Iqra National University, Peshawar
Department of Electrical Engineering

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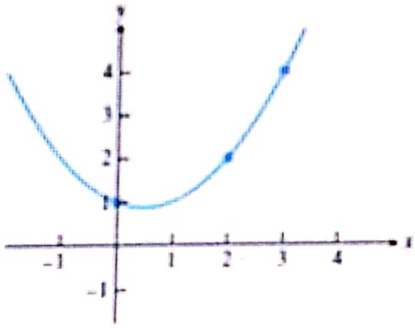
Mid – Term Examination Summer 2020
Date:19/08/2020

Course Code: MTH305 **Course Title:** Numerical Analysis
Prerequisite: ALI RAZA KHAN 12647 **Instructor:** Engr. Pir Meher Ali Shah
Module: 3 **Program:** BEE **Total Marks:** 30 **Time Allowed:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

Q1.	(a)	Find the polynomial of degree 3 or less that interpolates the point (0,2), (1,1), (2,0) and (3, -1)	Marks 5 CLO1
	(b)	Find the interpolating polynomial for the data points (0,1), (2,2) and (3,4) for the figure given below 	Marks 5 CLO1
Q2	(a)	Use the two-point forward difference formula with h=0.1 to approximate the derivate of $f(x)=1/x$ at $x=2$.	Marks 5 CLO1
	(b)	Use Newton's divided differences to find the interpolating polynomial passing through the points (0,1),(2,2) and (3,4).	Marks 5 CLO2
Q3	(a)	Solve the least square problem $\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$	Marks 5 CLO2
	(b)	Find the line that best fits the three data points (t, y)=(1,2),(-1,1) and (1,3) in the	Marks 5 CLO2

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Q1 (a):- Find the polynomial of degree 3 or less that interpolates the points $(0,2)$, $(1,1)$, $(2,0)$ and $(3,-1)$.

Sol

The Lagrange form is follow

$$P(x) = 2\frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} + 1\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}$$

$$+ 0\frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)\cdot(2-3)} - 1\frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$$= -\frac{1}{3}(x^3-6x^2+11x-6) + \frac{1}{2}(x^3-5x^2+6x) - \frac{1}{6}(x^3-3x^2+2x)$$

$$P(x) = -x+2$$

It means that there exists only one interpolating polynomial of degree 3 or less, but it may or maynot be degree 3. No interpolating polynomial of degree 2 or 3.

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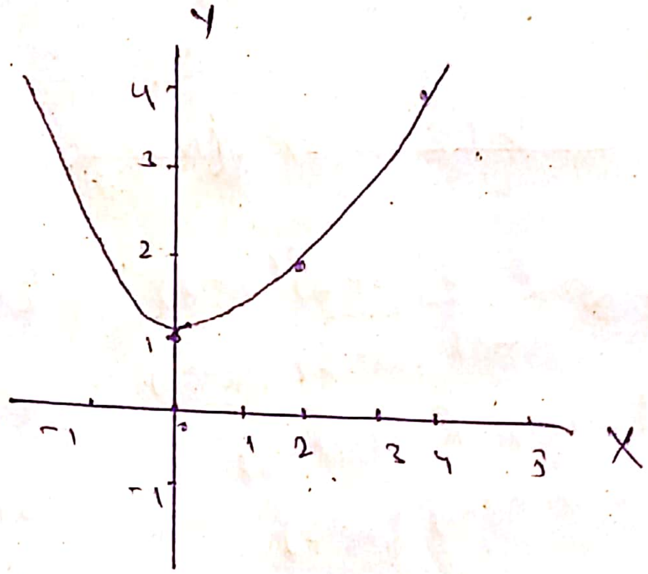
Q1(b) :- Find the interpolating polynomial for data points (0,1), (2,2) & (3,4) for the figure given below



Sol
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$$P_2(x) = y_1 \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + y_2 \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} + y_3 \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

Putting the values

$$P_2(x) = 1\frac{(x-2)(x-3)}{(0-2)(0-3)} + 2\frac{(x-0)(x-3)}{(2-0)(2-3)} + 4\frac{(x-0)(x-2)}{(3-0)(3-2)}$$

$$= \frac{1}{6}(x^2-5x+6) + 2\left(-\tfrac{1}{2}\right)(x^2-3x) + 4\left(\tfrac{1}{3}\right)(x^2-2x)$$

$$P_2(x) = \frac{1}{2}x^2 - \frac{1}{2}(x) + 1$$

(3)

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Check that $P_2(0) = 1$

$P_2(2) = 2$

$P_3(3) = 4$

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Q2(a):- Use the two points forward difference formula with $h = 0.1$ to approximate the derivative of $f(x) = 1/x$ at $x = 2$.

