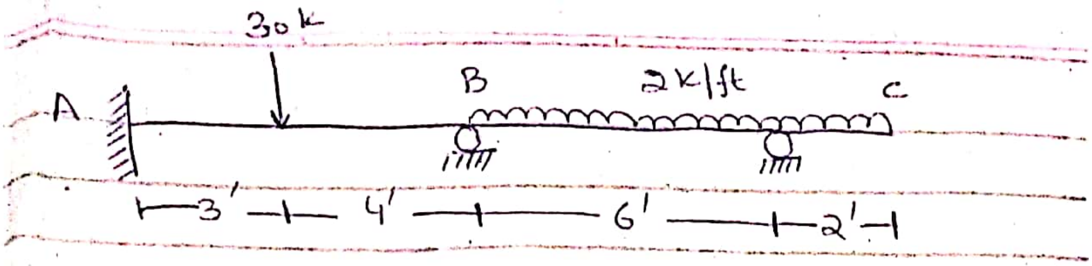


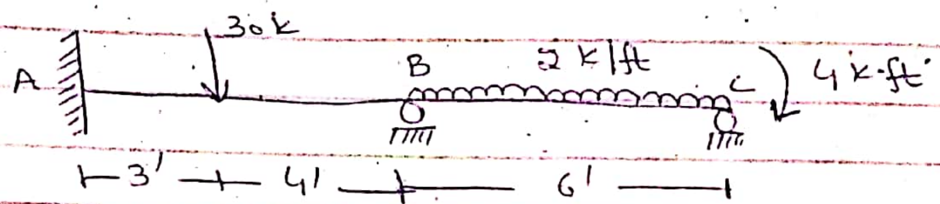

Sol:

## Step #1

Determining Kinematic Indeterminancy,

K.I = 5°

So we have to reduce the extended portion.



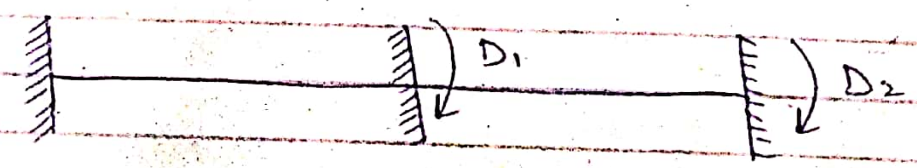

$$\Rightarrow \frac{2(2)}{1} = 4 \text{ k-ft}$$

Now :-

K.I = 2°

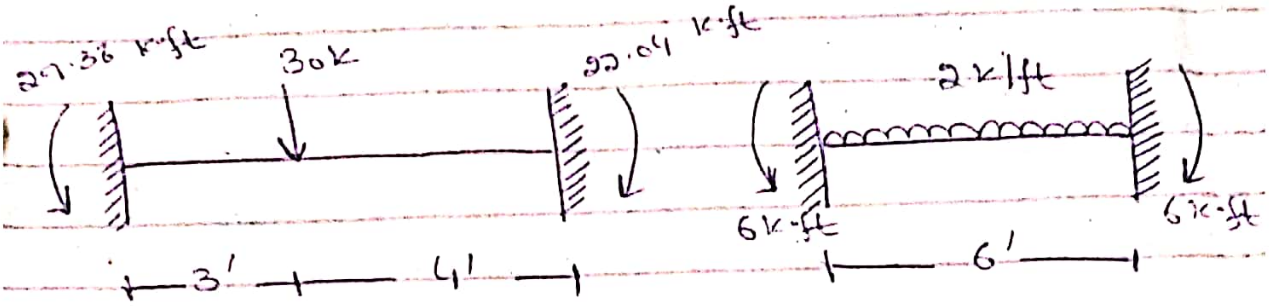
## Step #2

Determine Unknown Joint Displacement




$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \qquad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step #3 :-
Compute [ADL] Matrix.




=> For Pointed Load (not at mid).

=> For left end :-

$$= \frac{Pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ k-ft}$$

=> For Right end :-

$$= \frac{Pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k-ft}$$

=> For UDL :

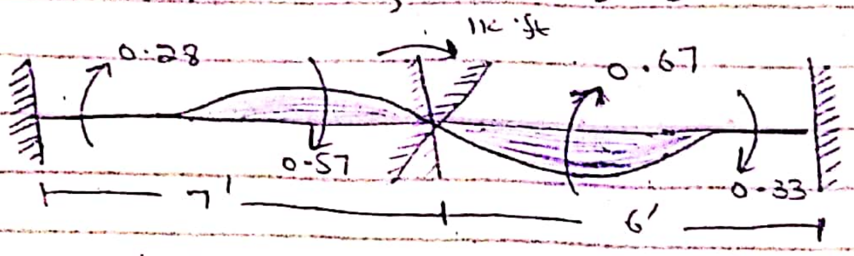
$$\frac{wL^2}{12} \rightarrow \frac{(2)(6)^2}{12} = 6 \text{ k-ft}$$

