sqrt{1-v})}{\sqrt{1+v} - \sqrt{1-v}}$$

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$$v + x \cdot \frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}} \times \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} + \sqrt{1-v}}$$

$$v + x \frac{dv}{dx} = \frac{1+v + 1-v + 2\sqrt{1-v^2}}{2v}$$

$$v + x \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v}$$

$$v + x \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v}$$

$$x \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v} - v$$

$$x \frac{dv}{dx} = \frac{1 + \sqrt{1-v^2}}{v} - v$$

$$x \frac{dv}{dx} = \frac{\sqrt{1-v^2} (1 + \sqrt{1-v^2})}{x}$$

$$\frac{v dv}{\sqrt{1-v^2} (1 + \sqrt{1-v^2})} = \frac{dx}{x}$$

taking integral Bds.

$$\int \frac{v dv}{\sqrt{1-v^2} (1 + \sqrt{1-v^2})} = \int \frac{dx}{x}$$

(b)

$$\text{put } 1 + \sqrt{1-v^2} = t$$

$$\frac{1}{2} (1-v^2)^{1/2} (-2v) dv = dt$$

$$\frac{v dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dv}{v}$$

$$-\ln t = \ln v + \ln c$$

$$-\ln (1 + \sqrt{1-v^2}) = \ln v + \ln c$$

$$\ln (1 + \sqrt{1-v^2}) = -\ln v + \ln c$$

$$\ln (1 + \sqrt{1-v^2}) = \ln (c/v)$$

$$1 + \sqrt{1-v^2} = 1/cv$$

$$1 + \sqrt{1-y^2} = 1/cv$$

$$1 + \sqrt{x^2 - y^2} = 1/c$$

$$x + \sqrt{x^2 - y^2} = 1/c$$

$$x + \sqrt{x^2 - y^2} = C \quad \therefore 1/c = C$$

which is Required

(7)

Q NO.3.

⇒ Solve or

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x.$$

Solution or

$$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$$

$$\Rightarrow F(D)y = f(x)$$

As it is non homogeneous linear equation.

So solution will be

$$y = y_c + y_p \quad \text{--- (1)}$$

Complementary solution  $y_c$ .

$$D^4 + D^2 = 0 \Rightarrow D^2(D^2 + 1) = 0$$

$$\text{Either } D^2 = 0 \Rightarrow D = 0$$

$$D^2 + 1 = 0 \Rightarrow D^2 = -1$$

$$D = \sqrt{-1} \rightarrow D = i, \text{ or } D = 0 + i$$

Roots are real and complex.



(8)

$$y_c = C_1 e^{0x} + e^{0x} (C_2 \cos x + C_3 \sin x)$$

$$y_c = C_1 + C_2 \cos x + C_3 \sin x$$

$$y_p = \frac{1}{f(D)} f(x)$$

$$y_p = \frac{1}{D^4 + D^2} (3x^2 + 4 \sin x - 2 \cos x)$$

$$= \frac{3x^2}{D^4 + D^2} + \frac{4 \sin x}{D^4 + D^2} - \frac{2 \cos x}{D^4 + D^2}$$

$$f(D) = D^4 + D^2$$

$$\text{at } D=0 \Rightarrow f(D) = 0$$

$$\text{So, } f'(D) = 4D^3 + 2D$$

$$\text{Now, also for } D=0 \Rightarrow f'(D) = 0$$

again differentiating

$$f''(D) = 12D + 2$$

$$\text{So for } D=0$$

$$f''(0) = 12(0) + 2 = 2$$

$$\text{So replacing } \frac{1}{f(D)} \text{ with } \frac{x^2}{f''(0)}$$

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$$\Rightarrow y_p = \frac{x^2 \cdot 3x^2}{12D+2} + \frac{x^2 \cdot 4 \sin x - 2x^2 \cdot 2 \cos x}{12D+2}$$

put  $D=0$  in all.

So,

$$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4 \sin x - 2x^2 \cdot 2 \cos x}{12(0)+2}$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2 \sin x - 2x^2 \cdot 2 \cos x}{2}$$

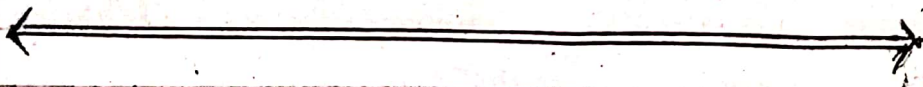
$$= \frac{3x^4}{2} + 2x^2 \sin x - x^2 \cos x$$

So,

putting in equation ①

$$y = C_1 + C_2 \cos x + C_3 \sin x + \frac{3}{2} x^4 + 2x^2 \sin x - x^2 \cos x$$

$$y = C_1 + (C_2 - x^2) \cos x + (C_3 + 2x^2) \sin x + \frac{3}{2} x^4$$



"THE END"