Sol:

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

$$= \frac{1/2.1 - 1/2}{0.1} \simeq -0.2381$$

The difference b/w this approximation and the correct derivative $f'(x) = -x^{-2}$ at $x = 2$ is error.

$$-0.2381 - (-0.2500) = 0.0119$$

Compare to the error predicted by formula $hf''(c)/2$ for some c b/w 2 and 2.1. Since $f''(x) = -2x^{-3}$ the error must b/w

$$(0.1)2^{-3} \simeq 0.0125$$

and $(0.1)(2.1)^{-3} \simeq 0.0108$

which is consistent with our result.

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A Second order formula can be develop by more advanced strategy According to taylor if f is three time continuously differentiable then

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(c_1)$$

and

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(c_2)$$

where $x-h < c_2 < x < c_1 < x+h$

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \frac{h^2}{12} f'''(c_1) - \frac{h^2}{12} f'''(c_2).$$

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Q2(b):- Use Newton's divided differences to find the interpolating Polynomial passing through points (0,1), (2,2) and (3,4).

Sol
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Applying the difinitions of divided differences leads to the following table:

0	1		
		$\frac{1}{2}$	
2	2		$\frac{1}{2}$
		2	
3	4		

After writing down the x, and y coordinate in seperate column will calculate the next column left to right as divided differences.

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$$\frac{2-1}{2-0} = \frac{1}{2}$$

$$\frac{2-\frac{1}{2}}{3-0} = \frac{1}{2}$$

$$\frac{4-2}{3-2} = 2$$

the cofficient of polynomial 1, 1/2, 1/2 can be read from the top edge of table. interpolating polynomial can be written as

$$P(x) = 1 + \frac{1}{2}(x-0) + \frac{1}{2}(x-0)(x-2)$$

or in nested form

$$P(x) = 1 + (x-0)\left(\frac{1}{2} + (x-2)\frac{1}{2}\right)$$

The base points for nested form are $x_1 = 0$, $x_2 = 2$ and interpolting polynomial as

$$P(x) = 1 + \frac{1}{2}x + \frac{1}{2}x(x-2)$$

$$\Rightarrow \frac{1}{2}x^2 - \frac{1}{2}x + 1 \quad \text{Ans}$$

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Q.3 (a):- Solve the least Square problem.

$$\begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 15 \\ 9 \end{bmatrix}$$

Sol #

The normal equations

$$A^T A x = A^T b \text{ are}$$

$$\begin{bmatrix} 9 & 6 \\ 6 & 29 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 45 \\ 75 \end{bmatrix}$$

So

$$\bar{x}_1 = 3.8 \text{ and } \bar{x}_2 = 1.8$$

The residual vector is

$$r = b - A\bar{x}$$

~~$$= \begin{bmatrix} -13 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} -3.4 \\ 13 \\ 11.2 \end{bmatrix}$$~~

$$r = \begin{bmatrix} -13 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 3.8 \\ 1.8 \end{bmatrix}$$

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$$= \begin{bmatrix} 13 \\ 15 \\ 9 \end{bmatrix} - \begin{bmatrix} -3.4 \\ 13 \\ 11.2 \end{bmatrix}$$

$$r = \begin{bmatrix} 0.4 \\ 2 \\ -2.2 \end{bmatrix}$$

which has Euclidean norm

$$\|e\|_2 = \sqrt{(0.4)^2 + (2)^2 + (-2.2)^2} = 3$$

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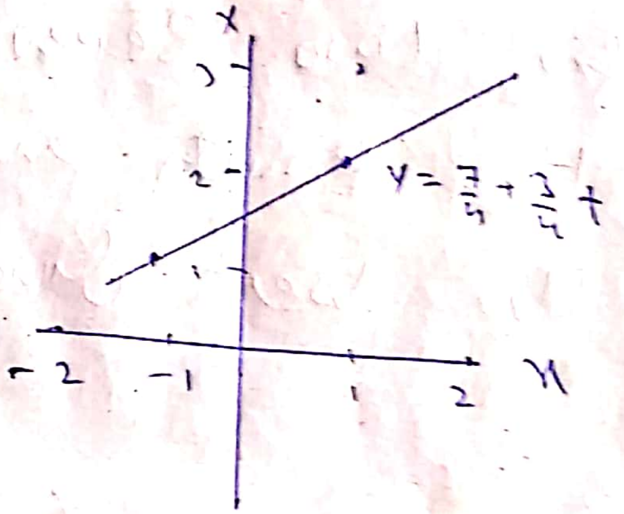
Q3 (b) Find the line that best fits the three data points (t, y) = (1, 2) (−1, 1) and (1, 3) in figure



Sol

The model is $y = c_1 + c_2 t$ and the goal is to find best c_1 and c_2. Substitution of the data points into model yields.

$$c_1 + c_2(1) = 2$$

$$c_1 + c_2(-1) = 1$$

$$c_1 + c_2(1) = 3$$

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or in matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

The system has no solution for two seperate reason.

the points are not collinear.

The best line is $y = 7/4 + 3/4 t$.

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End of Paper