$ADL_1 = +22.04 - 6 = 16.04$ k-ft
$ADL_2 = 6$ k-ft

Step # 4 :-
Compute [S] Matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a) $D_1 = 1k$, $D_2 = 0$




$\frac{4EI}{7} = 0.57$

$\frac{4EI}{6} = 0.67$

$\frac{2EI}{6} = 0.33$

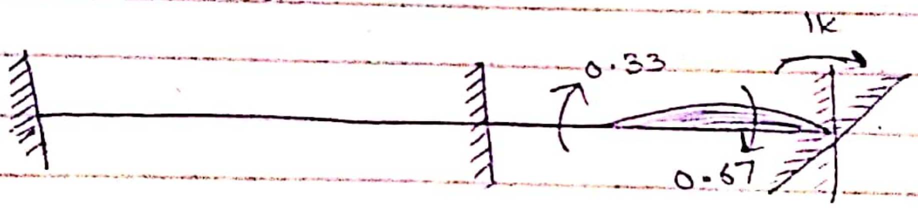
$\frac{2EI}{7} = 0.28$

$S_{11} = 0.57 + 0.67$

$= 1.24 \; EA$

$S_{21} = 0.33 \; EA$

b) $D_1 = 0$, $D_2 = 1k$.




$\frac{4EI}{6} = 0.67$

$\frac{2EI}{6} = 0.33$

$S_{12} = 0.33$

$S_{22} = 0.67$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step #5

Compute [D] matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ -0.33 & 0.67 \end{bmatrix}} \times \text{Adj } A \times \begin{bmatrix} \quad \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$
$$= 0.8308 - 0.1089$$
$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

Now:

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \frac{\begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}}{0.7219} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix}$$



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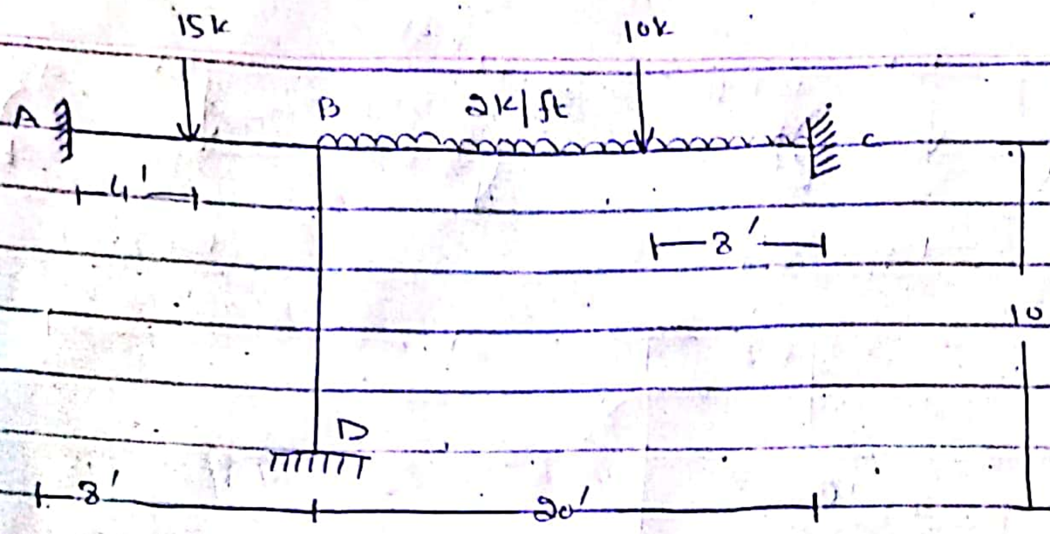
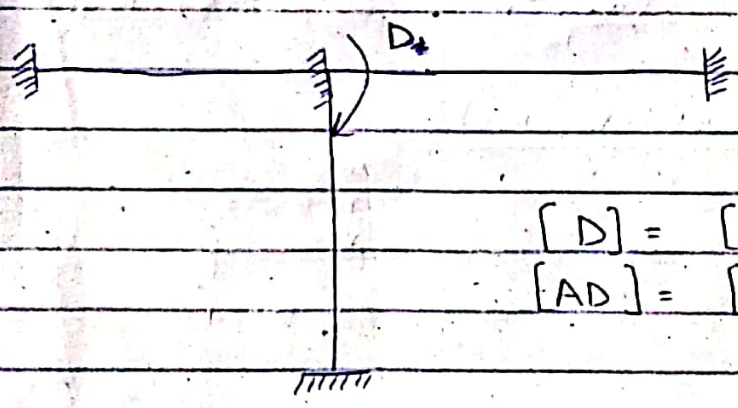



Sol.

