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Paper : Linear Algebra
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Q1 Given:-

$$x_1 - \overset{(3^{rd}\ ID)x_2}{\cancel{0x_2}} + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

So Findout:

Consistent or Not?

Solution:

Let: 3rd ID = 0

$$x_1 - 0x_2 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10$$

$$x_1 + x_3 = 0$$
$$2x_2 - 8x_3 = 8$$
$$5x_1 - 5x_3 = 10 \quad \therefore (5x_1 - 5x_3 = 10 \div 5)$$

$$x_1 + \cancel{x_3} = 2$$
$$x_1 + \cancel{x_3} = 0$$

$$2x_1 = 2$$

$$\boxed{x_1 = 1}$$

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$x_1 + x_3 = 0$ $(x_1 = 1)$

$1 + x_3 = 0$

$\boxed{x_3 = -1}$

$2x_2 - 8(-1) = 8$

$2x_2 + 8 = 8$

$2x_2 = 8 - 8$

$2x_2 = 0$

$\boxed{x_2 = 0}$

Now

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5/5 & 0/5 & -5/5 & 10/5 \end{array}\right] \quad R_3 \cdot 1/5$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 1 & 0 & -1 & 2 \end{array}\right]$$

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$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 2 & 0 & 0 & 2 \end{array}\right] \quad R_3 + R_1$$

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 2 & -8 & 8 \\ 1 & 0 & 1 & 0 \end{array}\right] \quad R_1 \longleftrightarrow R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & -8 & 8 \\ 1 & 0 & 1 & 0 \end{array}\right] \quad 1/2 R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 2 & -8 & 8 \\ 0 & 0 & 1 & -1 \end{array}\right] \quad R_3 - R_1$$

$\cancel{S.S = x_1 = 1, x_2 = 2x_2 - 8x_3 = 8}$

$\cancel{x_3 = 1 \quad x_2 = 2(+1) - 8}$

$S.S = \{ x_3 = -1, x_1 = 1$

$2x_2 - 8x_3 = 8$

$2x_2 - 8(-1) = 8$

$2x_2 + 8 = 8$

$2x_2 = 8 - 8 \Rightarrow 2x_2 = 0$

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$x_2 = 0$

$S.S = (x_1 = 1 , x_2 = 0 , x_3 = -1)$

It is consistent because it is has finite solution.

Ans 2 Given:

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4th\text{-}ID \\ 5 & -2 & 7 \end{bmatrix}$$

Findout: Adjoint Method..

Solution:

Let : 4th-ID = 4

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 4 \\ 5 & -2 & 7 \end{bmatrix}$$

Now we find the co-factors:

$A_{11} = (-1)^{1+1} = \begin{vmatrix} -1 & 4 \\ -2 & 7 \end{vmatrix} = 1(-7+8) = 1$

$A_{12} = (-1)^{1+2} = \begin{vmatrix} 2 & 4 \\ 5 & 7 \end{vmatrix} = -1(14-20) = +6$

$A_{13} = (-1)^{1+3} = \begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = 1(-4-(-5) = (-4+5) = 1$

$A_{21} = (-1)^{2+1} = \begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix} = -1(28-(-10) = -(28+10) = -38$

$A_{22} = (-1)^{2+2} = \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} = 1(21-25) = 4$

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$A_{23} = (-1)^{2+3} = \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = -1(-6-20) = +24$

$A_{31} = (-1)^{3+1} = \begin{vmatrix} 4 & 5 \\ -1 & 4 \end{vmatrix} = 1(16+5) = 21$

$A_{32} = (-1)^{3+2} = \begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} = -1(16-10) = -6$

$A_{33} = (-1)^{3+3} = \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = 1(-3-8) = -11$

Now:-

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

Puting the values:-

$$A \begin{bmatrix} 1 & 6 & 1 \\ -38 & 4 & 24 \\ 21 & -16 & -11 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & -38 & 21 \\ 6 & 4 & -16 \\ 1 & 24 & -11 \end{bmatrix} \leftarrow \text{Ans}$$

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Ans 3 Given:

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + 2y - 3z = 14$$

Findout =

Gauss Jorden Method

Solution:-

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 2 & -3 & 14 \end{bmatrix} \quad R_1 = 1/2 R_1$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 3 & 2 & -3 & 14 \end{bmatrix} \quad R_2 = R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -1 & -3 & -13 \end{bmatrix} \quad R_3 = R_3 - 3R_1$$

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Q3 Part

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$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -1 & -3 & -13 \end{bmatrix} \quad R_2 = 1/2 R_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -11 \end{bmatrix} \quad R_3 = R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -11 \end{bmatrix} \quad R_1 = R_1 - R_2$$

$$\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & \frac{11}{3} \end{bmatrix} \quad R_3 = -1/3 R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{-19}{3} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 11/3 \end{bmatrix} \quad R_1 = R_1 - 2R_3$$

$$\left\{ S.S : x = -\frac{19}{3}, \; y = 2, \; z = 11/3 \right\} \leftarrow \text{Ans}$$

