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Paper :- Electrical Network Analysis

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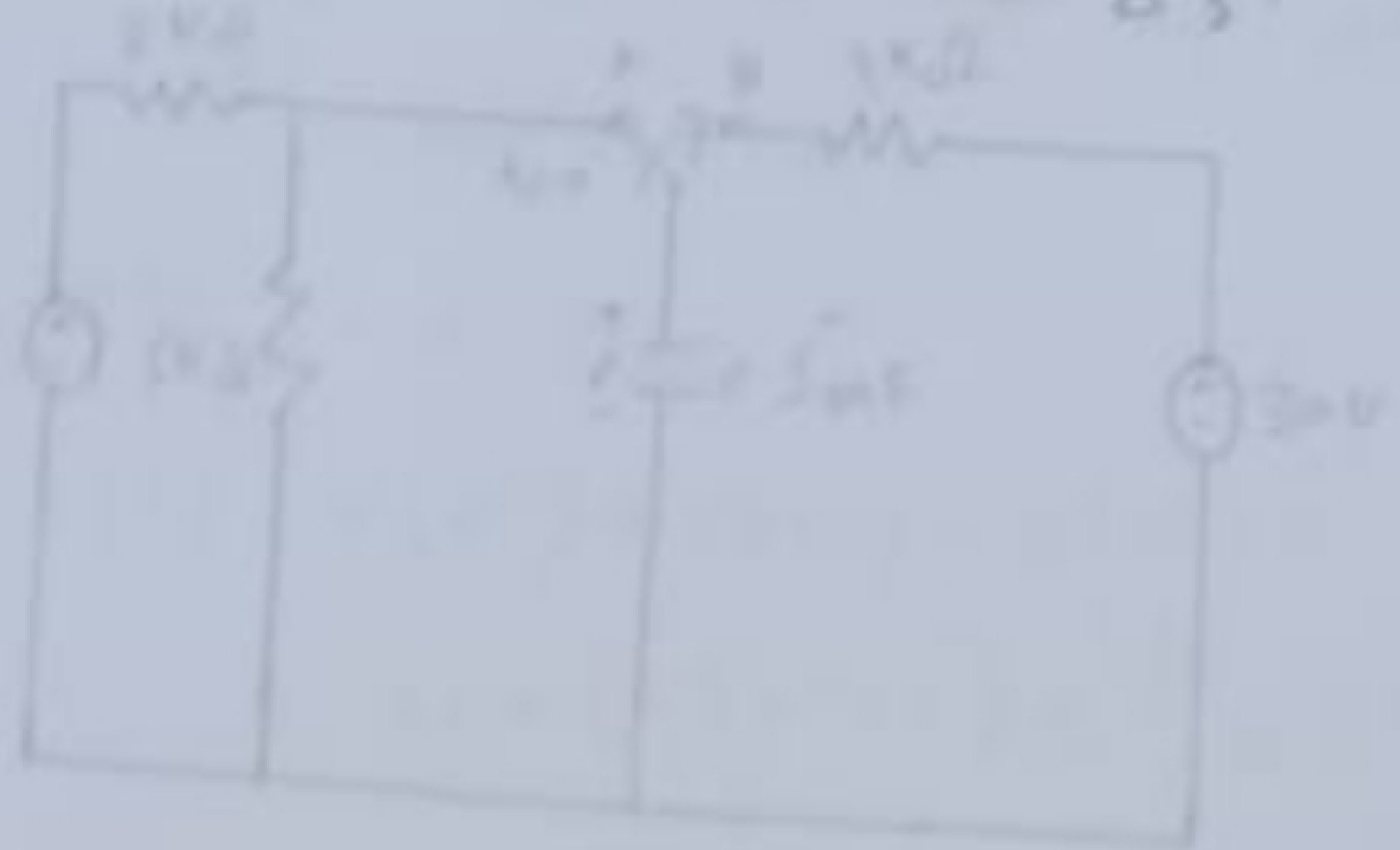
Q1:- The switch in fig. 1 has been in position A for a long time. At $t=0$ the switch moves to B. Determine $v(t)$ for $t>0$ and calculate its value at $t=2s$ and $8s$.



