Mid Term Summer 2020 Date: 20 August 2020
Multivariate Calculus
Time Allowed: Four Hours

Q.1.If (x + yi) / i = (7 + 9i) , where x and y are real, what is the value of (x + yi)(x - yi)?

Q.2. Find the values of x and y in the following equation, given further that x ∈ R, y ∈ R.

(x+iy)(2+i)=3-i

Q.3. Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in C$.

Q.4. Express $4 - \sqrt{5}i$ in polar form.

Q.5. Find the limit $\lim_{z \to 8} \frac{2z^2 - 17z + 8}{8 - z}$

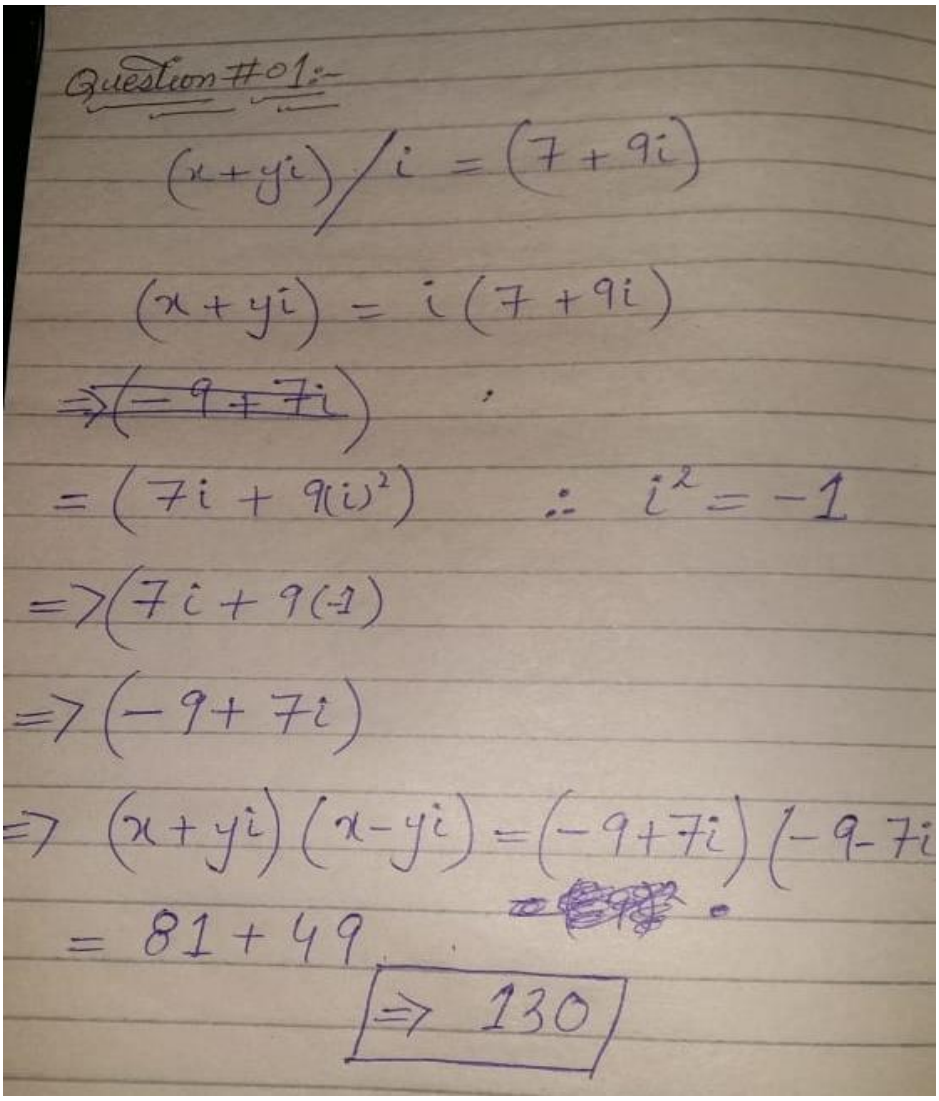
Q.6.