Step #1.-
Determine Kinematic Indeterminancy.

K.I = 1°

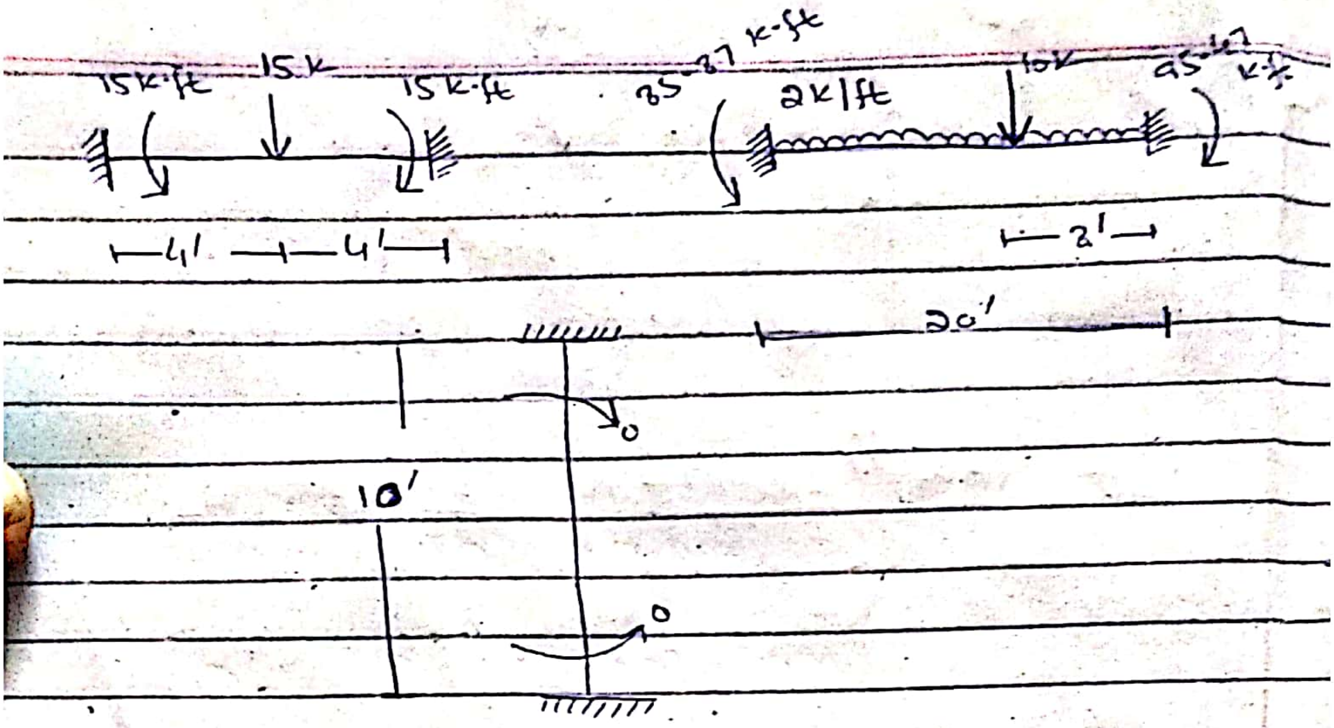
Step #2 :-
Determine Unknown Joint Displacement.




$[D] = [?]$

$[AD] = [0]$

Step #3 :-
Compute [ADL] Matrix.




=> Point Load at Center :-

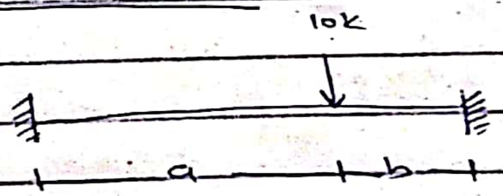
$$\frac{PL}{8} \Rightarrow \frac{(15)(8)}{8} = 15 \text{ kip-ft}$$

