

Mid Term Summer 2020
Multivariate Calculus
Time Allowed: Four Hours

Date: 20 August 2020

Q.1. If $(x + yi) / i = (7 + 9i)$, where x and y are real, what is the value of $(x + yi)(x - yi)$?

Q.2. Find the values of x and y in the following equation, given further that $x \in \mathbb{R}$, $y \in \mathbb{R}$.

$$(x+iy)(2+i)=3-i$$

Q.3. Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in \mathbb{C}$.

Q.4. Express $4 - \sqrt{5}i$ in polar form.

Q.5. Find the limit $\lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z}$

Q.6.

Differentiate

(i). $f(x) = (\ln x)^4$

(ii). $g(x) = x^2 \cdot \ln x$

Q.1. If $(x + yi) / i = (7 + 9i)$, where x and y are real, what is the value of $(x + yi)(x - yi)$?

Question #01:-

$$(x + yi) / i = (7 + 9i)$$
$$(x + yi) = i(7 + 9i)$$
$$\Rightarrow \cancel{(-9 + 7i)}$$
$$= (7i + 9i^2) \quad \because i^2 = -1$$
$$\Rightarrow (7i + 9(-1))$$
$$\Rightarrow (-9 + 7i)$$
$$\Rightarrow (x + yi)(x - yi) = (-9 + 7i)(-9 - 7i)$$
$$= 81 + 49$$
$$\Rightarrow \boxed{130}$$

Q.2. Find the values of x and y in the following equation, given further that $x \in \mathbb{R}$, $y \in \mathbb{R}$.

$$(x+iy)(2+i)=3-i$$

Question # 02:-
Solution:-

$$(x+iy)(2+i)=3-i$$
$$2x+ix+2iy+yi^2=3-i$$
$$2x+ix+2iy-y=3-i$$
$$(2x-y)+(x+2y)i=3-i$$
$$2x-y=3 \text{ --- (1) , } x+2y=-1 \text{ --- (2)}$$

multiply eq(2) by 2

$$\Rightarrow 2x+2y=-1 \text{ --- (3)}$$

subtract eq (3) & (1) we get

$$-3y=4$$
$$y = \frac{4}{-3} = -\frac{4}{3}$$

$y = -\frac{4}{3} \rightarrow$ put in eq (1) we get

$$2x - \frac{4}{3} = 3$$
$$2x = 3 + \frac{4}{3}$$
$$2x = \frac{10}{3}$$
$$x = \frac{10}{3} \times \frac{3}{2}$$

$x = 5$

Q.3. Solve the equation $2z^2 - 2iz - 5 = 0$, $z \in \mathbb{C}$.

Question # 03 :- $2z^2 - 2iz - 5 = 0$

Using ~~quard~~ quadratic formula

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 2$, $b = 2i$, $c = -5$

putting values.

$$z = \frac{-(-2i) \pm \sqrt{(-2i)^2 - 4(2)(-5)}}{2(2)}$$
$$= \frac{2i \pm \sqrt{4i^2 + 40}}{4} \quad \because i^2 = -1$$
$$= \frac{2i \pm \sqrt{-4 + 40}}{4}$$
$$= \frac{2i \pm \sqrt{36}}{4}$$
$$= \frac{2i \pm 6}{4}$$

$\Rightarrow z = \frac{2(i+3)}{4}$, $\frac{2(i-3)}{4}$

$$z = \frac{i+3}{2}, \quad z = \frac{i-3}{2}$$

Q.4. Express $4 - \sqrt{5}i$ in polar form.

Question # 04 :-

Express $4 - \sqrt{5}i$ in polar form.

Solution

$$4 - \sqrt{5}i$$

Polar form ~~is~~ $r \operatorname{cis} \theta$
we have to find r & θ
now for r

$$\Rightarrow r = \sqrt{a^2 + b^2} \quad \therefore a = 4, b = -\sqrt{5}$$

$$= \sqrt{(4)^2 + (-\sqrt{5})^2}$$

$$= \sqrt{16 + 5} \Rightarrow \sqrt{21}$$

$$r = \sqrt{21}$$

$$\theta = \tan^{-1} b/a$$

$$\theta = 2\pi - \tan^{-1} \sqrt{5}/4$$

$$= -2\pi \tan^{-1} \sqrt{5}/4$$

So

$$\boxed{r \operatorname{cis} \theta = \sqrt{21} \operatorname{cis} 2\pi \tan^{-1} \sqrt{5}/4}$$