Differentiate

(i). $f(x)=(\ln x)^4$

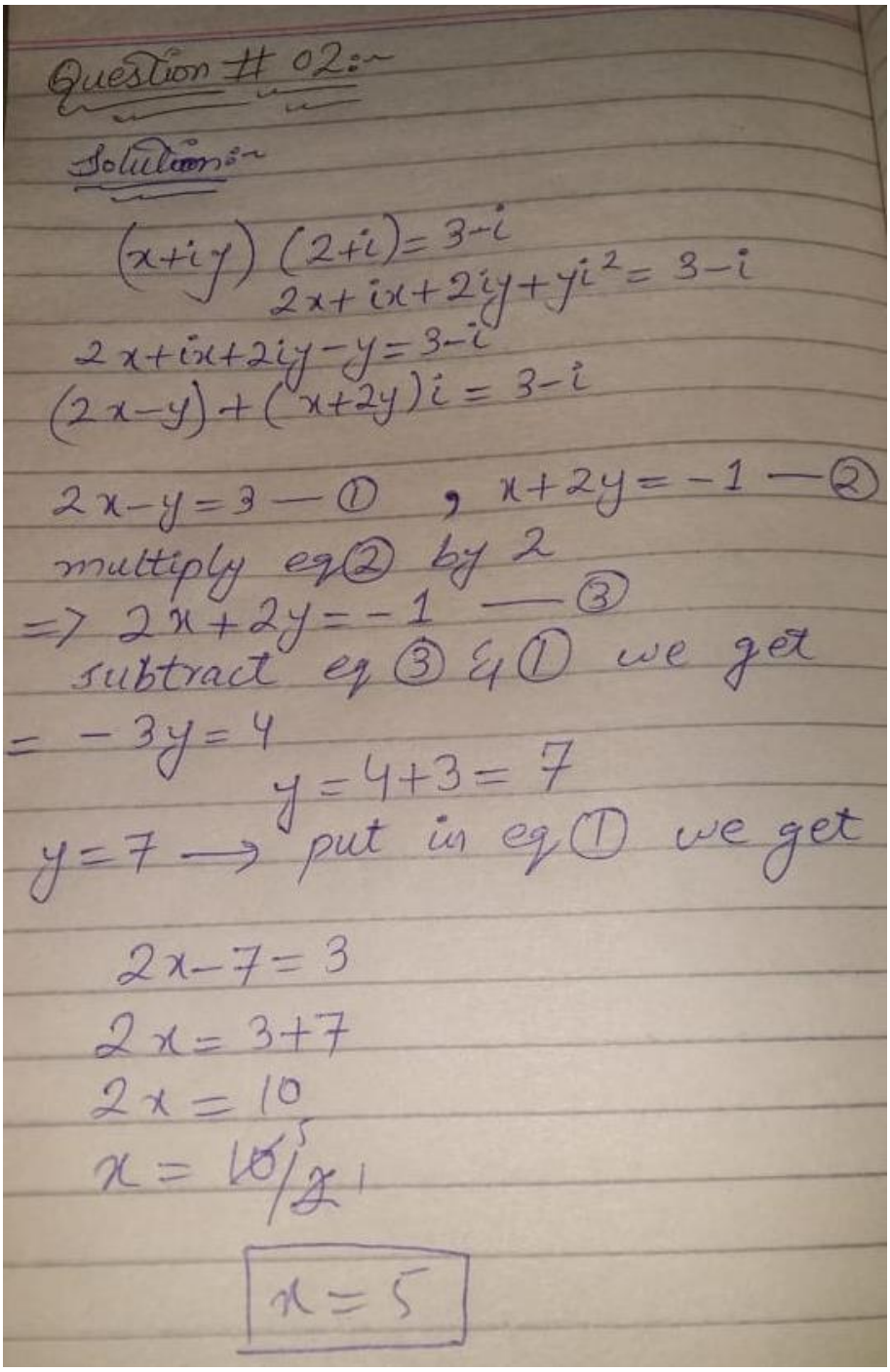
(ii). $g(x)=x^2.\ln x$

Q.1.If (x + yi) / i = (7 + 9i) , where x and y are real, what is the value of (x + yi)(x - yi)?