Ans 4 Given:

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

Solution =

Let $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

for eigen values Ist I find to

$A - \lambda I$.

$$A - \lambda I = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 2 & 2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{bmatrix}$$

So taking determint.

$$|A - \lambda I| = \begin{vmatrix} 4-\lambda & 2 & 2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{vmatrix}$$

Expand by Row 1

$$|A - \lambda I| = (4-\lambda)\left[(3-\lambda)(1-\lambda) - 8\right] - 2\left[-5(1-\lambda) + 4\right] - 2\left[-20 + 2(3-\lambda)\right]$$

$$= (4-\lambda)(3-\lambda)(1-\lambda) - 8(4-\lambda) + 10(1-\lambda) - 8 + 40 - 4(3-\lambda)$$

$$= (12 - 4\lambda - 3\lambda + \lambda^2)(1-\lambda)(- 32 + 8\lambda + 10 - 10\lambda - 8 + 40 - 12 + 4\lambda$$

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Ans 4 Part.

$= (12 - 7\lambda + \lambda^2)(1-\lambda) - 2 + 2\lambda$

$= 12 - 12\lambda - 7\lambda + 7\lambda^2 + \lambda^2 - \lambda^3 - 2 + 2\lambda$

$= 10 - 17\lambda + 8\lambda^2 - \lambda^3$

$= -\lambda^3 + 8\lambda^2 - 17\lambda + 10 = 0$

~~$(\lambda-1)(-\lambda^2 - 7\lambda - 10) = 0$~~

= ~~$\lambda - 1$~~ By synthitic division.

1	-1	8	-17	10
		1	7	

+1	-1	8	-17	10
		-1	7	-10
	-1	7	-10	0

$(\lambda-1)(-\lambda^2 + 7\lambda - 10) = 0$

$\lambda = 1$

$-\lambda^2 + 7\lambda - 10 = 0$

$-\lambda^2 + 5\lambda + 2\lambda - 10 = 0$

$\lambda(-\lambda + 5) - 2(-\lambda + 5) = 0$

$(\lambda - 2)(-\lambda + 5) = 0$

$\lambda - 2 = 0, \quad -\lambda + 5 = 0$

$\lambda = 2, \quad +\lambda = +5$

$\lambda = 2 \quad \lambda = 5$

Page 11 :-) Ans Q.4

Its eign values are 2, 1, 5
so all of its eign values are
distinict Matrix A is a diagonalizable
~~&~~ We can diagonalize it by
finding bases vector which is not
the part of Question.

Ans 5 Given:

$3x_1 + 5x_2 - 4x_3 = 0$

$-3x_1 - 25x_2 + 4x_3 = 0$

$6x_1 + x_2 - 8x_3 = 0$

Solution:-

Let $A = \begin{bmatrix} 3 & 5 & -4 \\ -3 & -25 & 4 \\ 6 & 1 & -8 \end{bmatrix}$

Expand by R_1

$= 3\begin{vmatrix} 25 & 6 \\ 1 & -8 \end{vmatrix} - 5\begin{vmatrix} -3 & 4 \\ 6 & 8 \end{vmatrix} - 4\begin{vmatrix} -3 & -25 \\ 6 & 1 \end{vmatrix}$

$= 3(200-4) - 5(24-24) - 4(-3+150)$

$= 3(196) - 5(0) - 4(147)$

$= 3(196) - 4(147)$

$= 588 - 588 = 0$

Here $|A| = 0$

Hence the solution of given system is trival i.e $S.S = \{, 0, 0, 0\}$.

OR

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Ans 5 Given:-

$3x_1 + 5x_2 - 4x_3 = 0$

$-3x_1 - 25x_2 + 4x_3 = 0$

$6x_1 + x_2 - 8x_3 = 0$

Solution:-

Write in Matrix form

~~Let A~~ = $\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & 8 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & -20 & 0 & 0 \\ 6 & 1 & 8 & 0 \end{array}\right]$ $R_2 + R_1$

$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & -9 & +16 & 0 \end{array}\right]$ $R_3 - 2R_1$

$\left[\begin{array}{ccc|c} 3 & 5 & +0 & 0 \\ 0 & -20 & 0 & 0 \\ 0 & -9 & 16 & 0 \end{array}\right]$ taking $x_3 = 0$ and x_2 free

$3x_1 = -5x_2$

$x_1 = -5/3x_2$

$x = \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix}$

Ans 6 Given :-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

Findout = Rank = ?

Solution :-

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 1 & 3 & 4 & 0 \end{bmatrix} \quad R_2 = R_2 - 3R_1$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 = R_3 - R_1$$

$C_2 \longrightarrow C_2 - 3C_1$

$C_3 \longrightarrow C_3 - 4C_1$

$C_4 \longrightarrow C_4 - 3C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Chang colmn 4 with 2

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Divide column 2 by -6

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

~~Ran~~ →

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank} = 2}$ ← Ans

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