Solution:-

For $t<0$ the switch is at position A the capacitor acts like an open circuit to dc, but $v$ is the same as the voltage across the $5k\Omega$ resistor. Hence, the voltage across the capacitor just before $t=0$ is obtained by voltage division as

$$v(0^-) = \frac{5}{5+3}(24) = 15\,v$$

using the fact that the capacitor voltage cannot change instantaneously,

$$v(0) = v(0^-) = v(0^+) = 15\,v$$

For $t>0$ the switch is in position B. The Thevenin resistance connected to the capacitor is $R_{Th}$

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$R_{Th} = 4k\Omega$ and the time constant is

$$\tau = R_{Th}C = 4\times10^{3}\times0.5\times10^{-3} = 2s$$

Since the capacitor act like an open circuit to dc at steady state.

$V(\infty) = 30\,V$ Then

$$v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$

$$= 30 + (15 - 30)e^{-t/2} = (30 - 15e^{-0.5t})\,V$$

At $t = 2$

$$V(2) = 30 - 15e^{-0.5(2)}$$

$$v(2) = 30 - 15e^{-1}$$

$$v(2) = 30 - 15(0.3678)$$

$$v(2) = 30 - 5.517$$

$$v(2) = 24.483\,V$$

At $t = 8$

$$v(8) = 30 - 15e^{-0.5(8)}$$

$$v(8) = 30 - 15e^{-4}$$

$$v(8) = 30 - 15(0.0183)$$

$$v(8) = 30 - 0.2745$$

$$v(8) = 29.7255\,V$$

2:- Detemine the inductor current for both $t > 0$ and $t < 0$ for the circuit in fig 2.


Solution :-

For $t < 0$

The switch is closed and inductor acts as short circuit therefore inductor current is