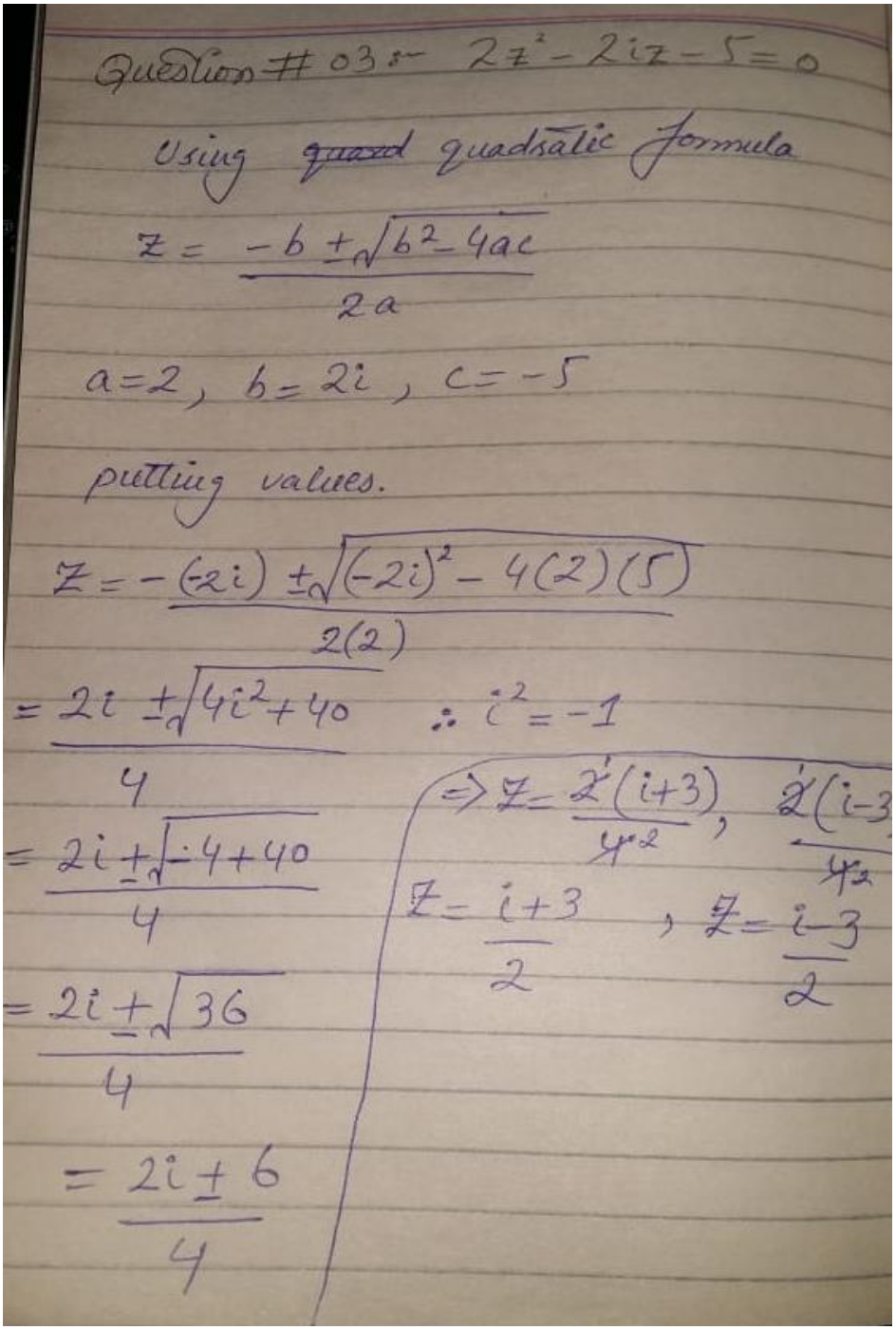


Q.2. Find the values of x and y in the following equation, given further that x ∈ R, y ∈ R.