=> Uniformly Distributed Load :-

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k-ft}$$

=> Point Load (Not at mid) :-

Suppose :-




For Left End :-

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k-ft}$$

For ~~Left~~ Right End :-

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k-ft}$$

So Total Moment at left end is

$19.2 + 66.67 = 85.87$ k.ft

Similarly at right End :-

$28.8 + 66.67 = 95.47$ k.ft

So $[AD_L] = -85.87 + 15 = -70.87$ k.ft

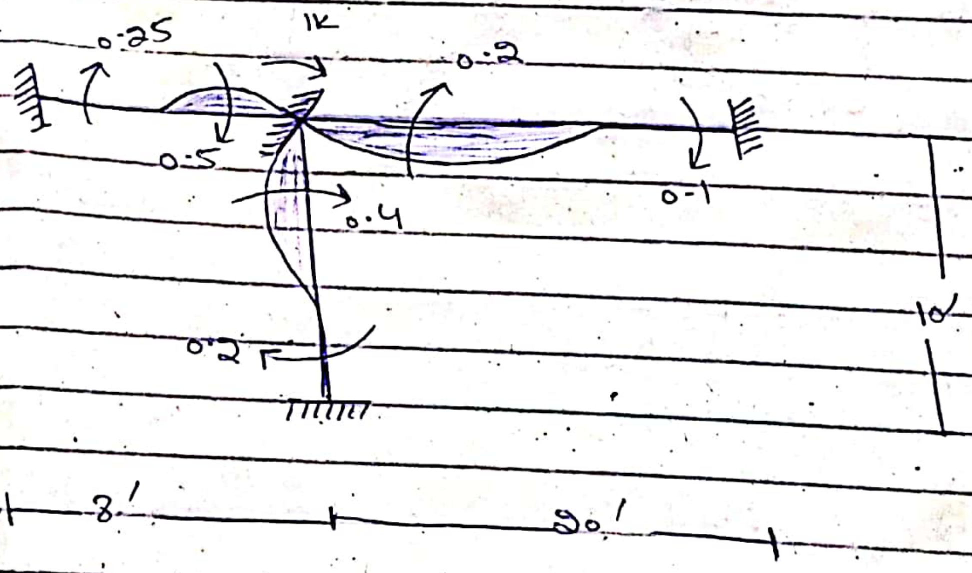
Step #4 :-

Determine [S] Matrix

$[S] = [S_{11}]$

Now,

D = 1k




$\Rightarrow \dfrac{4EI}{8} = 0.5 \qquad \dfrac{2EI}{8} = 0.25$

$\Rightarrow \dfrac{4EI}{20} = 0.2 \qquad \dfrac{2EI}{20} = 0.1$

$\Rightarrow \dfrac{4EI}{10} = 0.4 \qquad \dfrac{2EI}{10} = 0.2$



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$[S] = (0.5 + 0.4 + 0.2)\ EI$