$$\boxed{i = 6A}$$

For $t > 0$

The switch is open and and time constant

$$\tau = L/R$$

$$\tau = 3/2$$

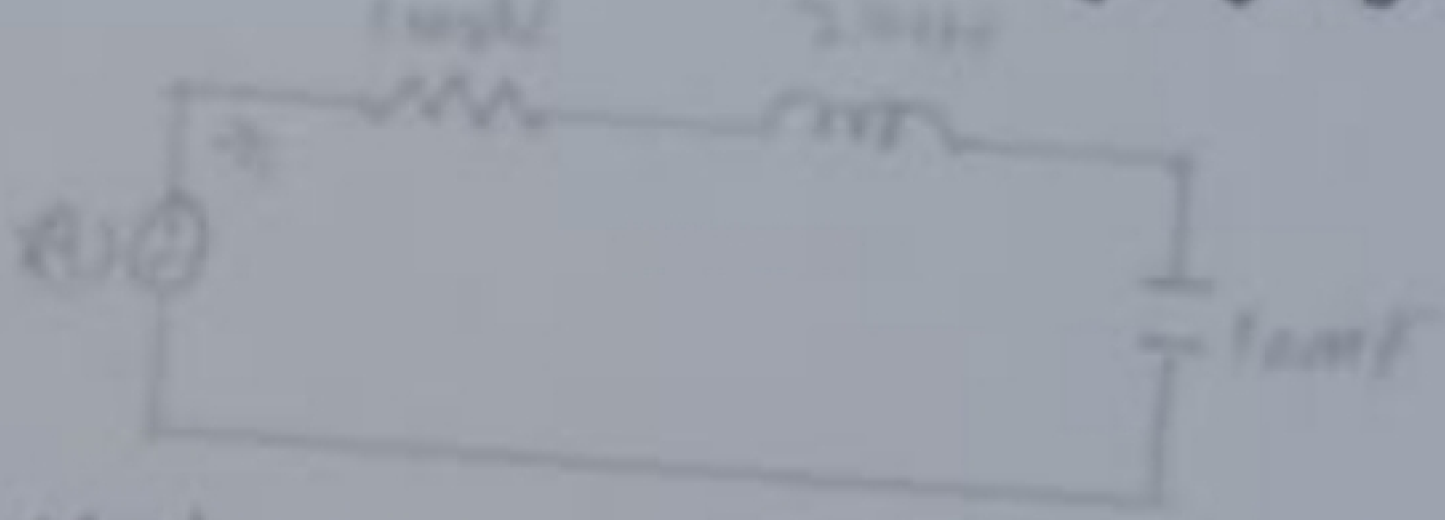
Now the inductor current

$$i(t) = 6e^{-t/\tau}$$

$$i(t) = 6e^{-t/3/2}$$

$$\boxed{i(t) = 6e^{-\frac{2t}{3}} u(t)\, A}$$

Q4:- A Series RLC Circuit has $R = 100\Omega$, $L = 240H$ and $C = 10mF$. If the input voltage is $v(t) = 10\cos 2t$ Find the current flowing through the circuit.



Solution:-

input voltage is $v(t) = 10\cos 2t\ V$

Here

Amplitude $= V_m = 10V$

Angular frequency, $\omega = 2$ rad/s

Phase angle, $\phi = 0$

So phasor for the voltage

$v(t) = 10 \angle 0° V$

Now for inductive reactance

$X_L = \omega L$

So $\omega = 2$ rad/s, $L = 240 H$

$X_L = 2(240)$

$= 480\,\Omega$

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Now for capacitive reactance

$X_c = \frac{1}{\omega C}$ , $\omega = 2$ rad/s

$C = 10\mu F$

$X_c = \frac{1}{2(10 \times 10^{-3})}$

$X_c = \frac{1}{2 \times 10^{-2}}$

$X_c = \frac{10^{2}}{2}$

$X_c = \frac{100}{2}$

$X_c = 50\Omega$

Now for impedence

$Z = R + jX_L - jX_c$

$R = 100\Omega$, $X_L = 480\Omega$, $X_c = 50\Omega$

$Z = (100 + 480 - 50)\Omega$

$Z = (100 + j430)\Omega$

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Represent Z in phasor form

$Z = (100 + j430)\,\Omega$

$Z = \sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left(\frac{430}{100}\right)$

$Z = \sqrt{10000 + 184900} \angle \tan^{-1}(4.3)$

$Z = \sqrt{194900} \angle 76.90^\circ\,\Omega$

$Z = 441.47 \angle 76.90^\circ\,\Omega$

Now for current flowing;

$i = \frac{v(t)}{Z}$

$v(t) = 10 \angle 0^\circ$ , $Z = 441.47 \angle 76.9$

$i = \frac{10 \angle 0^\circ\,V}{441.47 \angle 76.90^\circ\,\Omega}$

$i = \frac{10}{441.47} \angle [0 - 76.90^\circ]\,A$

$$i = 22.6\times10^{-3}\angle -76.90^{\circ} A$$

$$= 22.6\angle -76.90^{\circ} mA$$

So the general expression for $i$

$$i = 22.6\cos(2t - 76.90^{\circ}) mA.$$

Q3:- A series RLC circuit is described by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = 10$$

Find the response when $L = 0.5H$, $R = 4\Omega$ and $C = 0.2F$. Let $i(0)/dt = 0$

Solution:-

Given data

$L = 0.5H$

$R = 4\Omega$

$C = 0.2F$

$i(0) = 1A$

$$\frac{di(0)}{dt} = 0$$

Voltage of the given RLC circuit is described by

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$$\frac{Ld^2i}{dt} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Divided by $L$

$$\frac{d^2i}{dt^2} + \frac{R di}{L dt} + \frac{1}{LC} = \frac{10}{L}$$

For R.H.S of the equation multiply by $\frac{C}{C}$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10C}{LC}$$