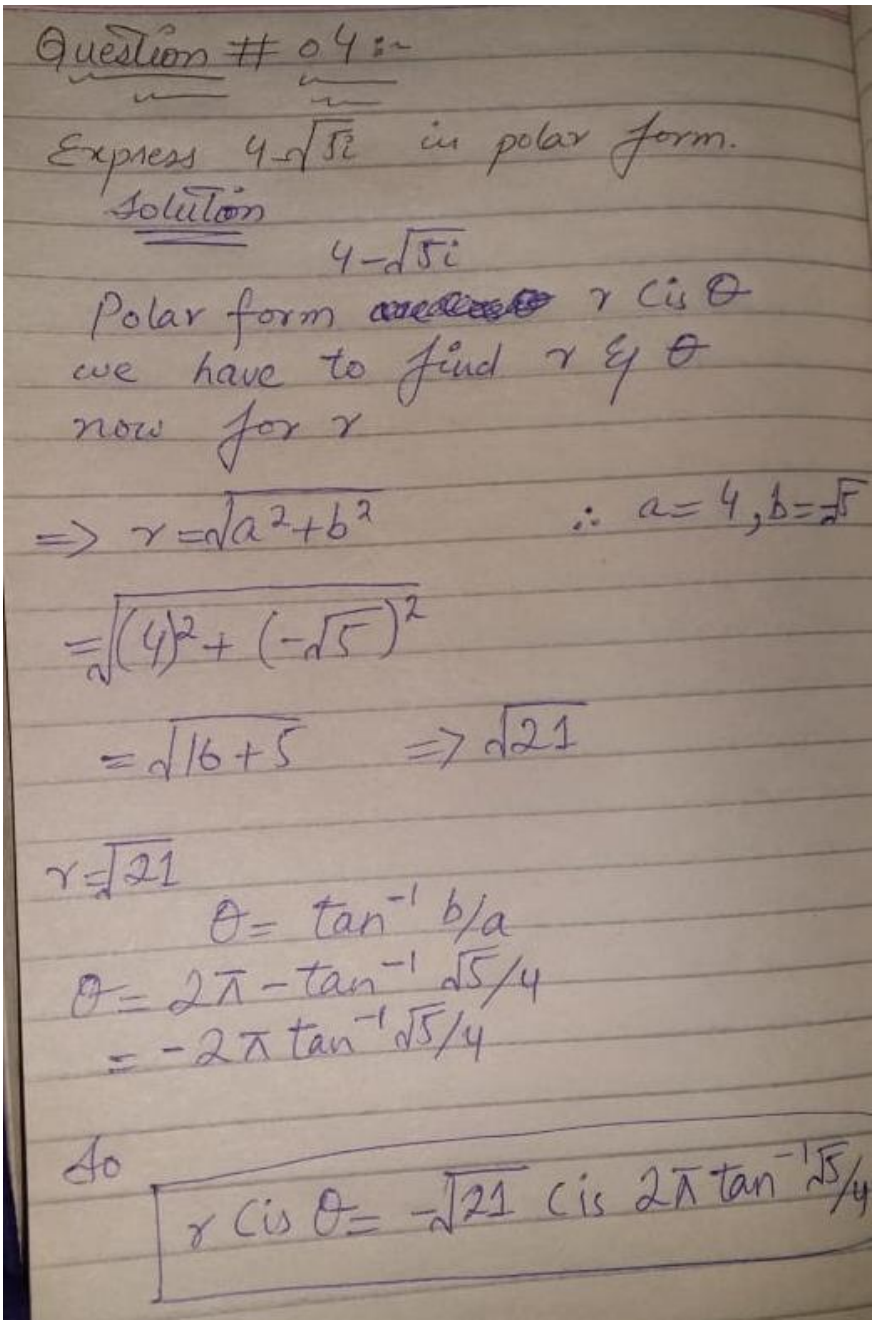
(x+iy)(2+i)=3-i



Q.3. Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in C$.