$= 1.1\ EI$

$[S] = 1.1\ EI$

Step #5:-

Compute [D] Matrix

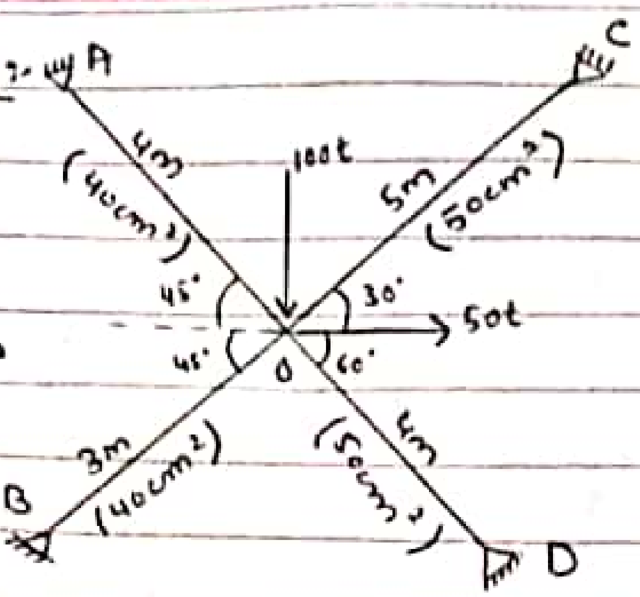
$[D] = [S]^{-1} \times [AD] - [ADL]$

$[D] = \frac{1}{1.1} \times [0] - [-70.87]$

$= \frac{70.87}{1.1}$

$[D] = \boxed{64.42}\ 1/EI$

**Problem #03:-**




$E = 2000\ t/cm^2$

**Sol:** **For A:**

$$\sin 45° = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828\ m$$

$$\cos 45° = \frac{b}{4}$$

$$\Rightarrow b = 2.828\ m$$

**For B:-**

$$\sin 45° = \frac{P}{3}$$

$$\Rightarrow P = 2.12\ m$$

$$\cos 45° = \frac{b}{h}$$

$$\Rightarrow b = 2.12\ m$$

**For C:-**

$$\sin 30° = \frac{P}{h=5}$$

$$\Rightarrow P \Rightarrow 2.5\ m$$

9

$\cos 30^\circ = \frac{b}{5}$

$\Rightarrow \quad b = 4.33m$

Now $EA_{(A)} = 2000 \times 40 = 80{,}000\ t$

$EA_{(B)} = 2000 \times 40 = 80{,}000\ t$

$EA_{(C)} = 2000 \times 50 = 100{,}000\ t$

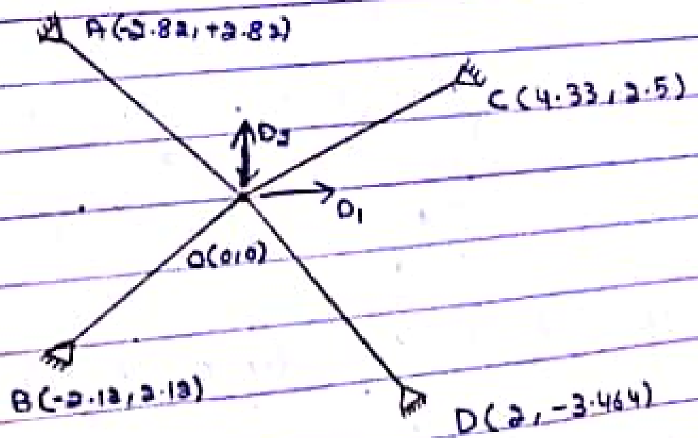
$EA_{(D)} = 2000 \times 50 = 100{,}000\ t$

Step#01:- K.I

$K.I = 2j - r$

$= 2(5) - 8 = 2^\circ$

Step#02: Select unknown joint displacement




$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \quad \begin{bmatrix} AD_1 \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$

Step#03: $[AMD]_{4\times2}$ & $[S]_{2\times2}$

i) $D_1 = 1$ , $D_2 = 0$

$$AMD = \frac{EA}{L^2}(X_k - X_j)$$

$$AMD_{11} = \frac{80{,}000}{(400)^2} \times (0+282) = 141$$

$$AMD_{21} = \frac{80{,}000}{(300)^2} \times (0+212) = 188.44$$

$$AMD_{31} = \frac{100{,}000}{(500)^2} \times (0-433) = -173.2$$

$$AMD_{41} = \frac{100{,}000}{(400)^2} \times (0-200) = -125$$

Now $S_{11} = \sum_{i=1}^{m} \frac{EA}{L^3}(X_k - X_j)^2$

$$= \frac{80{,}000 \times (282)^2}{400^3} + \frac{80{,}000 \times (212)^2}{(300)^3} + \frac{100{,}000 \times (-433)^2}{(500)^3}$$

$$+ \frac{100{,}000 \times (-200)^2}{(400)^3}$$

$$S_{11} = 99.405 + 133.167 + 149.991 + 62.5$$

$$\boxed{S_{11} = 445.063}$$

(11)

$$S_{12} = S_{21} = \sum_{i=1}^{m} \frac{EA}{L^3} \times (X_k - X_j)(Y_k - Y_j)$$

$$= \frac{80{,}000}{(400)^3} \times (282)(-282) + \frac{80{,}000}{(300)^3} \times (212)(212)$$

$$+ \frac{100{,}000}{(500)^3} \times (-433)(0-250) + \frac{100{,}000}{(400)^3} \times (-200)(0+346)$$

$$\boxed{S_{12} = S_{21} = 12.237}$$

ii) $D_1 = 0$ , $D_1 = 1K'$

$$AMD = \frac{EA}{L^2}(Y_k - Y_j)$$

$$AMD_{12} = \frac{80{,}000}{400^2}(-282) = -141$$

$$AMD_{22} = \frac{80{,}000}{300^2}(212) = 188.44$$

$$AMD_{32} = \frac{100{,}000}{500^2}(-250) = -100$$

$$AMD_{42} = \frac{100{,}000}{400^2}(346) = 216.25$$

Now, $S_{22} = \sum_{i=1}^{m} \frac{EA}{L^3}(Y_k - Y_j)^2$

$$= \frac{80{,}000}{400^3}(-282)^2 + \frac{80{,}000}{300^3}(212)^2 + \frac{100{,}000}{500^3}(-250)^2$$

$$+ \frac{100{,}000}{400^3}(346)^2$$

$$\boxed{S_{22} = 469.628}$$

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Step #04:

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step#06:- $[AM]$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$= \begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$