$C = 0.2F$ thus

$$\frac{d^2i}{dt^2} + \frac{R di}{L dt} + \frac{i}{LC} = \frac{2}{LC}$$

Subtitute.

$$\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 10i = 20 \longrightarrow (1)$$

general equation of RLC circuit is given by

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1\,i}{LC} = \frac{2 \cdot 5}{LC} \longrightarrow (2)$$

compare (1) and (2)

$$\frac{R}{L} = 8 \longrightarrow (3)$$

$$\frac{1}{LC} = 10 \longrightarrow (4)$$

$$\frac{Is}{LC} = 20 \longrightarrow (5)$$

From equation (3) $\alpha$ is given by

$$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/s} \longrightarrow (6)$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

From equation (4)

$$\omega_0 = \sqrt{10} \text{ rad/s} \rightarrow (7)$$

$\therefore$ ~~Therefore~~ The circuit is over damped

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 + \sqrt{6} \text{ rad/s}$$

and

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$= -4 + \sqrt{4^2 - \sqrt{10}^2}$$

$$= -4 - \sqrt{6} \text{ rad/s}$$

From eq (5) the steady state current is given by

$I_s = 20 \times LC = 20 \times 0.5 \times 0.2 = 2A$ — (8)

$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t > 0$ — (9)

$t = 0$

$i(0) = I_s + A_1 + A_2$

$1 = 2 + A_1 + A_2$

Thus $A_1 + A_2 = -1$ — (10)

From eq (9) Find $\frac{di}{dt}(t)$

$\frac{di(t)}{dt} = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}$

$t = 0$,

$\frac{di(0)}{dt} = A_1 s_1 + A_2 s_2$

substitute the value

$(-4 + \sqrt{6}) A_1 + (-4 - \sqrt{6}) A_2 = 0$

solve (10) & (11) simultaneously

$A_1 = -1.336$

$A_2 = 0.316$

Substitute in (9)

$i(t) = 2 - 1.316 e^{(-4+\sqrt{6})t} + 0.316 e^{(-4-\sqrt{6})t} A$

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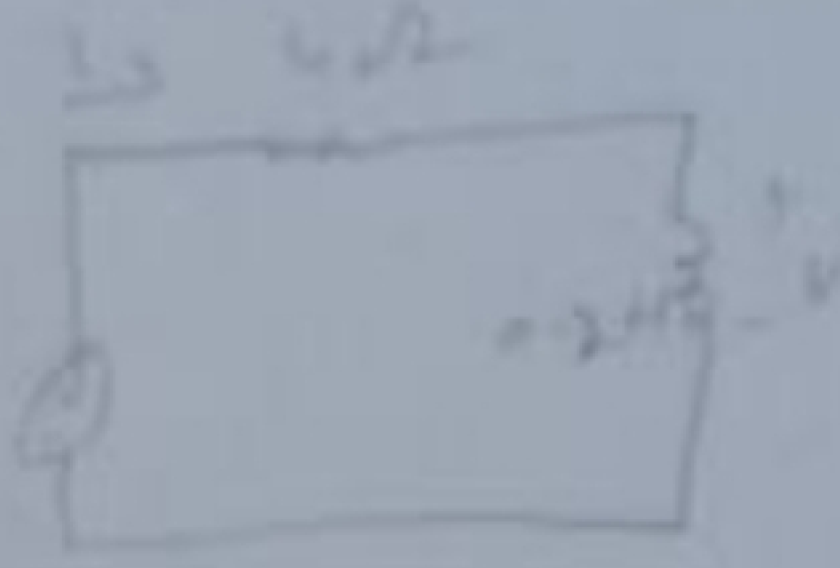
$$i(t) = 2 - 1.316\, e^{(-4+\sqrt{6})t} + 0.316\, e^{(-4-\sqrt{6})t} \text{ A.}$$