Q.4. Express 4 - $\sqrt{5}i$ in polar form.



Question # 04 :-

Express $4-\sqrt{5}i$ in polar form.

Solution

$4-\sqrt{5}i$

Polar form $r\,cis\,\theta$

we have to find r & θ

now for r

$\Rightarrow r=\sqrt{a^2+b^2}$ $\quad \therefore a=4, b=\sqrt{5}$

$=\sqrt{(4)^2+(-\sqrt{5})^2}$

$=\sqrt{16+5} \Rightarrow \sqrt{21}$

$r=\sqrt{21}$

$\theta = \tan^{-1} b/a$

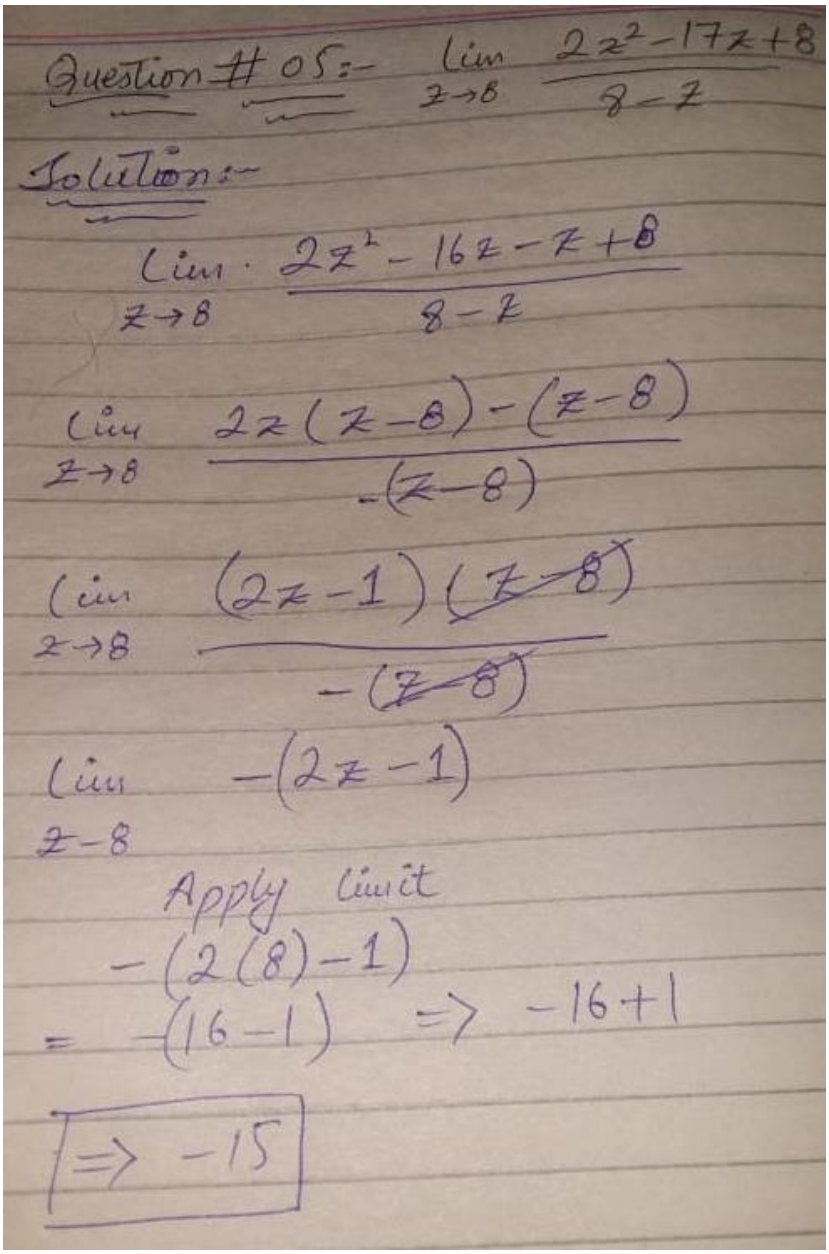
$\theta = 2\pi - \tan^{-1}\sqrt{5}/4$