Q.5. Find the limit

$$\text{Question \# 05: } \lim_{z \rightarrow 8} \frac{2z^2 - 17z + 8}{8 - z}$$

Solution:-

$$\lim_{z \rightarrow 8} \frac{2z^2 - 16z - z + 8}{8 - z}$$

$$\lim_{z \rightarrow 8} \frac{2z(z-8) - (z-8)}{-(z-8)}$$

$$\lim_{z \rightarrow 8} \frac{(2z-1)(\cancel{z-8})}{-(\cancel{z-8})}$$

$$\lim_{z \rightarrow 8} -(2z-1)$$

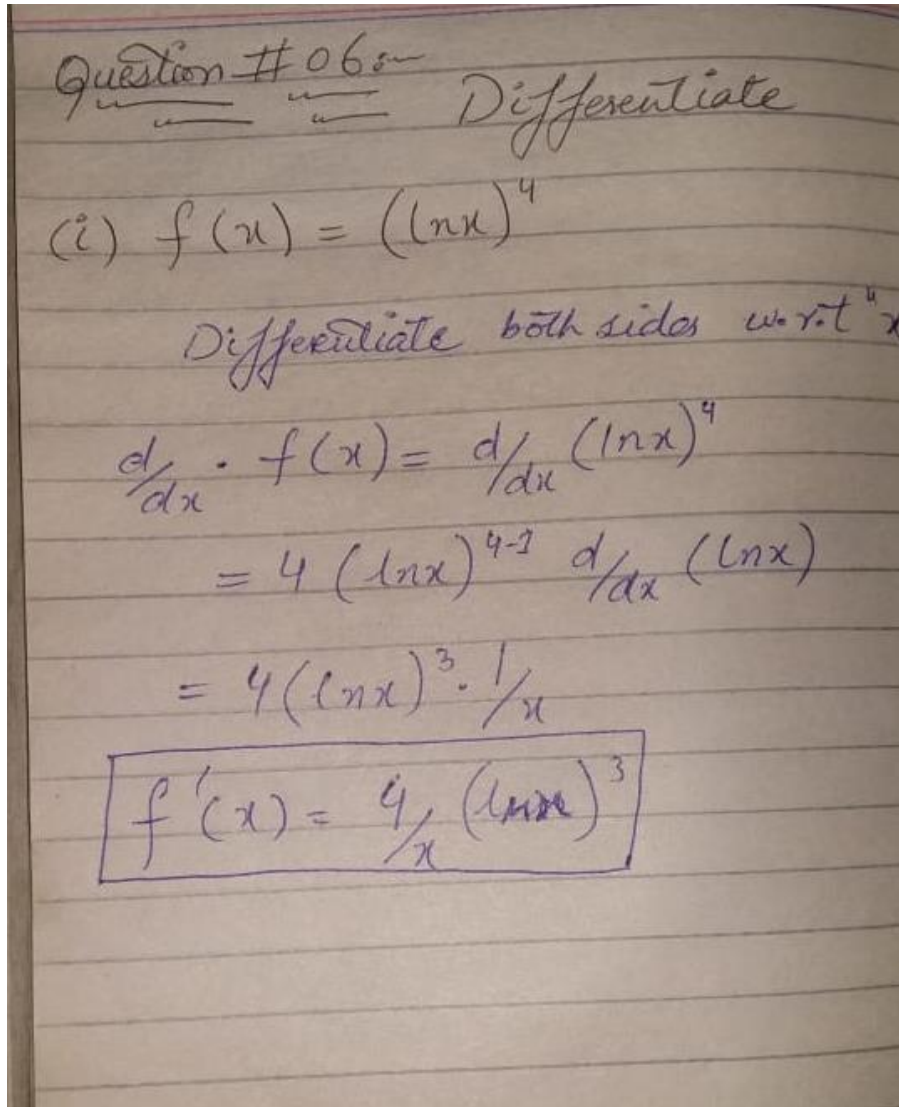
Apply limit

$$= -(2(8)-1) \Rightarrow -16+1$$

$$\boxed{\Rightarrow -15}$$

Q.6. Differentiate

(i). $f(x) = (\ln x)^4$



(ii). $g(x) = x^2 \cdot \ln x$

(ii) $g(x) = x^2 \ln x$

Differentiate w.r.t 'x'

$$\frac{d}{dx} g(x) = \frac{d}{dx} (x^2 \ln x)$$

$$g'(x) = x^2 \frac{d}{dx} \ln x + \ln x \frac{d}{dx} x^2$$

$$= \frac{x^2}{x} + \ln x \cdot 2x^{2-1} \frac{dx}{dx}$$

$$= x + \ln x \cdot 2x$$

$$= x + 2x \ln x$$

$$= x(1 + 2 \ln x)$$