Q25:-

Find $v(t)$ and $i(t)$ in the circuit shown in figure 3




$v_s = 20 \sin(10t + 30^\circ)$ v

solution:-

From the voltage source

$(v_s) = 20 \sin(10t + 30^\circ - 90^\circ)$ v

$(v_s) = 20 \sin(10t - 60^\circ)$ v

$v_s = 20 \angle -60^\circ$ V

$\omega = 10$ rad/s

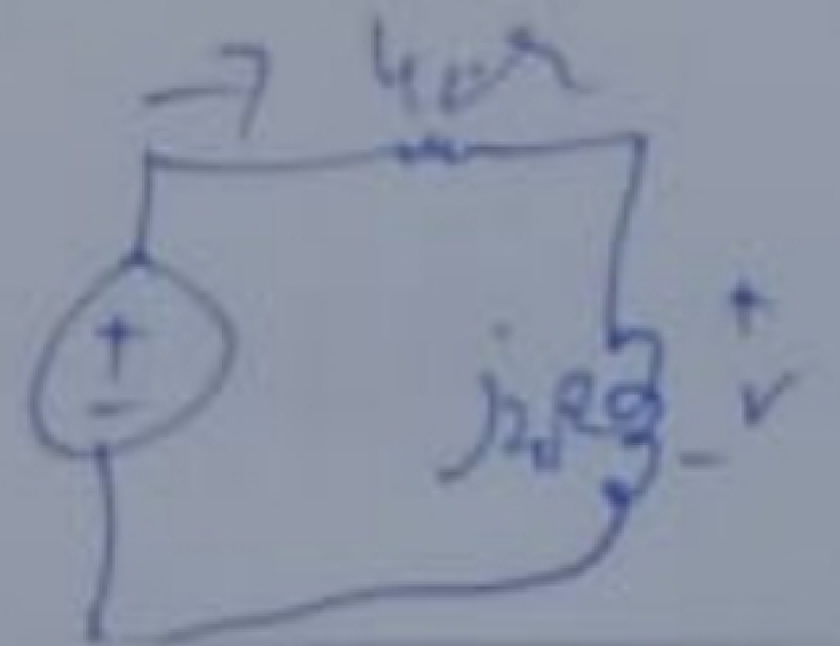
$X_L = j \omega L$

$0.2H = j \times 10 \times 0.2$

$0.2H = j2\,\Omega$

The given circuit can be represent

$20\angle -60^\circ$ v



From the circuit diagram

$z = 4 + j2\sqrt{2}$

Hence the current is

$$I = \frac{20\angle -60^\circ}{4 + j2}$$

$$I = \frac{20\angle -60}{\sqrt{4^2 + 2^2}\angle \tan^{-1}\left(\frac{2}{4}\right)}$$

$$I = \frac{20\angle -60}{2.472\angle 26.57^\circ}$$

$$I = 4.472\angle -86.57^\circ$$

convert into time domain

$i(t) = 4.472\cos(10t - 86.57^\circ)$

$i(t) = 4.472\sin(10t - 86.57^\circ + 90)$

$i(t) = 4.472\sin(10t + 3.43^\circ)$ A

From the circuit voltage across the inductor is

$v = j2 \times i$

$v = j2 \times (4.472\angle -86.57^\circ)$

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convert Polar form to retangular form we get

$V = j2 \times (0.26756 - j4.464)$

$V = 8.928 + j0.53512$

From rectangular to Polar

$\Rightarrow V = \sqrt{(8.926)^2 + (0.53512)^2} \angle \tan^{-1} \frac{0.53512}{8.926}$

Now into time domain

$v(t) = 8.944 \cos(10t + 3.4^\circ)$

$v(t) = 8.944 \sin(10t + 3.4^\circ + 90^\circ)$

$v(t) = 8.944 \sin(10t + 93.4^\circ)\,V$