$= -2\pi \tan^{-1}\sqrt{5}/4$

So

$$r\,cis\,\theta = -\sqrt{21}\,cis\,2\pi\tan^{-1}\sqrt{5}/4$$

Q.5. Find the limit



Question # 05:- $\lim_{z \to 8} \frac{2z^2 - 17z + 8}{8 - z}$

Solution:-

$$\lim_{z \to 8} \frac{2z^2 - 16z - z + 8}{8 - z}$$

$$\lim_{z \to 8} \frac{2z(z-8) - (z-8)}{-(z-8)}$$

$$\lim_{z \to 8} \frac{(2z-1)(z-8)}{-(z-8)}$$

$$\lim_{z \to 8} -(2z-1)$$

Apply limit

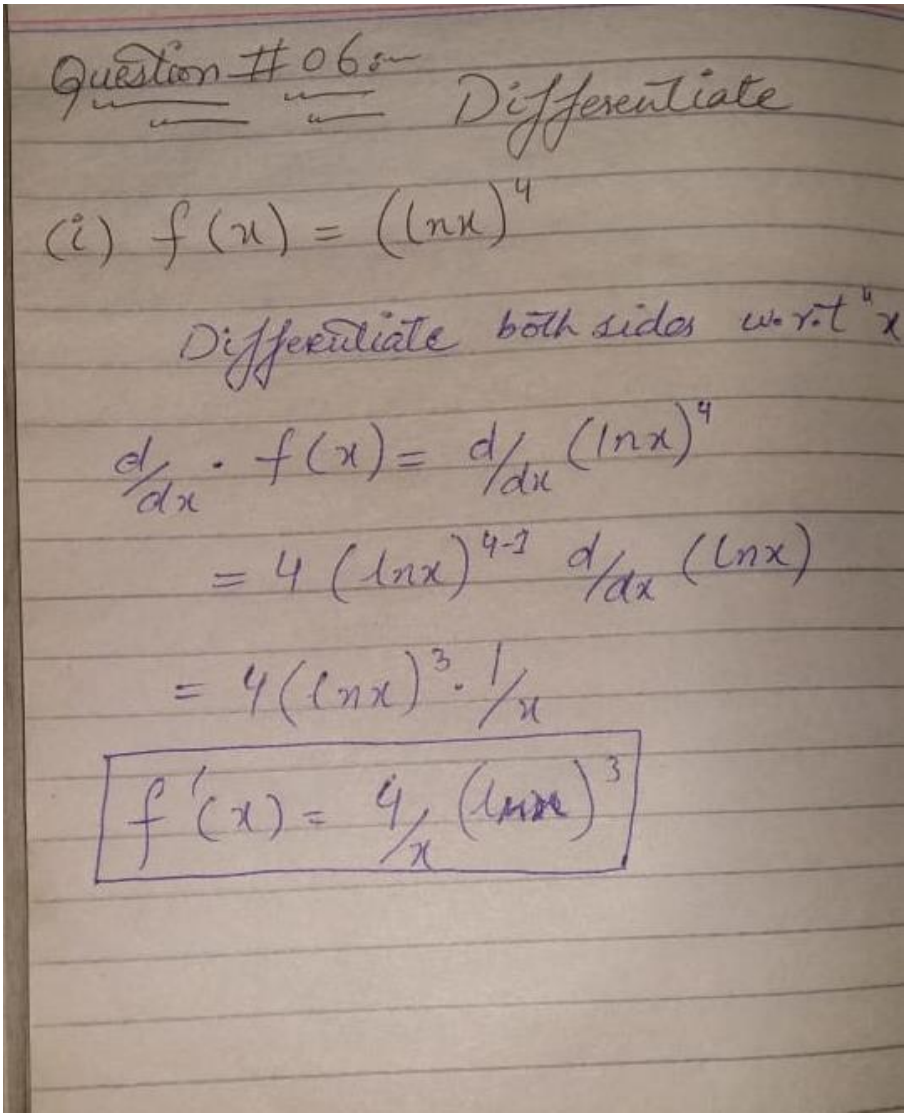
$$-(2(8)-1)$$

$$= -(16-1) \Rightarrow -16+1$$

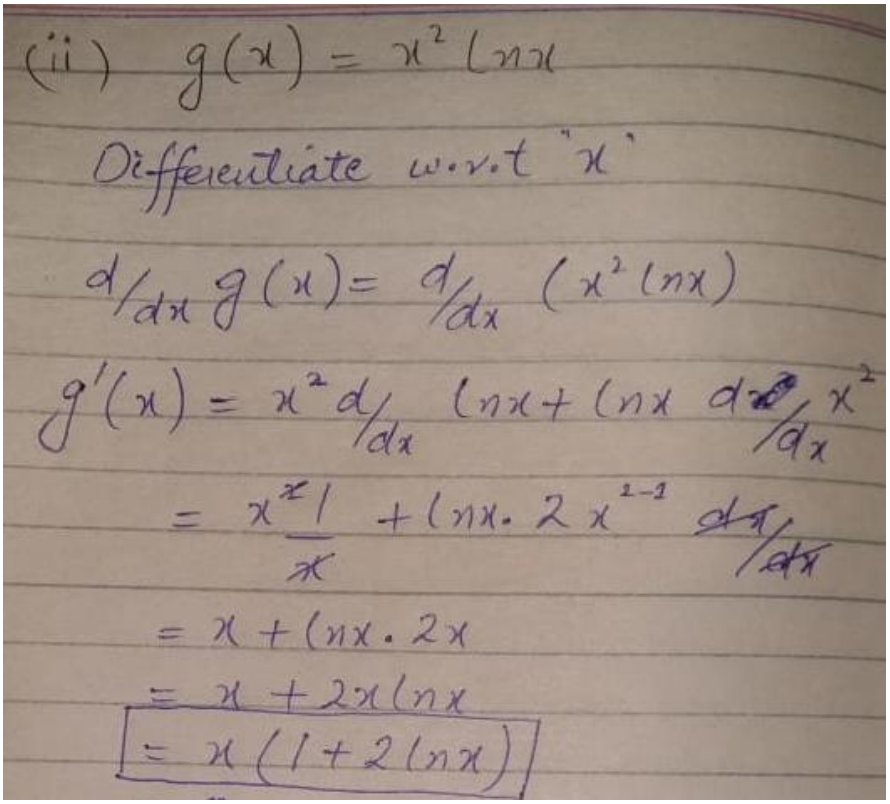
$$\boxed{\Rightarrow -15}$$

Q.6. Differentiate

(i). $f(x)=(\ln x)^4$



(ii). $g(x)=x^2.\ln x$



(ii) $g(x) = x^2 \ln x$

Differentiate w.r.t 'x'

$\frac{d}{dx} g(x) = \frac{d}{dx}(x^2 \ln x)$

$g'(x) = x^2 \frac{d}{dx} \ln x + \ln x \frac{d}{dx} x^2$

$= x^2 \frac{1}{x} + \ln x \cdot 2x^{2-1} \frac{dx}{dx}$

$= x + \ln x \cdot 2x$

$= x + 2x \ln x$

$= x(1 + 2\ln x)$